Prime Factorization and Exponents Lesson #1:
Prime Factors

Overview

In this unit we look at prime factorization of whole numbers and applications including greatest common factor, least common multiple, square root, and cube root. We review powers with whole number exponents where the base is numerical. We also extend these concepts to powers where the base is variable and where the exponents are integers using inquiry, analyzation, and reasoning.

Factors

The whole number 6 is exactly divisible by the whole numbers 1, 2, 3, and 6.

The numbers 1, 2, 3, and 6 are the **factors** of the whole number 6.

The number 6 has four factors.

In each case, state the number of factors of the given whole number.

a) 1 **b)** 2 **c)** 3 **d)** 4 **e)** 12

Factor Pairs

We say **2** and **3** are a **factor pair** of **6** because $2 \times 3 = 6$.
A **factor pair** is a set of two whole numbers which when multiplied result in a specific product.

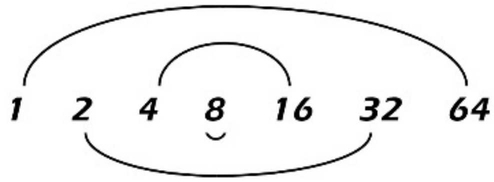
i) State another factor pair of 6.

We can use a diagram listing all the factors of a number to help determine all possible factor pairs of that number.

The diagram opposite shows there are four factor pairs of 64.

8 and 8 are a factor pair because $8 \times 8 = 64$.

ii) List the other factor pairs.



List the factors of the following and determine all the factor pairs for each.

a) 15 **b)** 16

Prime and Composite Numbers

A **prime number** is defined as a whole number which has exactly two distinct factors. The two factors are always 1 and the number itself, e.g. 3, 7.

A **composite number** is a whole number which has more than two factors, e.g. 10, 18.

The number 1 has only one factor and is neither prime nor composite.

In this course the number 0 is defined to have no factors.

- Complete the list of the first ten prime numbers:

2, 3, 5, __ , __ , __ , __ , __ , __ , __ , ...

Classify the following whole numbers as prime or composite.

a) 46 **b)** 37 **c)** 39 **d)** 101 **e)** 103

Prime Factors

The **prime factors** of a whole number are the factors of the number which are prime.

For example The factors of 6 are 1, 2, 3, and 6.

The prime factors of 6 are 2 and 3.

a) State the factors of 12.

b) State the prime factors of 12.

c) Express 12 as a **product of prime factors**.

Complete Assignment Questions #1 - #9

Prime Factorization

Every composite number can be expressed as a product of prime factors. Expressing a whole number as a product of prime factors is called the **prime factorization** of the number.

The prime factorization of small numbers like 12 can probably be done mentally. For larger numbers, a division table or a tree diagram can be used.

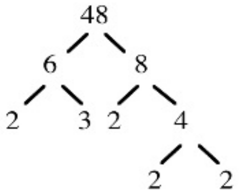
The diagrams below illustrate these techniques for the prime factorization of 48.

- Prime numbers must be used as divisors when using a division table.
- Any divisor may be used when using a tree diagram.

division table

2	48
2	24
2	12
2	6
3	3
	1

tree diagram



48 = 2 × 2 × 2 × 2 × 3

or using powers

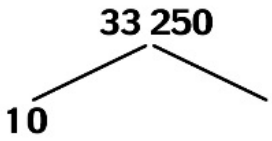
$48 = 2^4 \times 3$

Two students, Tantoo and Craig, work together to confirm that the prime factorization of 33 250 is $2 \times 5^3 \times 7 \times 19$. Tanto uses a division table and Craig uses a factor tree. Complete their work to verify that the prime factorization of 33 250 is $2 \times 5^3 \times 7 \times 19$.

Tantoo
Division Table

2	33 250

Craig
Tree Diagram



Complete Assignment Questions #10 - #17

Assignment

1. In each case, determine the number of factors of the given whole number.

a) 8 **b)** 11 **c)** 17 **d)** 33 **e)** 45

2. List the factors of the following numbers and determine all the factor pairs.

a) 21 **b)** 22

c) 25 **d)** 36

3. In some of the parts of question #2, the number of factor pairs is exactly one half of the number of factors and in other parts it is not. Explain.

4. State the numbers in question #1 which are

a) prime **b)** composite

5. Classify the following whole numbers as prime or composite.

a) 30 **b)** 41 **c)** 43 **d)** 57

e) 59 **f)** 121 **g)** 133 **h)** 169

6. Twin primes are defined to be consecutive odd numbers that are both prime (e.g. 5 and 7). List the seven other twin primes less than 80.

7. **a)** State the factors of 20.

b) State the prime factors of 20.

c) Express 20 as a product of prime factors.

8. State the prime factors of

a) 15

b) 24

c) 45

d) 66

9. Explain why the numbers 0 and 1 have no prime factors.

10. Use a division table to express the prime factorization of the following using powers where appropriate.

a) 140

b) 330

c) 1 911

d) 1 925

11. Use a tree diagram to determine the prime factorization of the following:

a) 390 **b)** 546

c) 3 705 **d)** 6 762

12. In each case write the number as a product of prime factors.

a) 189 **b)** 685

c) 4 235 **d)** 7 980

Multiple Choice

13. Which of the following numbers is **not** a prime factor of 1 925?

A. 5
B. 7
C. 11
D. 13

14. How many of the numbers in the list 2, 3, 9, 13 are **not** prime factors of 2 592?

A. 4
B. 3
C. 2
D. 1

Numerical Response

15. The sum of all of the prime factors of 373 065 is _______.

(Record your answer in the numerical response box from left to right)

16. There is only one set of prime triplets: three consecutive odd numbers which are all prime. If the prime triplets are a, b, and c, then the value of abc is ______.

(Record your answer in the numerical response box from left to right)

17. The number 686 can be expressed as a product of prime factors in the form $p \times q^r$. The value of $p + q + r$ is ______.

(Record your answer in the numerical response box from left to right)

Answer Key

1.a) 4 **b)** 2 **c)** 2 **d)** 4 **e)** 6

2.a) factors: 1, 3, 7, 21
factor pairs: 1 and 21, 3 and 7

b) factors: 1, 2, 11, 22
factor pairs: 1 and 22, 2 and 11

c) factors: 1, 5, 25
factor pairs: 1 and 25, 5 and 5

d) factors: 1, 2, 3, 4, 6, 9, 12, 18, 36
factor pairs: 1 and 36, 2 and 18, 3 and 12, 4 and 9, 6 and 6

3.
- If the whole number is not a perfect square, then each factor has another factor which forms a factor pair thus the number of factor pairs is exactly one half the number of factors.
- If the whole number is a perfect square, then the square root of the number is a factor which does not have a different factor to form a factor pair. Therefore the number of factors will be an odd number and the number of factor pairs will not be half that number.

4. a) 11, 17 **b)** 8, 33, 45

5.a) composite **b)** prime **c)** prime **d)** composite **e)** prime **f)** composite **g)** composite **h)** composite

6. 3 and 5, 11 and 13, 17 and 19, 29 and 31, 41 and 43, 59 and 61, 71 and 73.

7.a) 1, 2, 4, 5, 10, 20 **b)** 2, 5 **c)** $2 \times 2 \times 5$ **8. a)** 3, 5 **b)** 2, 3 **c)** 3, 5 **d)** 2, 3, 11

9. 0 is defined to have no factors and 1 has only one factor. Since a prime number has exactly two distinct factors neither 0 or 1 can be a prime number.

10.a) $2^2 \times 5 \times 7$ **b)** $2 \times 3 \times 5 \times 11$ **c)** $3 \times 7^2 \times 13$ **d)** $5^2 \times 7 \times 11$

11.a) $2 \times 3 \times 5 \times 13$ **b)** $2 \times 3 \times 7 \times 13$ **c)** $3 \times 5 \times 13 \times 19$ **d)** $2 \times 3 \times 7^2 \times 23$

12.a) $3^3 \times 7$ **b)** 5×137 **c)** $5 \times 7 \times 11^2$ **d)** $2^2 \times 3 \times 5 \times 7 \times 19$

13. D **14.** C

15.

6	2		

16.

1	0	5	

17.

1	2		

Prime Factorization and Exponents Lesson #2:
Applications of Prime Factors

Review

Express the numbers 48 and 72 as products of prime factors.

Greatest Common Factor

The **greatest common factor** (GCF) of a set of whole numbers is the largest whole number which divides exactly into each of the members of the set.

For example, the GCF of 8, 16, and 20 is ____.

State the greatest common factor of

a) 15, 25, and 35 **b)** 18 and 20 **c)** 36 and 54

In the example above parts a) and b) were fairly simple to do, but part c) was more complicated because each number had a large number of factors.

In cases like this we can use prime factorization to determine the GCF.

From the review $48 = 2 \times 2 \times 2 \times 2 \times 3$ and $72 = 2 \times 2 \times 2 \times 3 \times 3$.

To determine the GCF of 48 and 72 we find the product of each prime factor (including repeats) which is common to both prime factorizations.

GCF of 48 and 72 is $2 \times 2 \times 2 \times 3 =$ _____.

Use prime factorization to determine the greatest common factor of the given whole numbers.

a) 90 and 225 **b)** 154 and 198

Use prime factorization to determine the GCF of 245, 315, and 770.

Lowest Common Multiple

Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, ...

Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, ...

Common multiples of 6 and 8 are 24, 48, ...

The **lowest common multiple** (LCM) of 6 and 8 is ____.

Determine the lowest common multiple of the following:

a) 5 and 7 **b)** 10, 15, and 20 **c)** 10, 12, and 14

In the example above, parts a) and b) were fairly simple to do, but part c) was more complicated. Prime factors can be used to simplify the solution.

$10 = \mathbf{2} \times \mathbf{5}$

$12 = 2 \times \mathbf{2} \times \mathbf{3}$

$14 = 2 \times \mathbf{7}$

To determine the LCM, take all the prime factors of one of the numbers and multiply by any additional factors in the other numbers.

Take **2** and **5** from 10, another **2** and **3** from 12, and **7** from 14.

$\mathbf{2} \times \mathbf{5} \times \mathbf{2} \times \mathbf{3} \times \mathbf{7} =$ _______, the LCM of 10, 12, and 14.

Class Ex. #5

Use prime factorization to determine the LCM of

a) 15 and 35 **b)** 126 and 441 **c)** 22, 154, and 198

Complete Assignment Questions #1 - #7

Prime Factorization of a Perfect Square

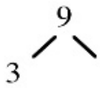
Perfect squares of whole numbers include 1, 4, 9, 16, 25, 36, 49, etc.

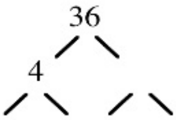
Every perfect square has two **square roots**: one positive and one negative. The square root which is positive is called the **principal square root**.

The principal square root of each number above is 1, 2, 3, 4, 5, 6, 7, etc.

In this lesson, where we are dealing only with whole numbers, we will use the term square root to mean the principal square root.

Complete the prime factorization of the following perfect squares: 9, 36, 49, and 64.





$9 = 3 \times 3$ The square root of 9 is

$36 = 2 \times 2 \times \quad \times$ The square root of 36 is $2 \times 3 =$

$49 =$ The square root of 49 is

$64 =$ The square root of 64 is $2 \times \quad \times \qquad =$

The prime factorization of a perfect square will involve sets of factors which each occur two times (or a multiple of two times).

If the prime factorization of a number does not result in sets of factors which each occur two times (or a multiple of two times), then we can say that the number is **not** a perfect square.

Class Ex. #6

Consider the number 44 100.

a) Use a calculator to find the square root of 44 100.

b) **Explain** how we can use the prime factorization of 44 100 to show that 44 100 is a perfect square. Verify your calculator answer by this method.

Class Ex. #7

In each case use prime factorization to determine if the number is a perfect square. If the number is a perfect square, state the square root of the number.

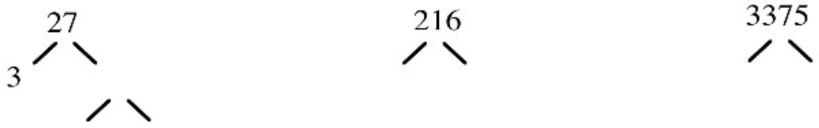
a) 1 225

b) 3 042

Prime Factorization of a Perfect Cube

Perfect cubes of whole numbers include 1, 8, 27, 64, 125, etc.
The **cube root** of each number above is 1, 2, 3, 4, 5, etc.

Complete the prime factorization of the following perfect cubes: 27, 216, and 3 375.



The prime factorization of a perfect cube will involve sets of factors which each occur three times (or a multiple of three times).

If the prime factorization of a number does not result in factors which each occur three times (or a multiple of three times), then we can say that the number is not a perfect cube.

Class Ex. #8

In each case use prime factorization (division table) to determine whether the number is a perfect cube. If the number is a perfect cube, state the cube root of the number.

a) 6 912

b) 970 299

Complete Assignment Questions #8 - #15

Assignment

1. State the greatest common factor of

a) 14 and 21 **b)** 30 and 40 **c)** 12, 30, and 54

2. Use prime factorization to determine the GCF of

a) 150 and 420 **b)** 126 and 189 **c)** 294 and 385

3. Use prime factorization to determine the greatest common factor of

a) 483 and 575 **b)** 180 and 504 **c)** 1 700 and 1 938

d) 663 and 910 **e)** 84 and 231 **f)** 525 and 850

4. Determine the GCF of 66, 495, and 2 541

5. State the lowest common multiple of

a) 4 and 6 **b)** 3 and 9 **c)** 9 and 15 **d)** 40, 60, and 100

6. Use prime factorization to determine the LCM of

a) 14 and 30 **b)** 28 and 60 **c)** 10 and 115

d) 18 and 63 **e)** 55 and 143 **f)** 125 and 175

7. Determine the LCM of

a) 72 and 252

b) 6, 10, and 42

c) 12, 30, and 105

8. In each case use prime factorization to determine whether the number is a perfect square.
If the number is a perfect square, state the square root of the number.
Verify your answer by using a calculator to determine the square root.

a) 216 **b)** 11 025 **c)** 882 **d)** 1 225

9. Consider the number 74 088.

a) Use a calculator to find the cube root of 74 088.

b) **Explain** how we can use the prime factorization of 74 088 to show that 74 088 is a perfect cube. Verify your calculator answer by this method.

10. In each case use prime factorization to determine whether the number is a perfect cube. Verify your answer by using a calculator to determine the cube root.

a) 216 **b)** 11 025

11. **Explain** how you could use prime factorization to determine if a particular whole number is both a perfect square and a perfect cube.

Multiple Choice

12. The greatest common factor of 399 and 462 is

A. 3
B. 7
C. 19
D. 21

13. The greatest common factor of two whole numbers x and y is 10. Which of the statements A, B, C, or D below is false?

A. x and y must be even numbers.
B. The product xy must be divisible by 100.
C. x and y are both divisible by 5.
D. Neither x nor y can be a prime number.
Answer **E** if none of the statements is false.

Numerical Response

14. The lowest common multiple of 35, 231, and 275 is _______.
(Record your answer in the numerical response box from left to right)

15. A new children's encyclopedia has 950 pages. Each page contains two background colours for illustrations. Page 8 and every 8th page thereafter has green as one of the background colours. Page 18 and every 18th page thereafter has orange as one of the background colours.
How many pages in the book have both green and orange as background colours?
(Record your answer in the numerical response box from left to right)

Answer Key

1a) 7 **b)** 10 **c)** 6 **2a)** 30 **b)** 63 **c)** 7
3a) 23 **b)** 36 **c)** 34 **d)** 13 **e)** 21 **f)** 25
4. 33 **5a)** 12 **b)** 9 **c)** 45 **d)** 600
6a) 210 **b)** 420 **c)** 230 **d)** 126 **e)** 715 **f)** 875
7a) 504 **b)** 210 **c)** 420 **8a)** no **b)** yes, 105 **c)** no **d)** yes, 35
9a) 42 **b)** If the prime factorization of 74 088 has factors which each appear three times, then 74 088 is a perfect cube.
10a) yes, 6 **b)** no
11. If the prime factorization of the number has factors which each appear six times, then the number will be both a perfect square and a perfect cube.
12. D **13.** E **14.**

5	7	7	5

15.

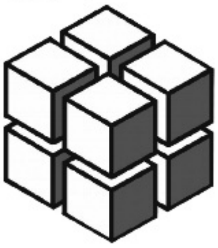
1	3		

Prime Factorization and Exponents Lesson #3:
Powers with Whole Number Exponents

In this lesson we review numbers written as powers, and the exponent laws applied to powers with numerical bases and whole number exponents.
We extend the work to consider bases which are variable.

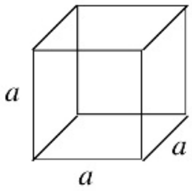
Exponents

In mathematics, exponents are used as a short way to write repeated multiplication.



The number of small cubes in the diagram can be calculated by the **repeated multiplication** $2 \times 2 \times 2$.
This can be written in **exponential form** as 2^3.

Exponents can also be used with variables.

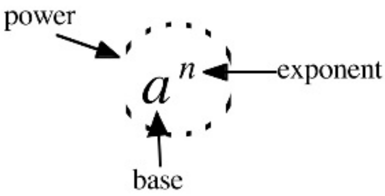


The volume of the cube to the left can be determined by repeated multiplication $a \times a \times a$ or in exponential form a^3.

Powers

A **power** is a number written in exponential form.

It consists of a **base** and an **exponent**.



State the base and the exponent in each of the following powers.

a) 4^5 **b)** $(-3)^6$ **c)** x^y

A number that multiplies a variable is called a **coefficient**.
In the expression $7p^3$ the coefficient is 7.

State the coefficient in each of the of the following.

a) $8x^2$ **b)** $-3z^9$ **c)** $\frac{a^8}{7}$

Note that written as a repeated multiplication $7p^3 = 7 \times p \times p \times p$,
whereas $(7p)^3 = 7p \times 7p \times 7p = 7 \times p \times 7 \times p \times 7 \times p = 7 \times 7 \times 7 \times p \times p \times p = 343p^3$.

Write each of the following as a repeated multiplication.

a) $3a^4b$ **c)** $3(ab)^4$

b) $3ab^4$ **d)** $(3ab)^4$

Evaluating Powers

$10^3 = 10 \times 10 \times 10 =$ $3^4 = 3 \times 3 \times 3 \times 3 =$

$(-6)^2 = (-6) \times (-6) =$ $-6^2 = -(6 \times 6) =$

The Zero Exponent

Complete the patterns below by adding one more row.

$10^4 = 10000$ $3^4 = 81$

$10^3 = 1000$ $3^3 = 27$

$10^2 = 100$ $3^2 = 9$

$10^1 = 10$ $3^1 = 3$

$10^0 =$

The results above are examples of a general rule when a base is raised to the exponent zero.
Complete: $a^0 =$ ____ .

Evaluate the following. **a)** 6^0 **b)** $(-9)^0$ **c)** -9^0 **d)** $2(6^2)^0$

Complete Assignment Questions #1 - #7

The Exponent Laws

The exponent laws with whole number exponents and numerical bases were covered in previous math courses.

The chart below extends the exponent laws to bases which are variables.

Complete the table as a review of the exponent laws.

Numerical Bases	Variable Bases	Exponent Laws
$8^3 \times 8^2 = (8 \cdot 8 \cdot 8)(8 \cdot 8)$ $= 8^5 \quad \text{or} \quad 8^{3+2}$	$a^3 \times a^2 = (a \cdot a \cdot a)(a \cdot a)$ $= a \quad \text{or} \quad a^{\ \ +\ \ }$	**Product Law** $(a^m)(a^n) =$
$8^3 \div 8^2 = \frac{8 \cdot 8 \cdot 8}{8 \cdot 8}$ $= 8^1 \quad \text{or} \quad 8^{\ \ -\ \ }$	$a^3 \div a^2 = \frac{a \cdot a \cdot a}{a \cdot a}$ $= a \quad \text{or} \quad a^{\ \ -\ \ }$	**Quotient Law** $a^m \div a^n = \frac{a^m}{a^n} =$ $(a \neq 0)$
$(8 \cdot 7)^3 = (8 \cdot 7)(8 \cdot 7)(8 \cdot 7)$ $= (8 \cdot 8 \cdot 8)(\qquad\qquad)$ $= 8^3 \cdot 7^3$	$(a \cdot b)^3 = (a \cdot b)(a \cdot b)(a \cdot b)$ $= (a \cdot a \cdot a)(\qquad\qquad)$ $= a \ \ b$	**Power of a Product Law** $(ab)^m =$
$\left(\frac{8}{7}\right)^3 = \left(\frac{\ }{\ }\right)\left(\frac{\ }{\ }\right)\left(\frac{\ }{\ }\right)$ $= \frac{8^3}{7^3}$	$\left(\frac{a}{b}\right)^3 = \left(\frac{\ }{\ }\right)\left(\frac{\ }{\ }\right)\left(\frac{\ }{\ }\right)$ $= \frac{a}{b}$	**Power of a Quotient Law** $\left(\frac{a}{b}\right)^n =$ $(b \neq 0)$
$(8^3)^2 = (8^3)(8^3)$ $= (\qquad\qquad)(\qquad\qquad)$ $= 8^6 \quad \text{or} \quad 8^{\ \ \times\ \ }$	$(a^3)^2 = (a^3)(a^3)$ $= (\qquad\qquad)(\qquad\qquad)$ $= a \quad \text{or} \quad a^{\ \ \times\ \ }$	**Power of a Power Law** $(a^m)^n =$

Use the exponent laws to simplify and then evaluate.

a) $3^4 \cdot 3^2$ **b)** $\dfrac{(-2)^5}{(-2)^3}$ **c)** $(5^2)^3$

Use the exponent laws to simplify.

a) $(a)^4(a)^3$ **b)** x^6x **c)** $b^7 \times b^0 \times b^4$ **d)** $\dfrac{x^8}{x^4}$ **e)** $\dfrac{y^{10}}{y^2}$ **f)** $(a^4)^3$

g) $(y^5)^5$ **h)** $\left(\dfrac{x}{y}\right)^9$ **i)** $\left(\dfrac{c}{4}\right)^2$ **j)** $(st)^6$ **k)** $(2a)^5$ **l)** $(-3pq)^4$

Complete Assignment Questions #8 - #22

Assignment

1. State the base and the exponent in each of the following powers.

a) 8^3 **b)** k^{15} **c)** 2^x **d)** $(-x)^4$ **e)** $\left(\dfrac{3}{4}\right)^6$

2. State the coefficient in each of the following.

a) $5x^7$ **b)** $-6z^2$ **c)** a^3 **d)** $\dfrac{y^3}{4}$ **e)** $\dfrac{5y^9}{8}$

3. Write each of the following as a repeated multiplication.

a) c^4 **b)** $5x^3$ **c)** $(ab)^2$ **d)** $(-5)^3$ **e)** s^2t

f) $2\left(\dfrac{5}{4}\right)^3$ **g)** $(4a)^3$ **h)** $3cd^2$ **i)** $3(cd)^2$ **j)** $(3cd)^2$

4. Evaluate.

a) 3^8 **b)** -5^2 **c)** $(-5)^2$

d) $(-5)^3$ **e)** -5^3 **f)** $\left(\frac{3}{5}\right)^3$

5. Evaluate without using a calculator.

a) -10^2 **b)** $(-10)^2$ **c)** -10^3 **d)** $(-10)^3$

6. Explain why -8^0 and $(-8)^0$ have different values.

7. Evaluate without using a calculator.

a) 32^0 **b)** -1^0 **c)** $\left(-\frac{1}{2}\right)^0$ **d)** $\frac{1}{2}(4)^0$ **e)** $\frac{1}{2}(4^2)^0$

8. Write in a simpler form and evaluate.

a) $9^3 \cdot 9^6$ **b)** $(7^2)^3$ **c)** $\frac{8^{15}}{8^{13}}$ **d)** $\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)^3$

e) $\frac{1.5^7}{1.5^5}$ **f)** $(-3^3)^2$ **g)** $(-5)^6 \times (-5)^2$ **h)** $4^3 \cdot 4^4 \cdot 4^2$

9. Explain using factors why $(x^2)(x^3) \neq (x^2)^3$.

10. Use the Product Law to simplify.

a) $a^4 \times a^2$ **b)** $m^6 \cdot m^3$ **c)** $(s^5)(s^5)$ **d)** x^6x^5 **e)** $y^{10} \times y^2$

11. Use the Quotient Law to simplify.

a) $\frac{t^8}{t^2}$ **b)** $x^6 \div x^4$ **c)** $\frac{p^{10}}{p^9}$ **d)** $d^{18} \div d^9$ **e)** $p^8 \div p$

12. Use the Power of a Product Law to simplify.

a) $(xy)^5$ **b)** $(mn)^4$ **c)** $(3x)^3$ **d)** $(10z)^3$ **e)** $\left(\frac{1}{2}c\right)^2$

f) $(2b)^4$ **g)** $(-x)^3$ **h)** $(-3y)^4$ **i)** $(-4pq)^2$ **j)** $(-4pq)^3$

13. Use the Power of a Quotient Law to simplify.

a) $\left(\frac{x}{y}\right)^2$ **b)** $\left(\frac{a}{b}\right)^6$ **c)** $\left(\frac{5}{c}\right)^4$ **d)** $\left(\frac{b}{5}\right)^3$ **e)** $\left(\frac{z}{y}\right)^{10}$

14. Use the Power of a Power law to simplify.

a) $(p^2)^2$ **b)** $(h^4)^5$ **c)** $(b^4)^3$ **d)** $(s^9)^{10}$ **e)** $(z^7)^3$

15. State the value of x in each of the following.

a) $(a^3)\,(a^x) = a^9$ **b)** $b^x \cdot b^4 = b^8$ **c)** $c^x \div c^4 = c^{12}$

d) $\frac{d^{10}}{d^x} = d^2$ **e)** $(e^x)^5 = e^{15}$ **f)** $(f^7)^x = f^7$

16. Use the exponent laws to simplify.

a) $\frac{x^{12}}{x^3}$ **b)** $(xy)^7$ **c)** $(t^3)^3$ **d)** t^3t^3

e) $y^4 \times y^8$ **f)** $\left(\frac{a}{b}\right)^{11}$ **g)** $\left(\frac{d}{2}\right)^3$ **h)** $(2st)^6$

17. Simplify.

a) $g^{12}g^3$ **b)** $\frac{a^7}{a^5}$ **c)** $(3bc)^4$ **d)** $\left(\frac{5}{y}\right)^2$

e) $(-a)^4$ **f)** $\left(-\frac{1}{3}pq\right)^4$ **g)** $(a^3)(a^4)(a^5)$ **h)** $\frac{x^6}{x^0}$

18. Simplify.

a) $\frac{y^7}{y^7}$ **b)** $(-ab)^5$ **c)** $(m^6)(m^6)$ **d)** $(r^0)^3$

e) $c^3c^4c^5c^6$ **f)** $(-ab)^6$ **g)** $\frac{1}{a^3a^5}$ **h)** $2(xy)^3$

19. After marking an exponents quiz a teacher recorded the most common errors made by students. In each case, identify the error made by the students and, where possible, provide the correct simplification.

a) $2^3 \times 2^4 = 2^{12}$ **b)** $(4^3)^2 = 4^9$

c) $3^4 \times 3^5 = 9^9$ **d)** $3^2 \times 2^3 = 6^5$

e) $(-5a^2b)^3 = -5a^6b^3$ **f)** $\left(\frac{1}{2}pq\right)\left(\frac{1}{2}pq\right) = p^2q^2$

Matching **20.** Match each item in column 1 on the left with the equivalent item in column 2 on the right. Each item in column 2 may be used once, more than once, or not at all.

Column 1

i) $(-a^2)^3$
ii) $(-a^3)^2$
iii) $a^3 \times a^2$
iv) $a^8 \div a^2$
v) $a^{30} \div a^6$
vi) $-(a^2)^3$
vii) $-(a^3)^2$

Column 2

A. a^4
B. a^5
C. a^6
D. a^{24}
E. $-a^5$
F. $-a^6$

Use the following information to answer the next question.

$(2^3)^p = 2^{12}$

$\dfrac{4^{10}}{4^q} = 4^2$

$2^r \cdot 2^r = 2^{16}$

$(3^s)^2 = 1$

Numerical Response **21.** Write the value of p in the first box.
Write the value of q in the second box.
Write the value of r in the third box.
Write the value of s in the fourth box.

(Record your answer in the numerical response box from left to right)

22. If $(a^n)\,(a^n)\,(a^n) = a^{27}$, where n is a whole number, then the value of n is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) base 8, exponent 3 **b)** base k, exponent 15 **c)** base 2, exponent x
d) base $-x$, exponent 4 **e)** base $\frac{3}{4}$, exponent 6

2. a) 5 **b)** -6 **c)** 1 **d)** $\frac{1}{4}$ **e)** $\frac{5}{8}$

3. a) $c \times c \times c \times c$ **b)** $5 \times x \times x \times x$ **c)** $a \times a \times b \times b$ **d)** $(-5) \times (-5) \times (-5)$
e) $s \times s \times t$ **f)** $2 \times \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4}$ **g)** $4 \times 4 \times 4 \times a \times a \times a$
h) $3 \times c \times d \times d$ **i)** $3 \times c \times c \times d \times d$ **j)** $3 \times 3 \times c \times c \times d \times d$

4. a) 6561 **b)** -25 **c)** 25 **d)** -125 **e)** -125 **f)** $\frac{27}{125}$
5. a) -100 **b)** 100 **c)** -1000 **d)** -1000

6. $-8^0 = -1$ since the exponent applies only to the base 8.
$(-8)^0 = 1$ since the exponent applies to the base -8.

7. a) 1 **b)** -1 **c)** 1 **d)** $\frac{1}{2}$ **e)** $\frac{1}{2}$

8. a) $9^9 = 387\ 420\ 489$ **b)** $7^6 = 117\ 649$ **c)** $8^2 = 64$ **d)** $\left(\frac{2}{3}\right)^4 = \frac{16}{81}$
e) $1.5^2 = 2.25$ **f)** $3^6 = 729$ **g)** $(-5)^8 = 390\ 625$ **h)** $4^9 = 262\ 144$

9. $(x^2)\,(x^3) = (x \times x)\,(x \times x \times x) = (x \times x \times x \times x \times x) = x^5$
$(x^2)^3 = x^2 \times x^2 \times x^2 = x \times x \times x \times x \times x \times x = x^6$
Therefore $(x^2)\,(x^3) \neq (x^2)^3$.

10. a) a^6 **b)** m^9 **c)** s^{10} **d)** x^{11} **e)** y^{12}

11. a) t^6 **b)** x^2 **c)** p **d)** d^9 **e)** p^7
12. a) x^5y^5 **b)** m^4n^4 **c)** $27x^3$ **d)** $1000z^3$ **e)** $\frac{1}{4}c^2$
f) $16b^4$ **g)** $-x^3$ **h)** $81y^4$ **i)** $16p^2q^2$ **j)** $-64p^3q^3$

13. a) $\frac{x^2}{y^2}$ **b)** $\frac{a^6}{b^6}$ **c)** $\frac{625}{c^4}$ **d)** $\frac{b^3}{125}$ **e)** $\frac{z^{10}}{y^{10}}$

14. a) p^4 **b)** h^{20} **c)** b^{12} **d)** s^{90} **e)** z^{21}

15. a) 6 **b)** 4 **c)** 16 **d)** 8 **e)** 3 **f)** 1

16. a) x^9 **b)** x^7y^7 **c)** t^9 **d)** t^6 **e)** y^{12} **f)** $\frac{a^{11}}{b^{11}}$ **g)** $\frac{d^3}{8}$ **h)** $64s^6t^6$
17. a) g^{15} **b)** a^2 **c)** $81b^4c^4$ **d)** $\frac{25}{y^2}$ **e)** a^4 **f)** $\frac{1}{81}p^4q^4$ **g)** a^{12} **h)** x^6
18. a) $y^0 = 1$ **b)** $-a^5b^5$ **c)** m^{12} **d)** $r^0 = 1$ **e)** c^{18} **f)** a^6b^6 **g)** $\frac{1}{a^8}$ **h)** $2x^3y^3$

19. **a)** The student multiplied 3 and 4, instead of adding 3 and 4. The correct simplification is $2^{3+4} = 2^7$.

b) The student squared 3 instead of multiplying 3 and 2. The correct simplification r is $4^{3 \times 2} = 4^6$.

c) The student multiplied the bases together. The correct simplification is 3^9.

d) The student multiplied the bases together. The exponent laws are only valid when the bases are the same. No simplification.

e) The student did not cube the 5. The correct simplification is $-125a^6b^3$.

f) The student added $\frac{1}{2}$ and $\frac{1}{2}$ instead of multiplying $\frac{1}{2}$ and $\frac{1}{2}$. The correct simplification is $\frac{1}{4}p^2q^2$.

20. **i)** F **ii)** C **iii)** B **iv)** C **v)** D **vi)** F **vii)** F

21.

4	8	8	0

22.

9			

Prime Factorization and Exponents Lesson #4:
Combining the Exponent Laws

Using Factors To Combine the Exponent Laws

Part One:

Three students are attempting to simplify the following expression:

$$3x^2 \times 5x^3$$

Their answers are shown below.

Harry $\Rightarrow 8x^5$ Janet $\Rightarrow 15x^6$ Laura $\Rightarrow 15x^5$

Explain using factors which student is correct.

Part Two:

Use factors to explain why $6a^6 \div 3a^2 = 2a^4$.

State the simplified form of the following.

a) $(-7a^8)(6a^{12})$ **b)** $3a^4 \times a^5 \times 6a^3$ **c)** $-16n^5 \div (-2n)$

d) $\dfrac{20y^5}{5y^5}$ **e)** $\dfrac{30b^{14}}{45b^{10}}$

Simplify the following.

a) $x^5y^8x^3y^4$ **b)** $\dfrac{x^5y^8}{x^3y}$ **c)** $(-3bc)(b^3c^2)(-4b^2c)$ **d)** $\dfrac{10e^8f^{12}}{4e^4f^7}$

Complete Assignment Questions #1 - #4

Combining the Exponent Laws

The following examples use two or more of the exponent laws in their solution.

Class Ex. #3

Simplify.

a) $(3x^2)^3$ **b)** $(-2a^2b^3)^2$ **c)** $\dfrac{x^3x^5}{x^2x}$ **d)** $\left(-\dfrac{2a}{y^3}\right)^3$

Class Ex. #4

Simplify the following.

a) $-(-n^2)^5$ **b)** $\left(\dfrac{4y^3 \times 3x^6}{6x^5}\right)^4$ **c)** $\dfrac{16(x^3y^5)^2}{(2x^2)^3}$ **d)** $(5ab^6)^2\,(4a^2b)$

Class Ex. #5

Write in simplest form.

a) $(-a)^6 \div (-a)^4$ **b)** $-a^6 \div (-a)^4$ **c)** $-a^7 \div (-a)^3$

Complete Assignment Questions #5 - #12

Extension

In higher level mathematics courses, you may meet variable bases and variable exponents including binomial exponents.

Use the exponent laws to simplify the following.

a) $\dfrac{b^{4x+y}}{b^{x-2y}}$ **b)** $\dfrac{x^{5a+7b} \cdot x^{3a+b}}{x^{a} \cdot x^{2a-7b}}$

Complete Assignment Question #13

Assignment

1. Simplify the following.

a) $3a^3 \times 3a^4$ **b)** $(10b^7)\,(3b^8)$ **c)** $3a^3 \cdot 5a^3$

d) $(-2x^4)\,(12x^9)$ **e)** $\left(-\dfrac{1}{2}e^7\right)\,(-14e^8)$ **f)** $0.4c^3 \times 0.5c$

2. Simplify.

a) $12x^4 \div 6x^2$ **b)** $(81e^9) \div (9e^8)$ **c)** $\dfrac{21d^6}{7d^2}$

d) $\dfrac{-80d^{80}}{8d^8}$ **e)** $(-10e^{10}) \div (-5e^5)$ **f)** $\dfrac{12f^6}{12f^5}$

3. Write in simplest form.

a) $(3a^2b^3)(5a^4b^8)$ **b)** $x^9y^0x^2y^4$ **c)** $\dfrac{6x^4y^7}{2x^3y^2}$

d) $\dfrac{5x^4y^7}{x^3y^2}$ **e)** $\dfrac{4f^{12}d^3}{12f^4d}$ **f)** $(7b^4c)(bc^2)(-2b^2c^6)$

4. Simplify.

a) $\dfrac{10e^8f^8}{15\,e^4f^2}$ **b)** $(2p^3)(4p^7)(-2p)$ **c)** $(-2xy)(x^2y^3)(-3xy)$

d) $(-8b^6c) \div (2b^3c)$ **e)** $(-10t^8y^6) \div (-2t^7y^3)$ **f)** $(4x^5z^7) \div (-16xz^6)$

5. Write in simplest form.

a) $(-a^2b^3)^4$ **b)** $(-a^2b^3)^5$ **c)** $\left(\dfrac{b^4}{a^3}\right)^3$ **d)** $\dfrac{c^5 \times c^2}{c^4 \times c}$

6. Simplify.

a) $(3ab^2)^4$ **b)** $(-4a^5c^2)^4$ **c)** $(-2m^3n^4)^5(m^2n^3)$

d) $(-4x^2y^3)^3(8xy^8)$ **e)** $(a^3b^4c^5)(3abc^2)^3$

7. Write each expression in simplest form without brackets.

a) $\left(\dfrac{2d^5 \times d^4}{4d^3}\right)^3$ **b)** $\left(\dfrac{-16a^5b^3 \cdot 2a^2b^6}{8ab^7}\right)^3$ **c)** $\left(\dfrac{-5k^3 \cdot k^2}{k}\right)^2\left(\dfrac{(-k)^5 \cdot k^2}{5k^2}\right)$

8. Write in a simpler form and evaluate.

a) $\dfrac{6^6 \times 6}{6^4}$ **b)** $(-3^3)^2$ **c)** $\left(\dfrac{2^{10}}{2^5}\right)^3$ **d)** $\dfrac{(0.7)^8}{(0.7)^4 \times (0.7)^2}$

e) $-5^6 \times 5^2$ **f)** $(-5)^6 \times (-5)^2$ **g)** $-10^{10} \div (-10)^8$ **h)** $\dfrac{-10^{10}}{-10^8}$

9. Write each expression in simplest form without brackets.

a) $(-x)^{12} \div (-x)^6$ **b)** $(-a)^6 \div (-a^4)$ **c)** $-p^{10} \div (-p)^2$

d) $c^5 \div (-c)^2$ **e)** $-(-t)^4 \div (-t)^3$ **f)** $-(-t^4) \div (-(t)^3)$

Multiple Choice

10. The simplified form of $\dfrac{1}{36}(2x^3)^2(-3yx^2)$ is

A. x^8y^2

B. $-\dfrac{1}{3}x^8y$

C. $-\dfrac{1}{3}x^7y$

D. $-\dfrac{1}{6}x^6y$

11. The expression $\dfrac{6(x^3y^5)^2}{(3xy)^4}$ is equivalent to the expression

A. $\dfrac{4}{9}x^2y^6$

B. $2x^5y^6$

C. $2x^2y^6$

D. $\dfrac{2}{27}x^2y^6$

Numerical Response **12.** If the expression $\dfrac{4x^{-4}}{8x^{-3}}$ is written in the form ax^b, then the value of $a - b$ is ______ .

(Record your answer in the numerical response box from left to right) $\boxed{}\boxed{}\boxed{}\boxed{}$

Extension **13.** Simplify each expression.

a) $a^{x+y}a^{2x+3y}$ **b)** $\dfrac{m^{x+9}}{m^3}$

c) $\dfrac{a^{3m+2}}{a^{m-3}}$ **d)** $\dfrac{x^{2y+7} \cdot x^{3y+2}}{x^{y+8}}$

Answer Key

1. a) $9a^7$ **b)** $30b^{15}$ **c)** $15a^6$ **d)** $-24x^{13}$ **e)** $7e^{15}$ **f)** $0.2c^4$

2. a) $2x^2$ **b)** $9e$ **c)** $3d^4$ **d)** $-10d^{72}$ **e)** $2e^5$ **f)** f

3. a) $15a^6b^{11}$ **b)** $x^{11}y^4$ **c)** $3xy^5$ **d)** $5xy^5$ **e)** $\frac{1}{3}f^8d^2$ **f)** $-14b^7c^9$

4. a) $\frac{2}{3}e^4f^6$ **b)** $-16p^{11}$ **c)** $6x^4y^5$ **d)** $-4b^3$ **e)** $5ty^3$ **f)** $-\frac{1}{4}x^4z$

5. a) a^8b^{12} **b)** $-a^{10}b^{15}$ **c)** $\dfrac{b^{12}}{a^9}$ **d)** c^2

6. a) $81a^4b^8$ **b)** $256a^{20}c^8$ **c)** $-32m^{17}n^{23}$ **d)** $-512x^7y^{17}$ **e)** $27a^6b^7c^{11}$

7. a) $\dfrac{d^{18}}{8}$ **b)** $-64a^{18}b^6$ **c)** $-5k^{13}$

8. a) $6^3 = 216$ **b)** $3^6 = 729$ **c)** $2^{15} = 32768$ **d)** $(0.7)^2 = 0.49$
e) $-5^8 = -390625$ **f)** $(-5)^8 = 390625$ **g)** $-10^2 = -100$ **h)** $10^2 = 100$

9. a) x^6 **b)** $-a^2$ **c)** $-p^8$ **d)** c^3 **e)** t **f)** $-t$

10. B **11.** D **12.** $\boxed{1}\boxed{.}\boxed{5}\boxed{}$

13. a) a^{3x+4y} **b)** m^{x+6} **c)** a^{2m+5} **d)** x^{4y+1}

Prime Factorization and Exponents Lesson #5:
Integral Exponents

The Negative Exponent

a) Complete the patterns below.

$10^3 = 1000$ $\quad$ $3^3 = 27$

$10^2 = 100$ $\quad$ $3^2 = 9$

$10^1 = 10$ $\quad$ $3^1 = 3$

$10^0 =$ $\quad$ $3^0 =$ $\quad$ $a^0 =$

$10^{-1} = \frac{1}{10} = \frac{1}{10^1}$ $\quad$ $3^{-1} =$ $\quad$ $a^{-1} =$

$10^{-2} =$ $\quad$ $3^{-2} =$ $\quad$ $a^{-2} =$

$10^{-3} =$ $\quad$ $3^{-3} =$ $\quad$ $a^{-3} =$

b) Write the following with positive exponents.
i) 10^{-7} **ii)** 3^{-5} **iii)** a^{-n}

Using the Exponent Laws to Define the Negative Exponent

Consider the expression $5^4 \div 5^7$.

a) Evaluate the expression as an exact value using a calculator.

b) Complete the following to evaluate the expression.

$$5^4 \div 5^7 = \frac{5 \cdot 5 \cdot 5 \cdot 5}{} = \frac{1}{5^{\square}} =$$

c) Use the quotient law to complete the following.

$$5^4 \div 5^7 = 5^{\square - \square} = 5^{\square}$$

d) The results in a) to c) are examples of a general rule when a base is raised to a negative exponent. Complete: $a^{-p} = \frac{1}{}$

e) Write the following with positive exponents and evaluate.
i) 2^{-1} **ii)** 3^{-2} **iii)** 4^{-3}

The Negative Exponent in the Denominator

Use the rule for division of fractions to show that $\frac{1}{4^{-3}} = 4^3$. Use a calculator to confirm.

Negative Exponent Law

A base (not including zero) raised to a negative exponent has the following properties:

$$a^{-n} = \frac{1}{a^n},\ a \neq 0 \qquad \text{and} \qquad \frac{1}{a^{-n}} = a^n,\ a \neq 0$$

Simplify, express with positive exponents, and evaluate without using a calculator.

a) $4^5 \times 4^{-3}$ **b)** $3^2 \times 3^{-5}$ **c)** $\frac{1}{2^{-5}}$ **d)** $\frac{6^{-7}}{6^{-5}}$ **e)** $(2^3)^{-1}$

Identify the following as true or false.

a) $\frac{8^3}{8^{-1}} = 8^4$ **b)** $\frac{8^3}{4^{-1}} = 2^4$ **c)** $a^{-3} = \frac{1}{a^3}$

Explain why $2p^{-3} \neq \frac{1}{2p^3}$.

a) $a^{-4} \times a^{-3}$

b) $6x^2 \div 2x^7$

c) $\dfrac{y^6}{2y^{-5}}$

d) $(-2x)^{-3}$

e) $\dfrac{8a^{-5}}{4b^{-3}}$

f) $\dfrac{(5p)^{-2}}{5q^4}$

Simplifying a Fractional Base with a Negative Exponent

Consider the expression $\left(\dfrac{2}{3}\right)^{-4}$.

a) Complete the following $\left(\dfrac{2}{3}\right)^{-4} = \dfrac{1}{\left(\dfrac{2}{3}\right)^{\square}} = \dfrac{1}{\ } = 1 \times \qquad =$

b) Evaluate $\left(\dfrac{3}{2}\right)^{4}$.

c) Classify the following statement as true or false. $\left(\dfrac{2}{3}\right)^{-4} = \left(\dfrac{3}{2}\right)^{4}$

d) Suggest a quick method for evaluating $\left(\dfrac{5}{2}\right)^{-3}$ without using a calculator.

In general, $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^{n} \quad a, b \neq 0.$

Complete Assignment Questions #1 - #15

Assignment

1. Write the following with positive exponents.

a) x^{-3} **b)** y^{-9} **c)** 4^{-1} **d)** $\dfrac{1}{a^{-5}}$ **e)** $\dfrac{1}{6^{-2}}$

2. Without using a calculator show that $\dfrac{3}{5^{-2}} = 75$.

3. Simplify, express with positive exponents, and evaluate without using a calculator.

a) $4^3 \times 4^{-4}$ **b)** $3^0 \times 3^{-3}$ **c)** $\dfrac{1}{7^{-2}}$ **d)** $\dfrac{10^{-3}}{10}$ **e)** $(3^2)^{-2}$

4. Express with positive exponents.

a) n^2m^{-5} **b)** $c^{-2}x^{-5}$ **c)** $16h^{-1}$ **d)** $\dfrac{2}{3}b^{-8}$ **e)** $(y^{-4})^{-2}$

f) $\dfrac{t^{-5}}{4}$ **g)** $\dfrac{1}{4x^{-9}}$ **h)** $\dfrac{4}{x^{-9}}$ **i)** $\dfrac{a^2}{b^{-7}}$ **j)** $\dfrac{a^{-2}}{b^7}$

5. Evaluate the following without using a calculator.

a) -3^{-2} **b)** $(-3)^{-2}$ **c)** $-7^2 \cdot 8^{-2}$ **d)** $(-8.3)^0$ **e)** $[-(3.9)^0]^{-2}$

6. Use a calculator to find the exact value of the following.

a) -4^{-4} **b)** $(-7)^{-3}$ **c)** $(0.75)^{-3}$ **d)** $(-0.025)^{-2}$ **e)** $\left(\dfrac{4}{7}\right)^{-3}$

7. State whether the following are true or false.

a) $6x^{-3} = \dfrac{6}{x^3}$ **b)** $5a^{-4} = \dfrac{1}{5a^4}$ **c)** $\dfrac{4}{b^{-6}} = 4b^6$ **d)** $\dfrac{x^{-3}}{2} = \dfrac{2}{x^3}$

e) $\dfrac{1}{5y^{-1}} = 5y$ **f)** $\dfrac{1}{\frac{1}{4}p} = \dfrac{1}{4}p^{-1}$ **g)** $(3x)^5 = \dfrac{1}{(3x)^{-5}}$ **h)** $\dfrac{1}{\left(\frac{1}{7}a\right)^{-2}} = 49a^2$

8. Simplify and write the answer with positive exponents.

a) $x^{10} \cdot x^{-5}$ **b)** $m^5 \div m^8$ **c)** $b^{-1} \cdot b^{-3}$ **d)** $-w^0 \div w^5$

9. Simplify and write the answer with positive exponents.

a) $a^8 \times a^{-10}$ **b)** $10x^2 \div 2x^{-1}$ **c)** $\dfrac{6y^{-6}}{2y^{-4}}$ **d)** $\dfrac{2a^{-5}}{4b^6}$

e) $-7x^{-2}$ **f)** $-(7x)^{-2}$ **g)** $(-7x)^{-2}$ **h)** $\dfrac{(-7x)^{-2}}{-7x^{-2}}$

10. Simplify each expression, writing the answer with positive exponents.

a) $a^{-3}a^{-3}$ **b)** $(5b^8b^{-12})(-10b^3b^{-12})$ **c)** $(-7x^3x^{-5})(x^2x^{-3})$

d) $(-2a^3)^{-3} \cdot 3a^{12}$ **e)** $\dfrac{16a^6b^{-3}}{-4a^6b^3}$ **f)** $(-3a^5b^{-3}c^0)^{-2}$

11. Simplify. Write the final answer with positive exponents.

a) $\dfrac{32a^2b^{-4}}{4a^{-8}b^{-2}} \times \dfrac{-8a^{-2}}{-3b^{-3}}$

b) $\dfrac{10(p^3q^2r^0)^{-3}}{(8p^{-3}q^5r^3)^{-2}}$

c) $(-2x^5y^3z^8)^{-2}(-2x^2y^{-8}z^{12})^3$

d) $(5a^3b^2)(-2a^{-2}b)^{-3} \div (-5a^8b^{-9})^{-2}$

12. Evaluate the following without using a calculator.

a) $\left(\dfrac{2}{3}\right)^{-3}$ **b)** $\left(\dfrac{1}{5}\right)^{-2}$ **c)** $\left(\dfrac{8}{5}\right)^{-1}$ **d)** $\left(\dfrac{3}{2}\right)^{-4}$

13. Simplify. Write the final answers with positive exponents.

a) $\left(\dfrac{c}{d}\right)^{-3}$ **b)** $\left(\dfrac{x}{4}\right)^{-3}$ **c)** $\left(\dfrac{p^2}{r^4}\right)^{-3}$ **d)** $\left(\dfrac{a^{-2}}{b^{-5}}\right)^{-3}$

e) $\left(\dfrac{-12x^{-3}}{6y^{-8}}\right)^{-1}$

f) $\left(\dfrac{12x^3y^{-1}}{-8x^{-1}y^5}\right)^{-2}$

Multiple Choice

14. The value of $\dfrac{1^{-3} + 3^0}{2^{-1}}$ is

A. 1
B. 4
C. 8
D. 12

15. Which of the following statements are true?

i) $3a^{-3} = \dfrac{1}{3a^3}$ ii) $8x^4 \div 4x^7 = \dfrac{1}{2x^3}$ iii) $\dfrac{1}{2a} = 2a^{-1}$

A. i) only
B. ii) only
C. iii) only
D. none of the statements are true

Answer Key

1. **a)** $\frac{1}{x^3}$ **b)** $\frac{1}{y^9}$ **c)** $\frac{1}{4}$ **d)** a^5 **e)** 6^2

2. $\frac{3}{5^{-2}} = 3 \times 5^2 = 3 \times 25 = 75$

3. **a)** $\frac{1}{4^1} = \frac{1}{4}$ **b)** $\frac{1}{3^3} = \frac{1}{27}$ **c)** $7^2 = 49$ **d)** $\frac{1}{10^4} = \frac{1}{10\,000}$ **e)** $\frac{1}{3^4} = \frac{1}{81}$

4. **a)** $\frac{n^2}{m^5}$ **b)** $\frac{1}{c^2x^5}$ **c)** $\frac{16}{h}$ **d)** $\frac{2}{3b^8}$ **e)** y^8

f) $\frac{1}{4t^5}$ **g)** $\frac{x^9}{4}$ **h)** $4x^9$ **i)** a^2b^7 **j)** $\frac{1}{a^2b^7}$

5. **a)** $-\frac{1}{9}$ **b)** $\frac{1}{9}$ **c)** $-\frac{49}{64}$ **d)** 1 **e)** 1

6. **a)** $-\frac{1}{256}$ **b)** $-\frac{1}{343}$ **c)** $\frac{64}{27}$ **d)** 1600 **e)** $\frac{343}{64}$

7. **a)** T **b)** F **c)** T **d)** F **e)** F **f)** F **g)** T **h)** F

8. **a)** x^5 **b)** $\frac{1}{m^3}$ **c)** $\frac{1}{b^4}$ **d)** $-\frac{1}{w^5}$

9. a) $\frac{1}{a^2}$ **b)** $5x^3$ **c)** $\frac{3}{y^2}$ **d)** $\frac{1}{2a^5b^6}$

e) $-\frac{7}{x^2}$ **f)** $-\frac{1}{49x^2}$ **g)** $\frac{1}{49x^2}$ **h)** $-\frac{1}{343}$

10. a) $\frac{1}{a^6}$ **b)** $-\frac{50}{b^{13}}$ **c)** $-\frac{7}{x^3}$ **d)** $-\frac{3}{8}a^3$ **e)** $-\frac{4}{b^6}$ **f)** $\frac{b^6}{9a^{10}}$

11. a) $\frac{64}{3}a^8b$ **b)** $\frac{640q^4r^6}{p^{15}}$ **c)** $-\frac{2z^{20}}{x^4y^{30}}$ **d)** $-\frac{125a^{25}}{8b^{19}}$

12. a) $\frac{27}{8}$ **b)** 25 **c)** $\frac{5}{8}$ **d)** $\frac{16}{81}$

13. a) $\frac{d^3}{c^3}$ **b)** $\frac{64}{x^3}$ **c)** $\frac{r^{12}}{p^6}$ **d)** $\frac{a^6}{b^{15}}$ **e)** $-\frac{x^3}{2y^8}$ **f)** $\frac{4y^{12}}{9x^8}$

14. B **15.** D

Prime Factorization and Exponents Lesson #6:
Scientific Notation

Scientific notation is included as an application of powers with integral exponents.

Scientific Notation - Large Numbers

The 2016 census shows Canada has over 260 000 First Peoples' languages speakers.
The distance from Earth to the sun is approximately 93 000 000 miles.
The number of molecules of a gas per m^3 is approximately 27 000 000 000 000 000 000.

All these large numbers are written in standard decimal notation and it would be easy when writing out these numbers to make a mistake with the number of zeros.

Scientific Notation is a way of writing numbers that are too large or too small to be conveniently written in standard decimal notation. Scientists, engineers and mathematicians, often use scientific notation to represent numbers and use the properties of exponents to simplify calculations.

In scientific notation all numbers are written in the form $\boldsymbol{a \times 10^n}$,
where the **exponent**, n, is an integer and the **coefficient**, a, is greater than or equal to 1 but less than 10. i.e. $1 \le a < 10$.

The number 32 000 in standard decimal notation can be written in product form as $3.2 \times 10 \times 10 \times 10 \times 10$ or in scientific notation as 3.2×10^4.

Complete the following table.

Standard Notation	Expanded Form	Scientific Notation
61 500		
500 000		
46.3		
	$2.81 \times 10 \times 10$	
	$7 \times 10 \times 10 \times 10 \times 10 \times 10$	
		9.2×10^2
		1.4×10^6

Complete the following statements from the top of the page using scientific notation.

a) The 2016 census shows Canada has over _________ First Peoples' languages speakers.

b) The distance from Earth to the sun is approximately _______________ miles.

c) The number of molecules of a gas per m^3 is approximately ________________.

Class Ex. #3

Convert the following numbers from scientific notation to standard notation.

a) 3.8×10^7 **b)** 2.51×10^{12} **c)** 2.9×10^3

Class Ex. #4

Simplify and write in scientific notation.

a) $7.1 \times 10^2 \times 1000$ **b)** $6.98 \times 10^7 \times 10$ **c)** $\frac{5 \times 10^6}{1000}$

Complete Assignment Questions #1 - #5

Scientific Notation - Small Numbers

An inch is about 0.000 025 4 km.
The mass of a particle of dust is about 0.000 000 000 75 kg.
The mass of an electron is about 0.000 000 000 000 000 000 000 000 000 9 g.

All these small numbers are written in standard decimal notation, and again it would be easy when writing out these numbers to make a mistake with the number of zeros.

The number 0. 000 63 in standard decimal notation can be written in the form

$$\frac{6.3}{10 \times 10 \times 10 \times 10} \text{ or } \frac{6.3}{10^4} \text{ which is equivalent to } 6.3 \times 10^{-4}.$$

Class Ex. #5

Complete the following table.

Standard Notation	Expanded Form	Scientific Notation
0.000 53		
0.000 000 07		
	$\frac{2.9}{10 \times 10 \times 10}$	
	$\frac{7.1}{10 \times 10 \times 10 \times 10 \times 10 \times 10}$	
		3.1×10^{-3}
		8.5×10^{-5}

Complete the following statements from the beginning of this section using scientific notation.

a) An inch is about ___________ kilometres.

b) The mass of a particle of dust is about _____________ kg.

c) The mass of an electron is about ____________ g.

a) Explain why 0.4×10^{-8} is not written in scientific notation.

b) Write 0.4×10^{-8} in scientific notation.

Complete Assignment Questions #6 - #10

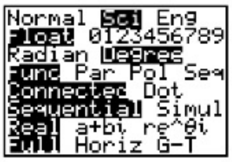
Scientific Notation on a Calculator

Using the Sci Mode

The `Sci` mode of a graphing calculator may be used to express numbers in scientific notation.

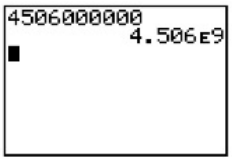
For example, use the following procedure to convert the approximate average distance that Neptune is from the sun (4 506 000 000 km):

1. Press `MODE`, scroll right to "`Sci`", and press `ENTER`.



```
Normal Sci Eng
Float 0123456789
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi re^θi
Full Horiz G-T
```

2. Then "`QUIT`" by pressing `2nd` `MODE`.

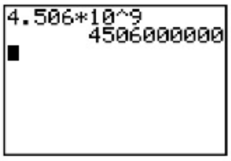


```
4506000000
          4.506E9
```

3. Enter the number 4 506 000 000 and press `ENTER`.

The number `4.506E9` is to be written 4.506×10^9 as an answer.

To convert back to standard notation, set the graphing calculator back to "`Normal`" mode, input the number 4.506×10^9, and press `ENTER`.



```
4.506*10^9
       4506000000
```

Using the EE feature

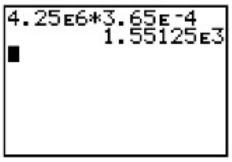
The `EE` feature represents "$\times 10^n$" on the calculator.
This feature may be useful in saving time.

Example: Use the following procedure to write the product $(4.25 \times 10^6) \times (3.65 \times 10^{-4})$ in scientific notation and in standard notation.

1. Place the calculator into scientific mode (`Sci`).

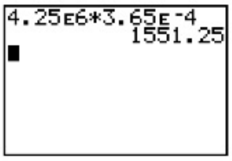
2. • Access the `EE` feature by pressing `2nd` `,` (the display will only show one `E`).

• Press 4.25 `2nd` `,` 6 `×` 3.65 `2nd` `,` `(-)` 4 `ENTER` . The answer 1.55125×10^3 is displayed.



```
4.25E6*3.65E-4
        1.55125E3
▮
```

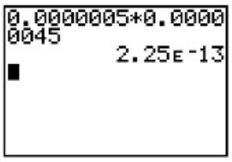
3. To give the answer in standard notation, place the calculator in normal mode (`Normal`) and repeat steps 1 and 2 above.



```
4.25E6*3.65E-4
          1551.25
▮
```

Note

• Sometimes a calculation is so large or small that the calculator automatically displays the answer in scientific notation, regardless of the mode it is in. To convert back to standard notation you will have to convert back manually.



```
0.0000005*0.0000
0045
         2.25E-13
▮
```

Class Ex. #8

Use a calculator to:

a) Write the product $(3.5 \times 10^5) \times (2.6 \times 10^9)$ in scientific notation.

b) Write the quotient $\dfrac{7.5 \times 10^{-17}}{5 \times 10^{-9}}$ in standard notation.

Complete Assignment Questions #11 - #16

Assignment

1. Complete the following table.

Standard Notation	Expanded Form	Scientific Notation
151 000		
23.4		
32 000		
	$8.3 \times 10 \times 10 \times 10 \times 10 \times 10$	
	$7.3 \times 10 \times 10$	
		8.4×10^2
		6.2×10^3

2. Express each number in scientific notation.

a) 2 300

b) 7 580 000

c) 41 000 000 000

d) 53.1

e) 4 320 000 000 000

f) 7.6

3. Express the number of km in scientific notation.

a) Mercury has an approximate average distance of 57.9 million km from the sun.

b) Saturn has an approximate average distance of 1.4 billion km from the sun.

4. Express each number in standard notation.

a) 1.8×10^{12}

b) 6.73×10^5

c) 9.99×10^7

d) Mars has an approximate average distance of 2.2794×10^8 km from the sun.

e) The Treaty 11 Land Agreement signed between First Nations and the Canadian Government covered approximately 9.5×10^5 km^2 of land.

5. Simplify and write in scientific notation.

a) $5.7 \times 10^3 \times 10\ 000$ **b)** $9.843 \times 10^8 \times 10$

c) $\dfrac{6.1 \times 10^8}{10\ 000}$ **d)** $\dfrac{20\ 000}{4 \times 10^2}$

6. Complete the following table.

Standard Notation	Expanded Form	Scientific Notation
0.000 000 9		
0.000 001		
	$\dfrac{3.5}{10 \times 10 \times 10 \times 10 \times 10}$	
	$\dfrac{9.9}{10}$	
		6.9×10^{-2}
		8.5×10^{-4}

7. Determine the direction and how many places the decimal point should be moved to change the following numbers, so that the number is between 1 and 10.

a) 15 **b)** 275 000 **c)** 0.009 8 **d)** 0.58 **e)** 997 123 784

8. Express each number in scientific notation.

a) 0.000 023 **b)** 0.005 **c)** 0.000 000 872 5

d) 79 300 **e)** 0.6 **f)** 789 000 000

g) 0.000 005 94 **h)** 0.025 1 **i)** 81 300 000

9. Express each number in standard notation.
a) 2.7×10^{-3} **b)** 1.28×10^{-4} **c)** 5.67×10^{6} **d)** 8.9×10

10. Express each of the following in scientific notation.
a) 23.2×10^{5} **b)** 0.7×10^{3} **c)** 0.089×10^{-7} **d)** 45.6×10^{-9}

11. Using your calculator answer the following in scientific notation.
a) $(3.6 \times 10^{8})(4.2 \times 10^{-5})$ **b)** $(1.8 \times 10^{5}) \div (3.2 \times 10^{2})$

c) $(0.089 \times 10^{-3})(0.1 \times 10^{-8})$ **d)** $(23.1) \div (0.5 \times 10^{8})$

12. Using your calculator answer the following in standard notation.
a) $(4.3\times 10^{2})(2.4 \times 10^{-7})$ **b)** $(8.1 \times 10^{2}) \div (0.75 \times 10^{-2})$

c) $(0.05 \times 10^{-3})(2.5 \times 10^{-3})$ **d)** $(6.8) \div (0.85 \times 10^{7})$

Multiple Choice

13. The tallest totem pole in Canada is located in Alert Bay, British Columbia. The traditional figures on the pole represent many of the families that belong to the Kwakwaka'wakw Nation and include the Bear, the Salmon, and the Whale. It is approximately 5640 cm high. Written using scientific notation, the height of the totem pole, in cm, is approximately

A. 5.64×10^{2}
B. 56.4×10^{2}
C. 5.64×10^{3}
D. 0.564×10^{4}

14. The length of the River Nile in Africa, is approximately 6 695 000 m. When this number is written in scientific notation in the form $a \times 10^{n}$, the value of n is

A. –6 **B.** 3 **C.** 6 **D.** 7

15. In January 2018, the highest grossing movie of all time was "Avatar", with world-wide takings of approximately 2 790 million dollars. In scientific notation, the number of dollars grossed by this movie is

A. 2.79×10^{10} **B.** 2.79×10^{9} **C.** 2.79×10^{6} **D.** 2.79×10^{3}

Numerical Response

16. The speed of light is 3×10^8 m/s. The sun is 1.5×10^{11} metres from the earth. If the number of seconds it takes for light to reach the earth is expressed in scientific notation in the form $a \times 10^n$, the value of $a + n$ is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

1.

Standard Notation	Expanded Form	Scientific Notation
151 000	$1.5 \times 10 \times 10 \times 10 \times 10 \times 10$	1.51×10^5
23.4	2.34×10	2.34×10^1
32 000	$3.2 \times 10 \times 10 \times 10 \times 10$	3.2×10^4
830 000	$8.3 \times 10 \times 10 \times 10 \times 10 \times 10$	8.3×10^5
730	$7.3 \times 10 \times 10$	7.3×10^2
840	$8.4 \times 10 \times 10$	8.4×10^2
6 200	$6.2 \times 10 \times 10 \times 10$	6.2×10^3

6.

Standard Notation	Expanded Form	Scientific Notation
0.000 000 9	$\frac{9}{10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10}$	9×10^{-7}
0.000 001	$\frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10}$	1×10^{-6}
0.000 035	$\frac{3.5}{10 \times 10 \times 10 \times 10 \times 10}$	3.5×10^{-5}
0.99	$\frac{9.9}{10}$	9.9×10^{-1}
0.069	$\frac{6.9}{10 \times 10}$	6.9×10^{-2}
0.000 85	$\frac{8.5}{10 \times 10 \times 10 \times 10}$	8.5×10^{-4}

2. a) 2.3×10^3 **b)** 7.58×10^6 **c)** 4.1×10^{10}
d) 5.31×10^1 **e)** 4.32×10^{12} **f)** 7.6×10^0
3. a) 5.79×10^7 km **b)** 1.4×10^9 km
4. a) 1 800 000 000 000 **b)** 673 000 **c)** 99 900 000
d) 227 940 000 **e)** 950 000
5. a) 5.7×10^7 **b)** 9.843×10^9 **c)** 6.1×10^4 **d)** 5×10^1
6. See table above
7. a) 1 to the left **b)** 5 to the left **c)** 3 to the right **d)** 1 to the right **e)** 8 to the left
8. a) 2.3×10^{-5} **b)** 5×10^{-3} **c)** 8.725×10^{-7} **d)** 7.93×10^4 **e)** 6×10^{-1}
f) 7.89×10^8 **g)** 5.94×10^{-6} **h)** 2.51×10^{-2} **i)** 8.13×10^7
9. a) 0.002 7 **b)** 0.000 128 **c)** 5 670 000 **d)** 89
10. a) 2.32×10^6 **b)** 7×10^2 **c)** 8.9×10^{-9} **d)** 4.56×10^{-8}
11. a) 1.512×10^4 **b)** 5.625×10^2 **c)** 8.9×10^{-14} **d)** 4.62×10^{-7}
12. a) 0.000 103 2 **b)** 108 000 **c)** 0.000 000 125 **d)** 0.000 000 8

13. C **14.** C **15.** B **16.**

7			

Prime Factorization and Exponents Lesson #7:
Practice Test

Use the space beside the question for your rough work.

1. Which of the following numbers is not a prime factor of 14 014?

A. 7
B. 11
C. 13
D. 17

2. How many numbers in the list $7, 11, 17, 21,$ are prime factors of 3 234?

A. 1
B. 2
C. 3
D. 4

Numerical Response **1.** The sum of all of the prime factors of 160 797 is ______.

(Record your answer in the numerical response box from left to right) ☐☐☐☐

3. The lowest common multiple of 33 and 110 can be written as a product of prime factors in the form $a \times b \times c \times d$ where a, b, c, and d are prime numbers with $a < b < c < d$. The value of c is

A. 3

B. 5

C. 11

D. 1 100

Numerical Response **2.** The greatest common factor of 6 699 and 8 265 is ______.

(Record your answer in the numerical response box from left to right) ☐☐☐☐

4. The lowest common multiple of 14 and 105 is equal to the greatest common factor of the numbers P and Q. Which of the following statements must be false?

A. P is a multiple of 7.

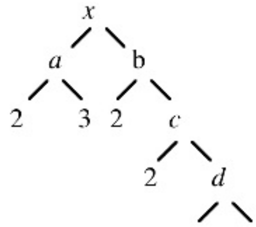
B. Q is a multiple of 21.

C. P could be less than 200.

D. Q could be greater than 2 000.

Use the following information to answer the next two questions.

The partial prime factorization of a number x is shown in the tree diagram.



5. If x is a perfect square, then the minimum value of d is

A. 2 **B.** 3 **C.** 6 **D.** 9

Numerical Response **3.** If x is a perfect cube, then the minimum value of x is ______.

(Record your answer in the numerical response box from left to right) ☐☐☐☐

6. $-a^0$ is equivalent to

A. 0 **B.** 1 **C.** $-a$ **D.** -1

7. Consider the following two statements about repeated multiplication.

Statement 1. $5pq^3 = 5 \times p \times q \times q \times q$ Statement 2. $6(xy)^2 = 6 \times 6 \times x \times x \times y \times y$

Which one of the following is true?

A. Statement 1 is correct and Statement 2 is incorrect.
B. Statement 2 is correct and Statement 1 is incorrect.
C. Both statements are correct.
D. Both statements are incorrect.

Use the following information to answer the next question.

$$(2^a)^4 = 2^8 \qquad (3^2)^b = 3^8$$

$$\frac{4^c}{4^2} = 4^3 \qquad 5^d \cdot 5^3 = 5^9$$

Numerical Response **4.** Write the value of a in the first box.
Write the value of b in the second box.
Write the value of c in the third box.
Write the value of d in the fourth box.

(Record your answer in the numerical response box from left to right)

8. Which of the following can be simplified to a^6?

I $a^3 \times a^2$ II $a^4 + a^2$ III $a^{12} \div a^2$ IV $a^8 - a^2$

A. I and III only **B.** II and IV only

C. I, II, III, and IV **D.** none of I, II, III, and IV

9. $5a^3 \times 2a^4$ can be simplified to

A. $7a^{12}$ **B.** $10a^7$ **C.** $10a^{12}$ **D.** $7a^7$

10. $\dfrac{5a^{16}}{10a^4}$ can be written as

A. $\frac{1}{2}a^{12}$ **B.** $2a^{12}$ **C.** $\frac{1}{2}a^4$ **D.** $2a^4$

Use the following information to answer the next two questions.

$(-2p^3q)(-3pq^2)(-4q^6)^2$ can be written in the form $a\,p^xq^y$ where a, x, and y are integers.

11. The value of a is

A. 96

B. −96

C. 24

D. −24

Numerical Response **5.** The value of $x + y$ is ______ .
(Record your answer in the numerical response box from left to right)

12. $\dfrac{6x^3}{2x^{-4}}$ can be simplified to

A. $3x^{-1}$ **B.** $3x^7$ **C.** $4x^7$ **D.** $4x^{-1}$

13. $5x^{-2}$ is equivalent to

A. $\dfrac{1}{5x^2}$ **B.** $-10x$ **C.** $\dfrac{1}{25x^2}$ **D.** $\dfrac{5}{x^2}$

14. If $(4.6 \times 10^{-3}) \times (3.5 \times 10^7) = 1.61 \times 10^n$, then the value of n is

A. 3

B. 4

C. 5

D. 6

15. A piece of paper is 8.1×10^{-5} m thick. Approximately how many pieces of paper are required to make a stack two metres high?

A. 25 000
B. 40 000
C. 250 000
D. 400 000

Written Response - 5 marks

Use the following information to answer the next question.

The average person is 1.65 m tall.
The average human has 1.25×10^5 hairs on his/her head.
The current population of the earth is approximately 6.80×10^9.
The circumference of the Earth is about 4.00×10^7 metres.
The mass of the Earth is about 5.98×10^{24} kg.
The mass of the Sun is about 1.99×10^{30} kg.
The mass of the lightest planet Mercury is about 3.30×10^{23} kg.
The mass of an electron is approximately 9.11×10^{-31} kg.

1.

- Determine, to the nearest million, the number of people laid end to end it would it take to encircle the earth.

- Determine an estimate for the total number of hairs on the heads of all the people on the earth. Answer in scientific notation to the nearest hundredth.

- Approximately how many electrons would have the same mass as the planet Mercury. Answer in scientific notation to the nearest hundredth.

- How many times heavier is the Sun than the combined mass of the Earth and Mercury? Answer in standard decimal form to the nearest one thousand.

Answer Key

1. D **2.** B **3.** B **4.** C **5.** C

6. D **7.** A **8.** D **9.** B **10.** A

11. A **12.** B **13.** D **14.** C **15.** A

Numerical Response

1. 7 3

2. 8 7

3. 2 1 6

4. 2 4 5 6

5. 1 9

Written Response

1.
- 24 000 000
- 8.50×10^{14}
- 3.62×10^{53}
- 315 000

Trigonometry Lesson #1:
Trigonometric Ratios

Overview of Unit

Trigonometry (from the Greek trigonon = three angles and metro = measure) is a branch of mathematics dealing with angles, triangles and trigonometric functions such as sine, cosine and tangent. In this unit we study relationships between sides and angles in right triangles.

Investigation #1

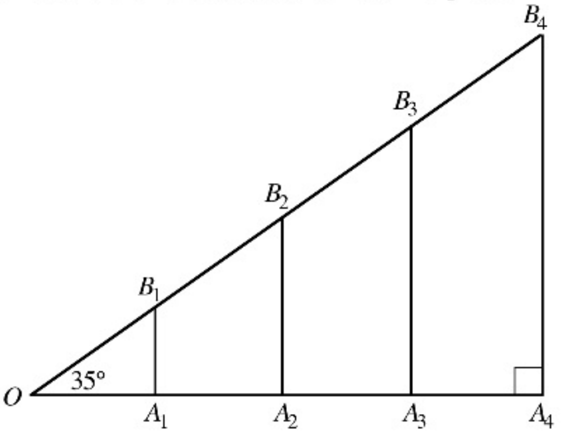
The diagram shows a series of similar right triangles, ⊿ OA_1B_1, ⊿ OA_2B_2, etc.
Complete the work below using a ruler to measure the indicated sides to 1 decimal place.
Calculate each ratio to 1 decimal place.

$\frac{A_1B_1}{OA_1} = \frac{1.4}{2} = 0.7$

$\frac{A_2B_2}{OA_2} = \frac{\quad}{4} =$

$\frac{A_3B_3}{OA_3} = \qquad =$

$\frac{A_4B_4}{OA_4} = \qquad =$



We can conclude that the ratio ______ is somehow connected to the angle of 35°.

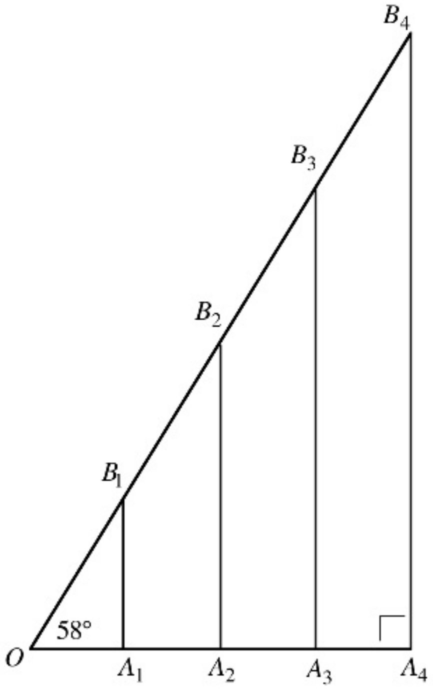
Investigation #2

$\frac{A_1B_1}{OA_1} = \frac{2.4}{1.5} = 1.6$

$\frac{A_2B_2}{OA_2} = \frac{\quad}{3} =$

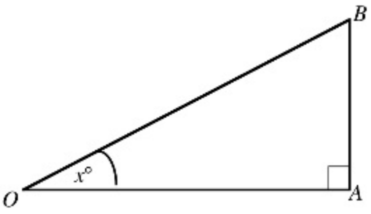
$\frac{A_3B_3}{OA_3} =$

$\frac{A_4B_4}{OA_4} =$



We can conclude that the ratio ______ is somehow connected to the angle of 58°.
We will investigate these ratios further in the rest of the lesson.

Ratios of Sides in a Right Triangle



Consider the right triangle AOB shown.

Let angle $AOB = x°$.

Each of the sides of the triangle is given a special name relative to the angle of $x°$.

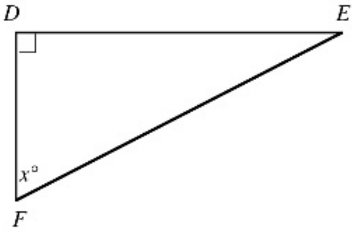
The longest side, OB, is called the HYPOTENUSE (hyp).

The side opposite the angle of $x°$, AB, is called the OPPOSITE (opp).

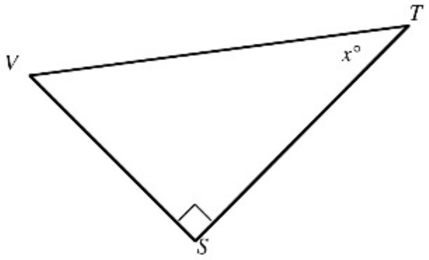
The third side of the triangle, OA, is called the ADJACENT (adj).

Mark on each of these triangles the hypotenuse (hyp), the opposite (opp), and the adjacent (adj) relative to the angle of $x°$.

a)



b)



- The ratio $\frac{AB}{OA}$ in the diagram at the top of this page is the same ratio we used in Investigations #1 and #2.
- This ratio can be written as $\frac{\text{opposite}}{\text{adjacent}}$ or $\frac{\text{opp}}{\text{hyp}}$ and is known as the tangent ratio.
- There are five other ratios possible using two of the sides of triangle AOB. All six ratios are listed below.

$\frac{AB}{OB} = \frac{\text{opposite}}{\text{hypotenuse}} =$ the sine ratio

$\frac{OA}{OB} = \frac{\text{adjacent}}{\text{hypotenuse}} =$ the cosine ratio

$\frac{AB}{OA} = \frac{\text{opposite}}{\text{adjacent}} =$ the tangent ratio

The three ratios above left are the primary trigonometric ratios and will be studied in this course.

$\frac{OB}{AB} = \frac{\text{hypotenuse}}{\text{opposite}} =$ the cosecant ratio

$\frac{OB}{OA} = \frac{\text{hypotenuse}}{\text{adjacent}} =$ the secant ratio

$\frac{OA}{AB} = \frac{\text{adjacent}}{\text{opposite}} =$ the cotangent ratio

The three ratios above right are the reciprocal trigonometric ratios and will be studied in **higher level math courses**.

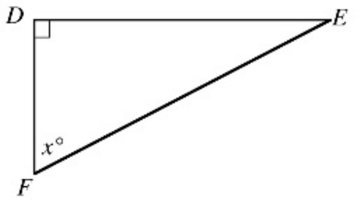
SOH CAH TOA

The rules for determining the sine ratio, cosine ratio and tangent ratio for an angle in a right triangle can be memorized by using the acronym SOH CAH TOA.

Class Ex. #2

The diagrams shown below are from Class Example #1. Use these diagrams to complete the work below to list the sine ratio, the cosine ratio, and the tangent ratio for the angle of $x°$.

a)

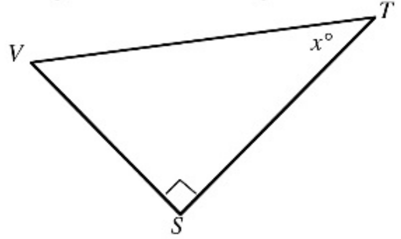


sine ratio for the angle of $x°$ $= \dfrac{\text{opposite}}{\text{hypotenuse}} =$

cosine ratio for the angle of $x°$ $= \dfrac{\text{adjacent}}{\text{hypotenuse}} =$

tangent ratio for the angle of $x°$ $= \dfrac{\text{opposite}}{\text{adjacent}} =$

b)



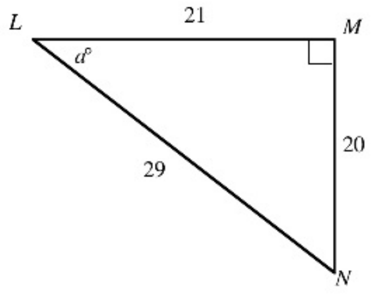
sine ratio for the angle of $x°$ $=$

cosine ratio for the angle of $x°$ $=$

tangent ratio for the angle of $x°$ $=$

Class Ex. #3

Complete the work below to express the three primary trigonometric ratios, relative to angle $a°$, as rational numbers and in decimal form to the nearest thousandth.



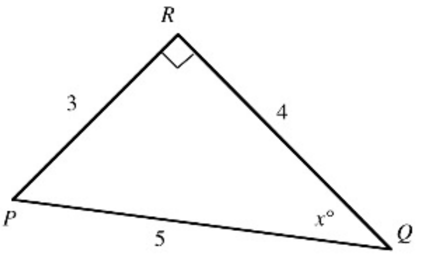
sine ratio relative to angle $a°$ $= \dfrac{\text{opp}}{\text{hyp}} =$

cosine ratio relative to angle $a°$ $=$

tangent ratio relative to angle $a°$ $=$

Complete Assignment Questions #1 - #3

Using sin, cos, tan



Class Ex. #4

Consider right triangle PQR shown.

The sine ratio for the angle of $x°$ is $\frac{PR}{PQ} = \frac{3}{5}$.

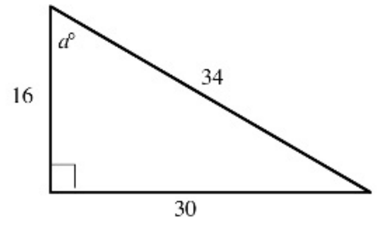
In short we write $\sin x° = \frac{3}{5}$.

- Similarly $\cos x° =$ and $\tan x° =$

Note Sometimes the trigonometric ratios for an angle are given in terms of the letter at which the angle is located. For example, in $\triangle PQR$ above we could write $\sin Q = \frac{3}{5}$.

Class Ex. #5

Complete the following, writing the ratio in simplest rational form.

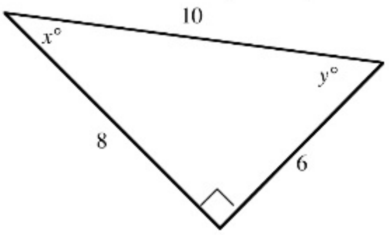


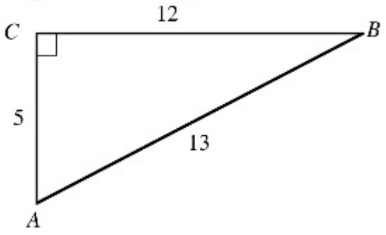
a) $\sin a° =$ **b)** $\cos a° =$

c) $\tan a° =$

Class Ex. #6

Write the rational number, in simplest form, which represents the trigonometric ratio.





i) $\sin x° =$ **ii)** $\tan y° =$ **iii)** $\cos A =$ **iv)** $\tan B =$

v) $\sin y° =$ **vi)** $\cos x° =$ **vii)** $\cos B =$ **viii)** $\sin A =$

Complete Assignment Questions #4 - #11

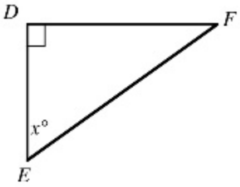
Assignment

1. Consider the eight triangles below. For each triangle, complete the tables for the angle $x°$.

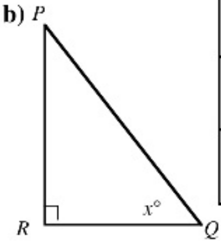
Triangle	Opposite Side	Adjacent Side	Hypotenuse
a)		DE	EF
b)			
c)			
d)			
e)			
f)			
g)			
h)			

Triangle	sine ratio	cosine ratio	tangent ratio
a)		$\frac{DE}{EF}$	
b)			
c)			
d)			
e)			
f)			
g)			
h)			

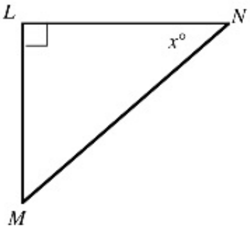
a)



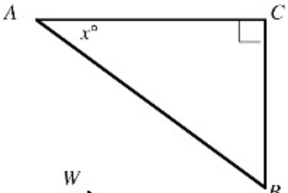
b)



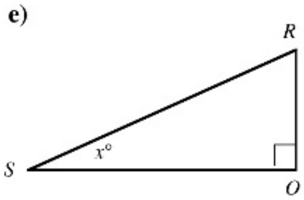
c)



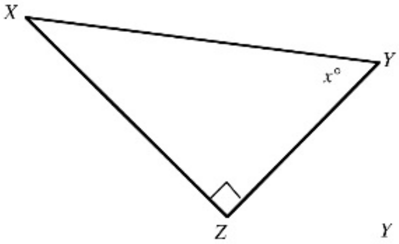
d)



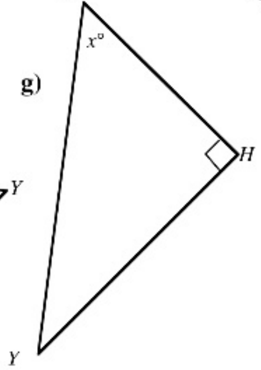
e)



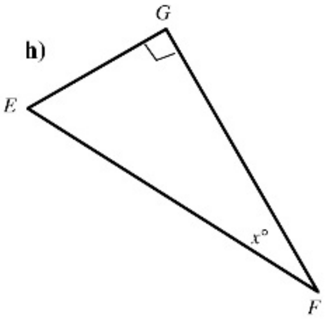
f)



g)



h)

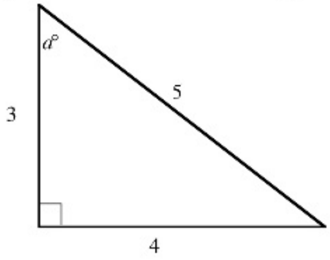


2. On each triangle mark the (hyp), (opp) and (adj) relative to angle $a°$ and determine the trigonometric ratios associated for angle $a°$.

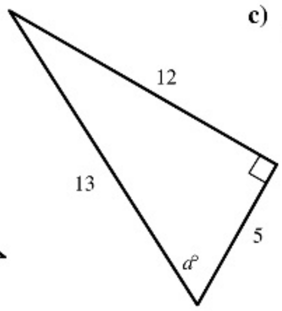
Complete the table.

Triangle	**sine ratio**	**cosine ratio**	**tangent ratio**
a)	$\frac{4}{5}$		
b)			
c)			

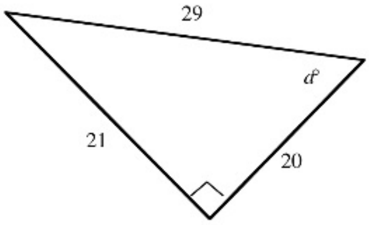
a)



b)

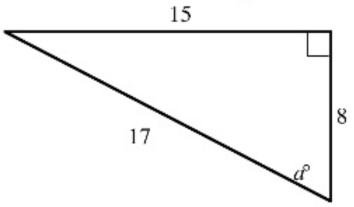


c)

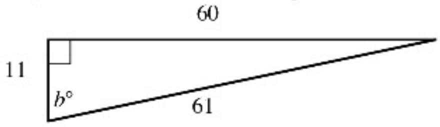


3. Determine, as a fraction in simplest form, the value of the trigonometric ratio indicated.

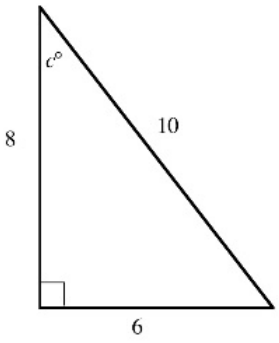
a) cosine ratio for the angle $a°$



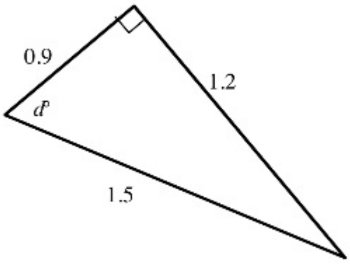
b) sine ratio relative to angle $b°$



c) tangent ratio relative to angle $c°$

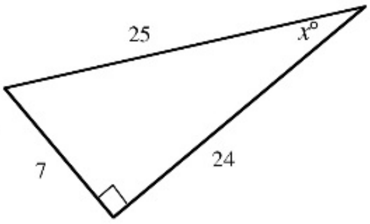


d) sine ratio for the angle $d°$

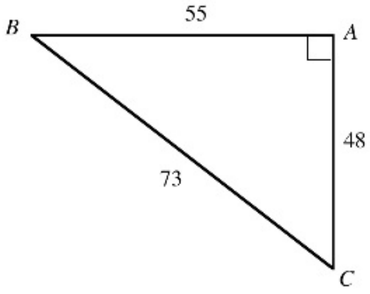


4. On the triangle, mark the hyp, opp, and adj for the angle $x°$ and determine the values of $\sin x°$, $\cos x°$, and $\tan x°$.

Write each answer as a decimal to the nearest hundredth.



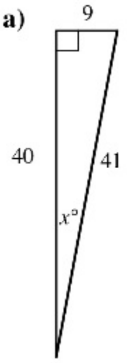
5. In the triangle, determine the exact values of $\sin B$, $\cos B$, and $\tan B$.



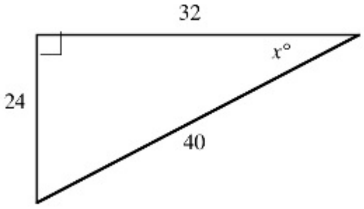
6. In each triangle, determine the values of $\sin x°$, $\cos x°$, and $\tan x°$ in simplest rational form.

Triangle	**sin $x°$**	**cos $x°$**	**tan $x°$**
a)			
b)			
c)			
d)			
e)			
f)			

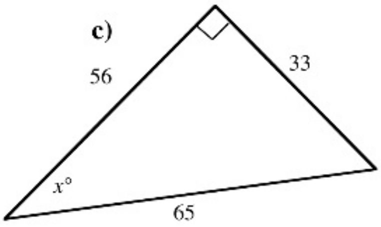
a)



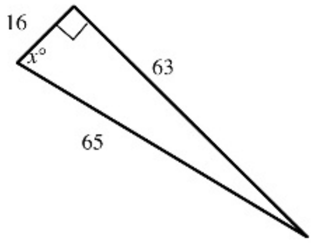
b)



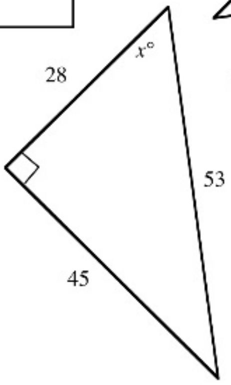
c)



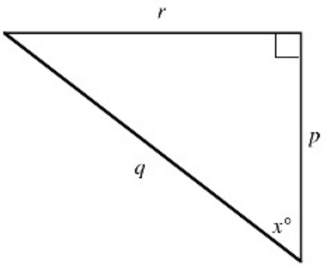
d)



e)

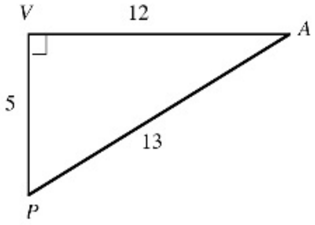


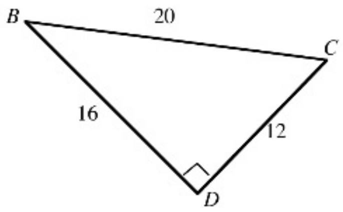
f)

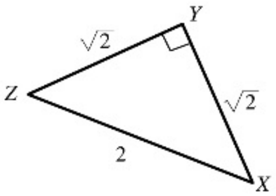


7. In each case, write the rational number which represents the trigonometric ratio.

a) $\sin A =$ **b)** $\cos B =$ **c)** $\tan X =$







Multiple Choice

8. In right triangle ABC, $AB = 52$ units, $AC = 48$ units and $BC = 20$ units. The value of $\cos B$ and $\sin B$ are respectively

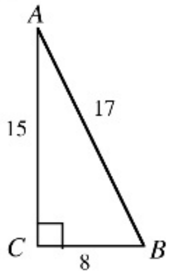
A. $\frac{5}{13}$ and $\frac{12}{13}$

B. $\frac{12}{13}$ and $\frac{5}{13}$

C. $\frac{5}{12}$ and $\frac{12}{5}$

D. $\frac{5}{13}$ and $\frac{12}{5}$

9 For the right angled triangle ABC, only one of the following ratios is correct. The correct ratio is



A. $\sin A = \frac{8}{15}$

B. $\cos A = \frac{8}{17}$

C. $\tan B = \frac{8}{15}$

D. $\sin B = \frac{15}{17}$

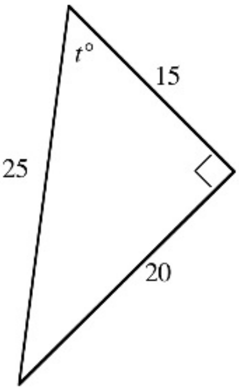
10. In a right triangle $\tan x° = \frac{7}{5}$. A student claims this indicates that in the right triangle the side opposite to the angle $x°$ is 7 units and the side adjacent to the angle $x°$ is 5 units.

The student's claim

A. is always true **B.** is always false

C. may be true or false. **D.** depends on the value of $x°$

Numerical Response **11.** In the diagram, to the nearest tenth, the value of $\frac{\sin t°}{\cos t°}$ is ________.



(Record your answer in the numerical response box from left to right)

Answer Key

1 .

Triangle	Opposite Side	Adjacent Side	Hypotenuse
a)	*DF*	*DE*	*EF*
b)	*PR*	*QR*	*PQ*
c)	*LM*	*LN*	*MN*
d)	*BC*	*AC*	*AB*
e)	*OR*	*OS*	*RS*
f)	*XZ*	*YZ*	*XY*
g)	*HY*	*HW*	*WY*
h)	*EG*	*FG*	*EF*

Triangle	sine ratio	cosine ratio	tangent ratio
a)	$\frac{DF}{EF}$	$\frac{DE}{EF}$	$\frac{DF}{DE}$
b)	$\frac{PR}{PQ}$	$\frac{QR}{PQ}$	$\frac{PR}{QR}$
c)	$\frac{LM}{MN}$	$\frac{LN}{MN}$	$\frac{LM}{LN}$
d)	$\frac{BC}{AB}$	$\frac{AC}{AB}$	$\frac{BC}{AC}$
e)	$\frac{OR}{RS}$	$\frac{OS}{RS}$	$\frac{OR}{OS}$
f)	$\frac{XZ}{XY}$	$\frac{YZ}{XY}$	$\frac{XZ}{YZ}$
g)	$\frac{HY}{WY}$	$\frac{HW}{WY}$	$\frac{HY}{HW}$
h)	$\frac{EG}{EF}$	$\frac{FG}{EF}$	$\frac{EG}{FG}$

Trigonometry Lesson #2:
Trigonometric Ratios on a Calculator and Patterns in Trigonometric Ratios

Review

Recall the following from the previous lesson.

SOH CAH TOA

sine ratio = $\frac{\text{opposite}}{\text{hypotenuse}}$

cosine ratio = $\frac{\text{adjacent}}{\text{hypotenuse}}$

tangent ratio = $\frac{\text{opposite}}{\text{adjacent}}$

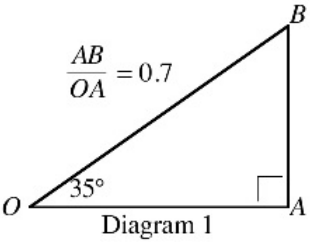


Diagram 1

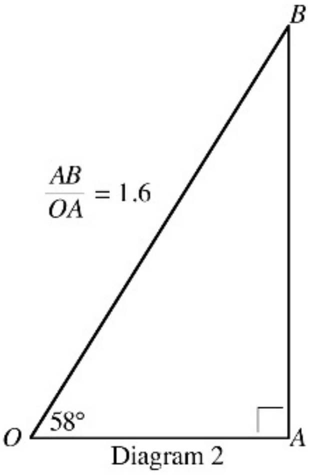


Diagram 2

Using a Calculator to Determine Trigonometric Ratios

From the investigations in the previous lesson we can now say that the ratio 0.7 is the approximate tangent ratio for an angle of 35° and that the ratio 1.6 is the approximate tangent ratio for an angle of 58°.

We can write the ratios in the form $\tan 35° = 0.7$ and $\tan 58° = 1.6$ (to 1 decimal place).

More accurate approximations can be found using a calculator.

To four decimal places, $\tan 35° =$ ____________ and $\tan 58° =$ ____________ .

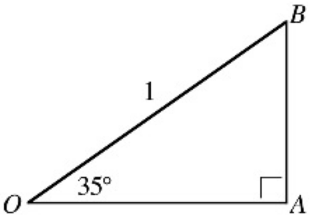
a) Use a calculator to determine the value of each trigonometric ratio to four decimal places.

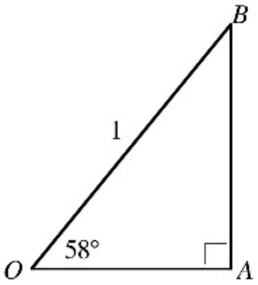
i) $\sin 35° =$ **ii)** $\cos 35° =$

iii) $\sin 58° =$ **iv)** $\cos 58° =$

b) With respect to the diagrams at the top of the page, which ratio of sides is represented by each of the ratios in **a)**?

The diagrams below show triangles similar to the ones on the previous page in which the two hypotenuses are equal to 1 unit.





As the angle *AOB* increases from 35° to 58° explain, using SOHCAHTOA, why the sine ratio increases in value but the cosine ratio decreases in value.

Determine what happens to the tangent ratio.

Without using a calculator, state which of the following ratios has the greater value.

a) sin 14°, sin 20°

b) cos 7°, cos 62°

c) tan 15°, tan 75°

d) sin 80°, cos 80°, tan 80°

The trigonometric ratios for all angles between 0° and 90° can be approximated using a calculator.

Use a calculator to determine the value of each trigonometric ratio to four decimal places.

a) sin 81° =

b) tan 14° =

c) cos 44° =

Determining the Angle Given a Trigonometric Ratio

We have discovered that $\tan 35° = 0.7$ and $\tan 58° = 1.6$, but is there an angle for which the tangent ratio is 1.2?

Use a "guess and check" technique to discover the measure of this angle to 1 decimal place.

This method is time consuming and the calculator provides us with a quicker method by using the inverse trigonometric function.

If $\tan x° = 1.2$, then the measure of the angle $x°$ can be determined by using the inverse tangent function $\tan^{-1}$.

If $\tan x° = 1.2$, then $\tan^{-1}(1.2) = x°$.

On a calculator, access the inverse tangent function, $\tan^{-1}$, by pressing

2nd	tan

.

2nd	tan	1	.	2	)	ENTER

determines the value of x.

Part i: Write the keying sequence which will determine the acute measure of the angle $a°$ if $\sin a° = 0.3584$

Part ii: In each case, determine the measure of the acute angle to the nearest degree.

a) $\sin a° = 0.3584$

b) $\cos b° = 0.5389$ $b° = \quad °$

c) $\tan P = 3.2106$ $\angle P = \quad °$

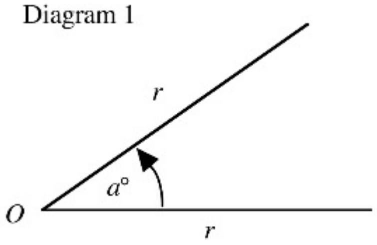
d) $\sin C = \frac{1}{2}$ $\angle C = \quad °$

Complete Assignment Questions #1 - #4

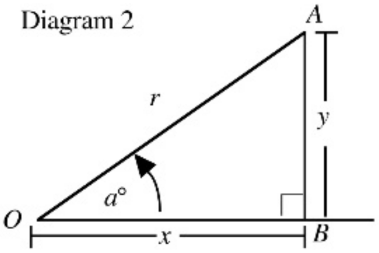
Trigonometric Ratios for Angles of 0° and 90°

The trigonometric ratios for angles of 0° and 90° can be determined using the idea of rotation angles.

A rotation angle of $a°$ is formed when a line of length r is rotated counterclockwise about a point O, as shown in Diagram 1.



Right triangle AOB, where AB is perpendicular to OA, is formed from Diagram 1 as shown in Diagram 2.



$\sin a° = \frac{y}{r}$.

Similarly $\cos a° =$

and $\tan a° =$

As the rotation angle, $a°$, is made smaller, then the opposite side of the triangle, y, decreases in value and the adjacent side, x, increases in value.

If $a = 0$, then the opposite side $y = 0$ and the adjacent side $x = r$.

So $\sin 0° = \frac{y}{r} =$ $\cos 0° = \frac{x}{r} =$ $\tan 0° = \frac{y}{x} =$

As the rotation angle, $a°$, is made larger, then the opposite side of the triangle, y, increases in value and the adjacent side, x, decreases in value.

If $a = 90°$, then the adjacent side $x = 0$ and the opposite side $y = r$.

So $\sin 90° = \frac{y}{r} =$ $\cos 90° = \frac{x}{r} =$ $\tan 90° = \frac{y}{x} =$

Patterns in Trigonometric Ratios

Class Ex. #6

Complete the following table, rounding if necessary to 2 decimal places.

Angle $x°$	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
sin $x°$										

Complete the following statement:

As the angle $x°$ increases from 0° to 90°, sin $x°$ increases from _____ to _____.

Class Ex. #7

Draw a right triangle LMN in which $\angle L = 90°$ and $\angle N = x°$.

a) Write an expression for the measure of $\angle M$ in terms of $x°$.

b) Explain, using ratios, why $\sin x° = \cos(90 - x)°$.

c) Complete the following, using an angle between 0° and 90°.

i) $\sin 20° = \cos$ ______

ii) $\sin 8° = \cos$ ______

Complete Assignment Questions #5 - #14

Assignment

1. Use a calculator to determine the value of each trigonometric ratio to four decimal places where necessary.

a) sin 68° = **b)** tan 30° = **c)** cos 19° =

d) cos 22° = **e)** tan 85° = **f)** sin 7° =

g) tan 60° = **h)** sin 18° = **i)** cos 72° =

j) cos 45° = **k)** sin 45° = **l)** tan 45° =

2. Use a calculator to determine the value of each trigonometric ratio to four decimal places.

a) sin 37.9° = **b)** tan 4.8° = **c)** cos 83.2° =

d) cos 5.7° = **e)** tan 63.6° = **f)** sin 17.4° =

3. In each case, use a calculator to determine the measure of the acute angle to the nearest degree.

a) $\sin a^\circ = 0.7612$, $a^\circ = \quad ^\circ$ **b)** $\cos b^\circ = 0.5398$, $b^\circ = \quad ^\circ$

c) $\tan P = 1.2173$, $\angle P = \quad ^\circ$ **d)** $\sin C = \frac{1}{3}$, $\angle C = \quad ^\circ$

e) $\tan t^\circ = \sqrt{3}$, $t^\circ = \quad ^\circ$ **f)** $\cos x^\circ = \frac{\sqrt{2}}{2}$, $x^\circ = \quad ^\circ$

g) $\sin A = \frac{\sqrt{2}}{2}$, $\angle A = \quad ^\circ$ **h)** $\tan X = \frac{1}{\sqrt{3}}$, $\angle X = \quad ^\circ$

4. In each case, use a calculator to determine the indicated acute angle to the nearest tenth of a degree.

a) $\sin A = 0.6789$ **b)** $\cos X = 0.1234$ **c)** $\tan P = 0.55$

d) $\sin a^\circ = 0.09$ **e)** $\cos e^\circ = \frac{7}{24}$ **f)** $\tan y^\circ = \frac{4}{3}$

g) $\cos K = 0.3$ **h)** $\tan M = 50$ **i)** $\sin R = \frac{5}{6}$

5. Explain, using diagrams, why the sine ratio for an angle of 20° is smaller than the sine ratio for an angle of 40°.

6. Explain, using diagrams, why the cosine ratio for an angle of 30° is greater than the cosine ratio for an angle of 70°.

7. Explain, using diagrams, why the tangent ratio for an angle of 15° is smaller than the tangent ratio for an angle of 50°.

8. Without using a calculator, state which of the following ratios has the greater value.

a) sin 58°, sin 32°

b) cos 67°, cos 35°

c) tan 56°, tan 86°

d) sin 29°, cos 29°

e) sin 74°, cos 74°

f) sin 60°, cos 60°, tan 60°

9. Complete the following table, rounding if necessary to 2 decimal places

Angle $x°$	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
$\cos x°$										

Complete the following statement:

As the angle $x°$ increases from 0° to 90°, $\cos x°$ ____________ from _____ to _____.

10. a) Complete the following table, rounding if necessary to 2 decimal places

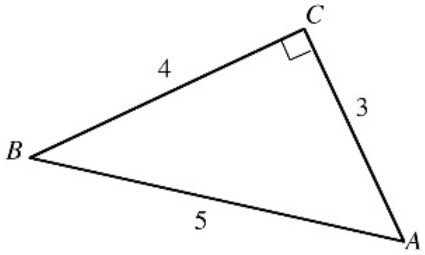
Angle $x°$	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
$\tan x°$										

b) Complete: $\tan 89° =$ _________ $\tan 89.9° =$ ________ $\tan 89.99° =$ ________

c) Complete the following statement:

As the angle $x°$ increases from 0° to 90°, $\tan x°$ ______________ from _____ to _______

11. Consider right triangle ABC shown.



a) Referring to triangle ABC, write the rational number which represents the indicated trigonometric ratio.

$\sin A =$

$\sin B =$

$\cos A =$

$\cos B =$

$\tan A =$

$\tan B =$

b) Which pairs of trigonometric ratios have the same value?

c) How do the values of $\tan A$ and $\tan B$ relate to each other?

Trigonometry Lesson #3:
Calculating The Length of a Side in Right Triangles

Review - The Pythagorean Theorem

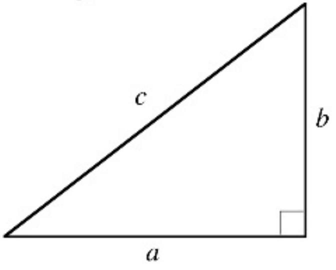
The Pythagorean Theorem can be applied in a right triangle to determine the length of the third side of a triangle in which the lengths of the other two sides are given.

In the triangle shown

$c^2 = a^2 + b^2$

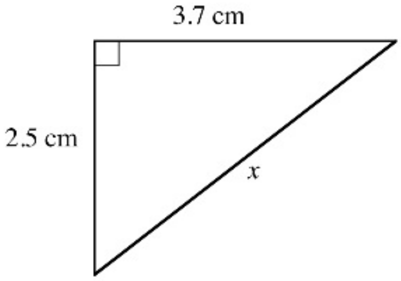
$a^2 = c^2 - b^2$

$b^2 = c^2 - a^2$

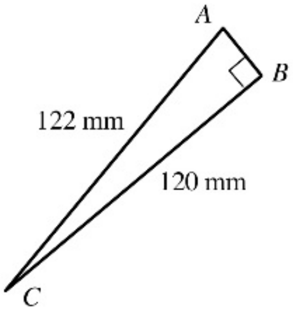


Calculate the length of the third side of each triangle, to the nearest tenth if necessary.

a)



b)



Using the Trigonometric Ratios to Calculate the Length of a Side

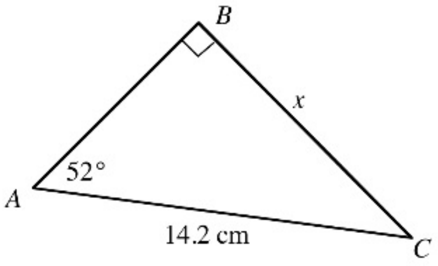
At the beginning of this lesson we reviewed the procedure for calculating the third side of a right triangle in which the other two sides were given.

It is also possible to determine the length of a side in a right triangle if the length of one side and the measure of one of the acute angles are given.

The procedure is demonstrated on the next page.

In right triangle ABC we are given the length of one side and the measure of an acute angle.

The procedure for determining the length of the side marked x is started below.



The given side, 14.2 cm, is the hypotenuse and the side to be determined, x, is opposite the given angle of 52°.

We use the ______________ ratio.

_____ $52° = \frac{x}{14.2}$ Cross multiply to get 14.2 _____ $52° = x$

To 1 decimal place, $x =$ ______ cm.

Class Ex. #2

Caribou jerky, a traditional Inuit food, can be seen drying on wooden racks along the shores of the Arctic Ocean. A piece of wood, 2.8 m long, that could be used to build a drying rack rests against a wall. The angle between the piece of wood and the ground is 40°.

Make a rough sketch and determine the distance between the bottom of the piece of wood and the bottom of the wall to the nearest tenth of a metre.

Class Ex. #3

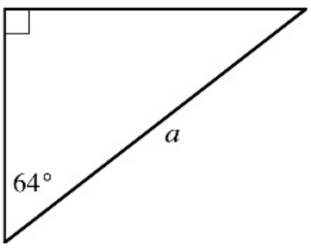
Determine the height of the wall in Class Ex. #2 to the nearest tenth of a metre.

a) using trigonometric ratios

b) using the Pythagorean Theorem

Complete Assignment Questions #1 - #4

In all of the previous work, the side which had to be determined was in the numerator of the trigonometric ratio. Complete the work below to determine the length of a side which appears in the denominator of the trigonometric ratio.



In the diagram, $\_\_\_\_\_ 64^\circ = \frac{7.9}{a}$

cross multiply to get

$$a \ \_\_\_\_\_ \ 64^\circ = 7.9$$

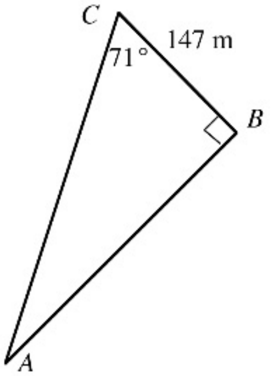
divide both sides by _____64° to get

$$a = \frac{7.9}{\_\_\_\_\_ \ 64^\circ}$$

$$= \ \_\_\_\_\_\_\_\_\_ \text{ (to the nearest tenth)}$$

Determine, to one decimal place, the length of the hypotenuse of triangle PQR in which angle $PQR = 90^\circ$, angle $PRQ = 28^\circ$, and $PQ = 15.6$ mm.

Use trigonometric ratios to determine the lengths of AB and AC in the given triangle. Answer correct to the nearest metre.

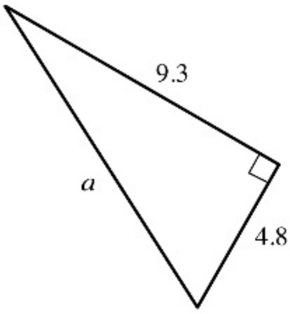


Complete Assignment Questions #5 - #12

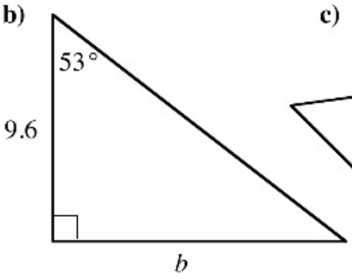
Assignment

1. Calculate, to the nearest tenth, the length of the indicated side in each triangle.

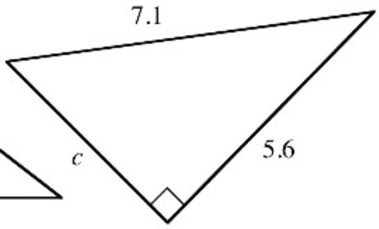
a)



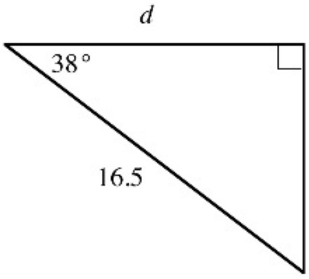
b)



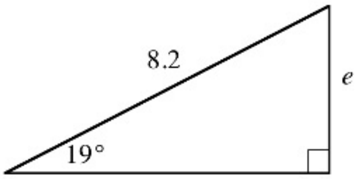
c)



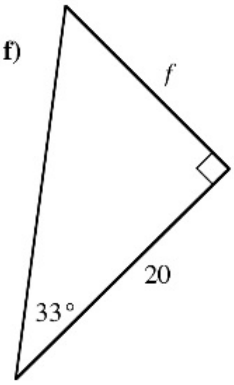
d)



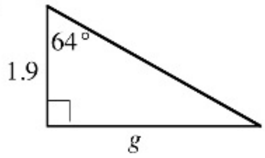
e)



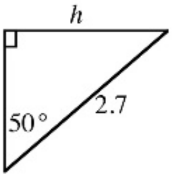
f)



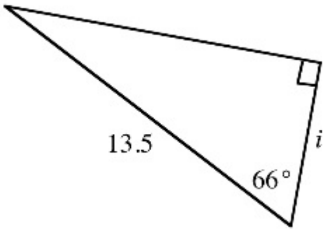
g)



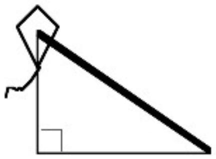
h)



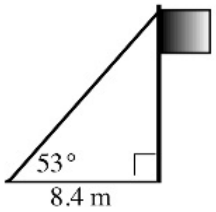
i)



2. A kite string is 65 metres long and makes an angle of 32° with the ground. Calculate, to the nearest metre, the vertical height, h, of the middle of the kite above the ground.



3. Use the measurements in the diagram to determine the height of the flagpole to the nearest tenth of a metre.



4. A ladder 5.3 m long is inclined at an angle of 72° to the ground.

a) How far up the wall, to the nearest tenth of a metre, does the ladder reach?

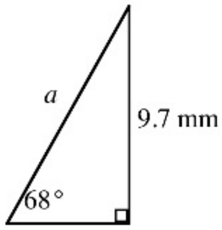
b) Use trigonometry to determine, to the nearest tenth of a metre, the distance between the bottom of the ladder and the bottom of the wall.

c) Use the answer to a) and the Pythagorean Theorem to determine, to the nearest tenth of a metre, the distance between the bottom of the ladder and the bottom of the wall.

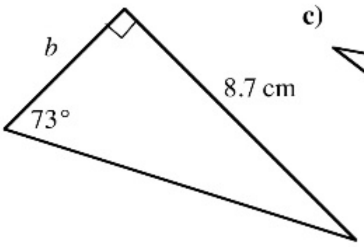
d) Explain why the answers to b) and c) are different.

5. Calculate, to the nearest tenth, the length of the indicated side in each triangle.

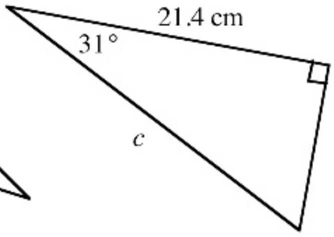
a)



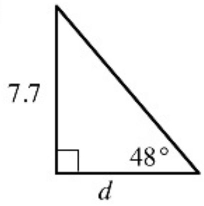
b)



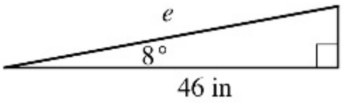
c)



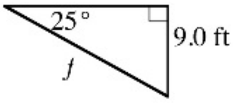
d)



e)

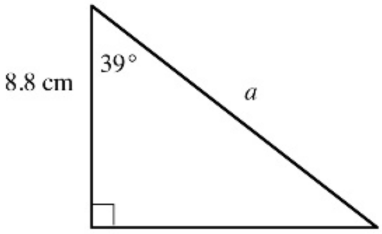


f)

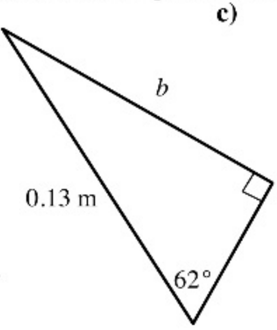


6. Calculate, to the nearest tenth of a cm, the length of the indicated side in each triangle.

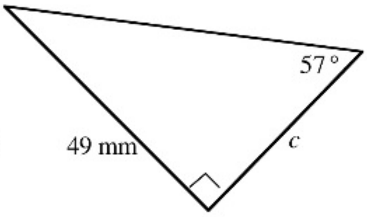
a)



b)

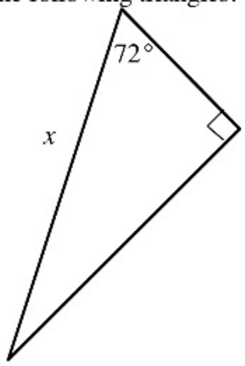


c)

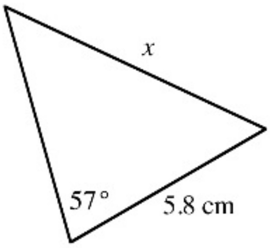


7. Explain why trigonometric ratios could not be used to calculate the side marked x in each of the following triangles.

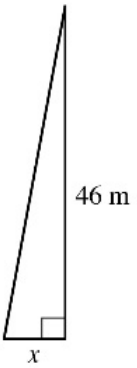
a)



b)



c)

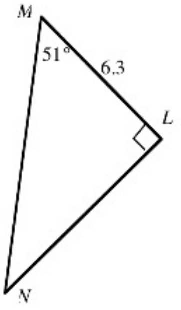


8. Determine, to the nearest tenth, the length of the hypotenuse of the following triangles.

a) Triangle PQR in which angle $QPR = 90°$, angle $PRQ = 47°$ and $PQ = 34.1$ mm.

b) $\triangle ABC$ in which $\angle ABC = 90°$, $\angle BCA = 29°$ and $BC = 8.4$ cm.

9. Calculate, to the nearest tenth, the lengths of LN and MN.



Multiple Choice

10. Triangle DEF is right-angled at F. Angle $DEF = 36°$ and $DF = 15$ cm. The length of DE, in cm, is given by

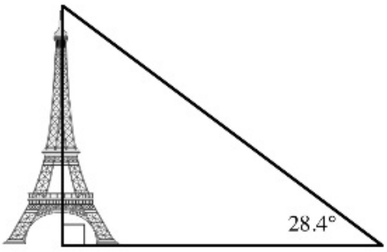
A. $15 \sin 36°$

B. $\dfrac{15}{\cos 36°}$

C. $15 \cos 54°$

D. $\dfrac{15}{\cos 54°}$

Numerical Response **11.** On a particular day, the Eiffel Tower in Paris casts a shadow of 599 m. Use the sketch to determine the height of the tower. To the nearest metre, the height of the tower is



(Record your answer in the numerical response box from left to right)

12. In right triangle ABC, angle $ABC = 90°$, angle $BAC = 70°$, and $AC = 29$ units.

To the nearest whole number, the perimeter of the triangle is _____ units.

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** 10.5 **b)** 12.7 **c)** 4.4 **d)** 13.0 **e)** 2.7 **f)** 13.0 **g)** 3.9 **h)** 2.1 **i)** 5.5

2. 34 m **3.** 11.1 m **4.** **a)** 5.0 m **b)** 1.6 m **c)** 1.8 m
d) Using a rounded length leads to a less accurate answer.

5. **a)** 10.5 mm **b)** 2.7 cm **c)** 25.0 cm **d)** 6.9 **e)** 46.5 in **f)** 21.3 ft

6. **a)** 11.3 cm **b)** 11.5 cm **c)** 3.2 cm

7. **a)** There is no side length given. **b)** The triangle is not right angled .
c) We need the measure of one of the acute angles.

8. **a)** 46.6 mm **b)** 9.6 cm **9.** $LN = 7.8$, $MN = 10.0$ **10.** D

11.	3	2	4		**12.**	6	6		

Trigonometry Lesson #4:
Calculating the Measure of an Angle in Right Triangles

Review

In Lesson #2 we learned how to determine the measure of an angle, when given a trigonometric ratio for the angle. For review, complete the following example.

Determine, to the nearest degree, the acute angle for which

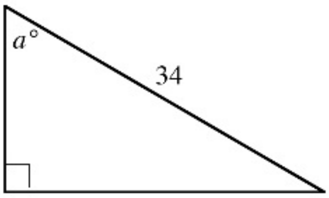
a) $\sin x° = 0.45$ **b)** $\cos y° = 0.1624$ **c)** $\tan z° = 5.2$

$x° =$ $y° =$ $z° =$

Calculating the Measure of an Angle

In order to use the trigonometric ratios to determine the measure of an angle in a right triangle, we need to know the lengths of two of the sides of the triangle.

Complete the following work for the diagram shown.



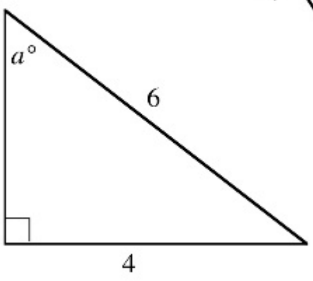
Relative to the angle $a°$, 16 is the length of the ADJACENT side and 34 is the length of the ______________.

The trigonometric ratio which involves the ADJACENT and the _____________ is the ____________ ratio.

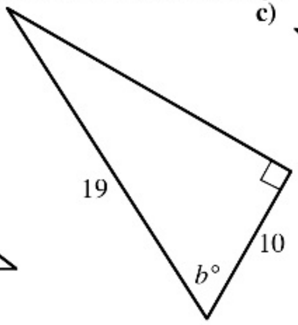
______ $a° = \frac{16}{34}$, so $a°$ = _____ to the nearest degree.

In each case, calculate the measure of the indicated angle to the nearest degree.

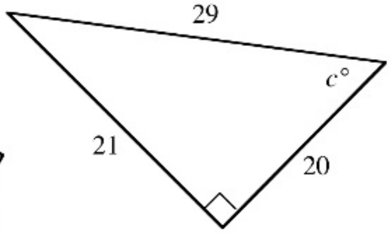
a)



b)



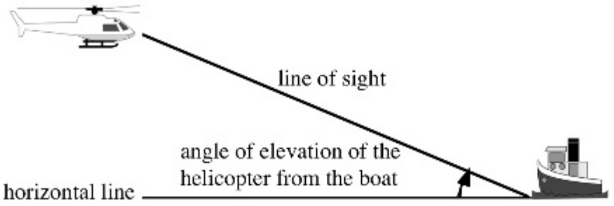
c)



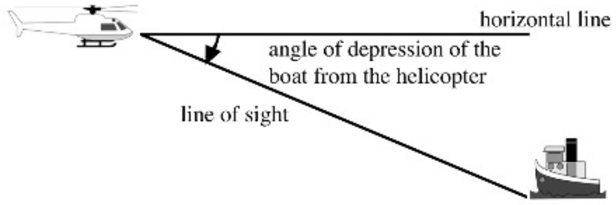
Complete Assignment Questions #1 - #5

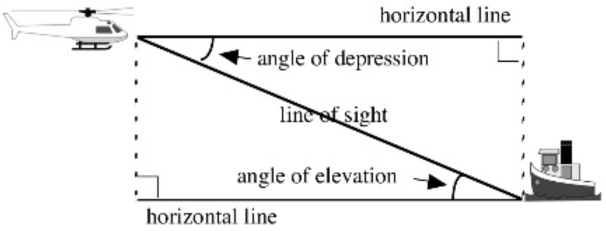
Angle of Elevation, Angle of Depression

- The **angle of elevation** is measured **upwards** from the **horizontal**. The sketch at the right illustrates the angle of elevation of the helicopter from the boat.


line of sight
angle of elevation of the helicopter from the boat
horizontal line

- The **angle of depression** is measured **downwards** from the **horizontal**. The sketch at the right illustrates the angle of depression of the boat from the helicopter.

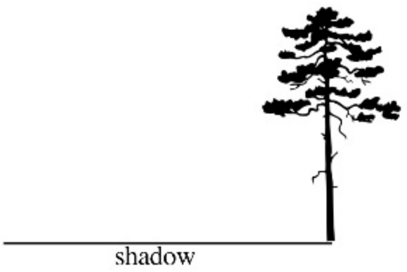

horizontal line
angle of depression of the boat from the helicopter
line of sight


horizontal line
angle of depression
line of sight
angle of elevation
horizontal line

- Note that angles of elevation and angles of depression are determined from the horizontal and NOT from the vertical.

- The sketch above shows that: angle of elevation of the helicopter from the boat = angle of depression of the boat from the helicopter

A tree 5.1 m tall casts a shadow 7.5 m long. Calculate the angle of elevation of the sun to the nearest tenth of a degree.



Class Ex. #3

A boat is 130 m from the base of a cliff. The cliff is 80 m high.

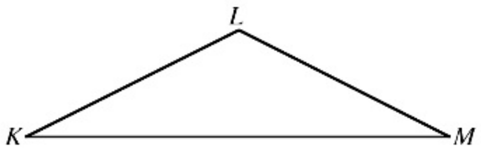
a) Draw a diagram to represent this scenario and mark the angle of depression of the boat from the top of the cliff.

b) Determine, to the nearest degree, the angle of depression of the boat from the top of the cliff.

Isosceles Triangles

Class Ex. #4

Consider isosceles triangle *KLM* in which $KL = 20$ cm, $LM = 20$ cm, and $KM = 30$ cm.



a) Why can't we use SOHCAHTOA in triangle *KLM* to determine the measure of angle *KLM*?

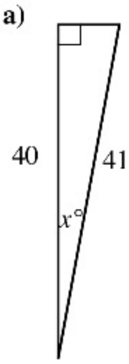
b) Determine, to the nearest degree, the measure of angle *KLM* by splitting triangle *KLM* into two congruent triangles.

Complete Assignment Questions #6 - #14

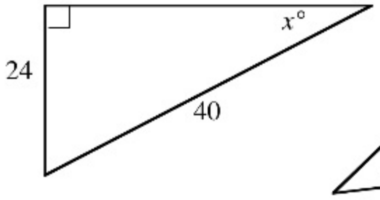
Assignment

1. In each case, calculate the measure of the indicated angle to the nearest degree.

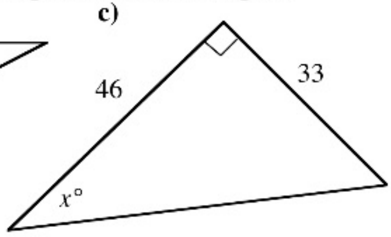
a)



b)

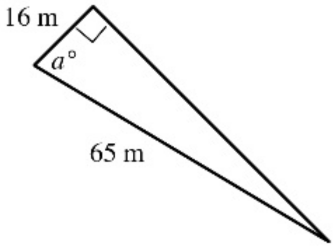


c)

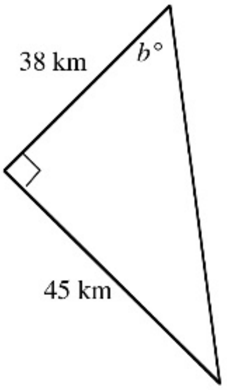


2. In each case, calculate the measure of the indicated angle to the nearest tenth.

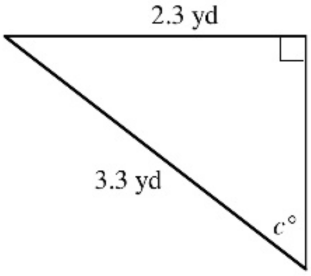
a)



b)

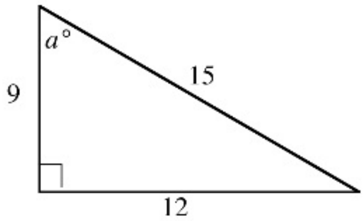


c)

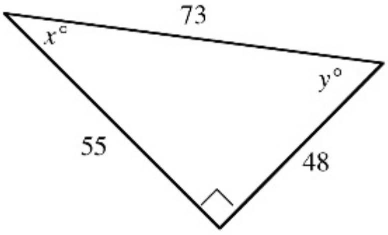


3. In each case, calculate the measure of the indicated angle to the nearest degree.

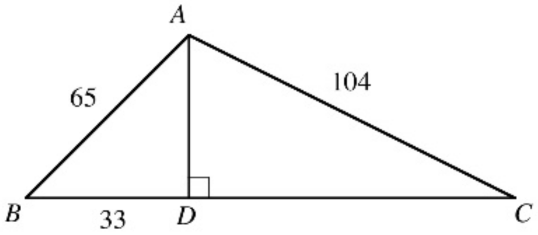
a)



b)



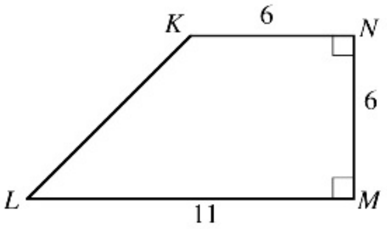
4. Consider the diagram consisting of two right triangles with a common side AD.



a) Use the Pythagorean Theorem to calculate the length of AD.

b) Determine, to the nearest degree, the measure of angle BCA.

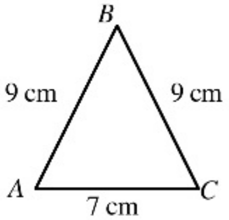
5. Determine the measure of angle LKN to the nearest degree.



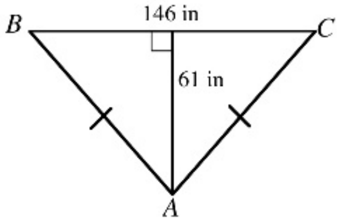
6. An electricity pylon 23.7 m high casts a shadow 46.8 m long. Determine the angle of elevation of the sun to the nearest tenth of a degree.

7. In each case, calculate the size of $\angle BAC$ to the nearest degree.

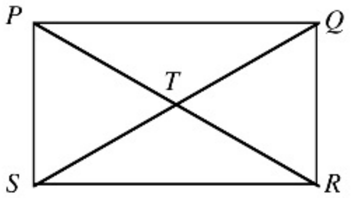
a)



b)

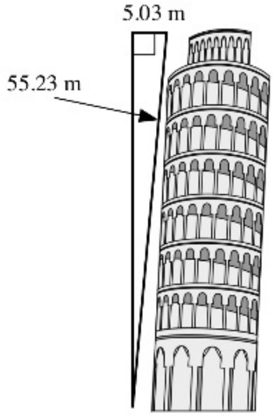


8. In the rectangle, $PQ = 12.8$ cm and $QR = 7.4$ cm. Determine the measure of angle PTQ to the nearest degree.



9. The term "% grade" is sometimes used to describe the slope of a road. For example, a road with a 7% grade has a vertical rise of 7 m for every horizontal distance of 100 m. Calculate, to the nearest degree, the angle a road with a 7% grade makes with the horizontal.

10. The Leaning Tower of Pisa is a building in Italy which leans due to the instability of the ground underneath it. At different points in history the tower has leaned at different angles. Use the measurements in the sketch to determine the angle of lean from the vertical to the nearest hundredth of a degree.



Multiple Choice **11.** A set of stairs has a vertical rise of 15 cm for every 28 cm horizontal run. To the nearest degree, the angle between the stairs and the floor is

A. 28°
B. 32°
C. 62°
D. 64°

Numerical Response **12.** A tourist at the top of a lighthouse spots a boat in the water below. The angle of depression of the boat from the tourist is 35°. At the same moment in time, the angle of elevation of the tourist from the boat, to the nearest degree, is _____°.

(Record your answer in the numerical response box from left to right) ☐☐☐☐

Numerical Response **13.** Susan Point, a Coast Salish artist from the Musqueam Nation, created three red cedar portals welcoming visitors to the Brockton Point Centre and the traditional lands of the Coast Salish people located in Vancouver's Stanley Park. The carvings are the gateway to British Columbia's most visited tourist attraction, a collection of 9 totem poles.

(http://vancouver.ca/parks-recreation-culture/totems-and-first-nations-art.aspx).

The height of one of the gateways is approximately 20 feet and it casts a shadow of 34.6 feet. To the nearest degree, the angle of elevation of the sun is _____ .

(Record your answer in the numerical response box from left to right) ☐☐☐☐

Numerical Response **14.** A submarine goes into a dive at a certain angle of depression and travels 275 m during the descent. When the submarine stops its dive, it has dropped a vertical distance of 150 m. To the nearest degree, the angle of depression of the dive is _____°.

(Record your answer in the numerical response box from left to right) ☐☐☐☐

Answer Key

1. **a)** 13° **b)** 37° **c)** 36° **2.** **a)** 75.7° **b)** 49.8° **c)** 44.2°

3. **a)** 53° **b)** $x = 41°, y = 49°$ **4.** **a)** 56 **b)** 33° **5.** 130° **6.** 26.9°

7. **a)** 67° **b)** 100° **8.** 120° **9.** 4° **10.** 5.23° **11.** A

12.

3	5		

13.

3	0		

14.

3	3	.	1

Trigonometry Lesson #5:
Determining Angles and Sides in Right Triangles

Overview

In Lesson #3 we learned how to calculate the length of a side in a right triangle using the Pythagorean Theorem and using trigonometry.

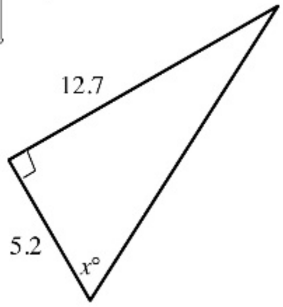
In Lesson #4 we learned how to calculate the measure of an angle in a right triangle using trigonometry.

In this lesson we combine both procedures in solving right triangles.

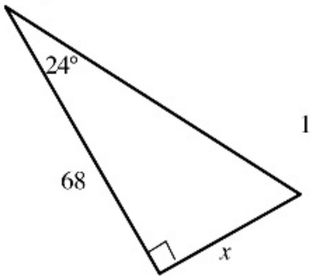
Class Ex. #1

In each case, calculate the measure of x to the nearest whole number.

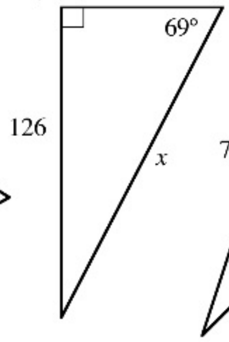
a)



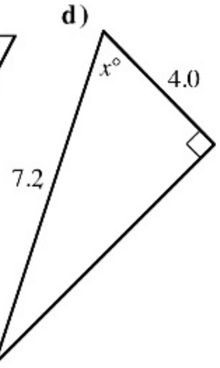
b)



c)



d)



Class Ex. #2

A telephone pole is 11 m high, and is supported by a wire 13 m long fixed to the top of the pole and to the ground.

a) Calculate, to the nearest degree, the angle between the wire and the ground.

b) Calculate the distance, to the nearest 0.1 m, between the point where the wire is fixed to the ground and the foot of the pole using

i) the Pythagorean Theorem **ii)** trigonometry.

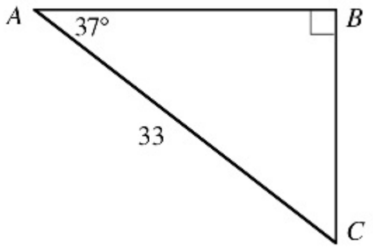
Complete Assignment Questions #1 - #6

Solving Right Triangles

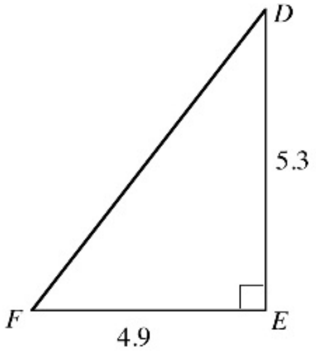
Solving a right triangle means that we have to determine the length of each side and the measure of each angle. In addition to the right angle, we need to know the length of one side and one other piece of information.

Depending on the information given, we can use the angle sum of a triangle, the Pythagorean Theorem and/or trigonometry to solve the triangle.

Solve the given triangle.
Give all answers correct to the nearest whole number.



Solve triangle *DEF*. Give all sides correct to the nearest tenth and all angles correct to the nearest whole number.

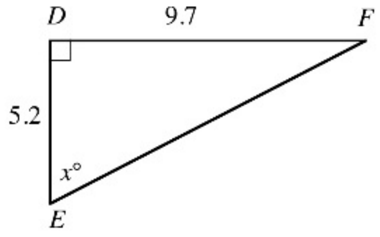


Complete Assignment Questions #7 - #17

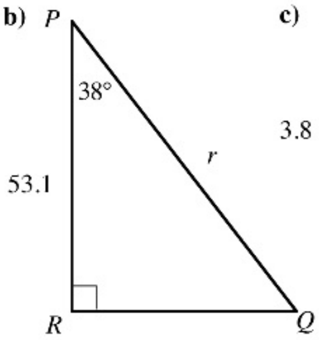
Assignment

1. In each case calculate the indicated measure. Give angles to the nearest degree and sides to the nearest tenth.

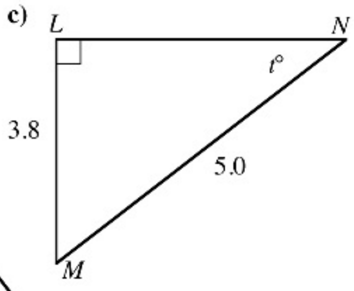
a)



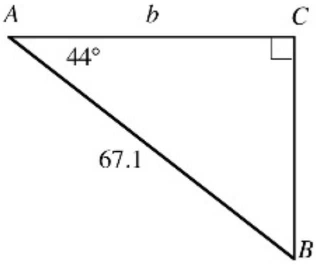
b)



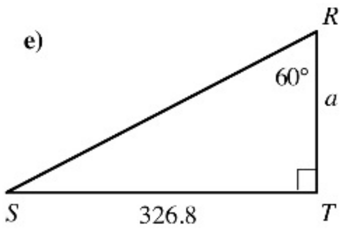
c)



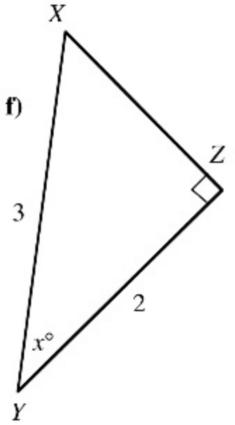
d)



e)

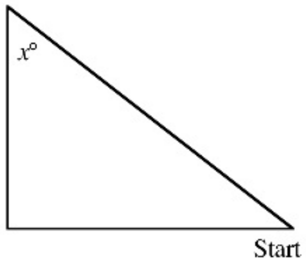

R
60°
a
S
326.8
T

f)



2. A mine shaft which slopes at an angle of 19° to the horizontal is driven into a hillside for 400 m. How much lower, to the nearest metre, is the end of the shaft than the beginning?

3. In a yacht race over a triangular course the instructions are: "Sail due west to a buoy 6 km away, then due north to a buoy 5 km away, and then return directly to the starting point".



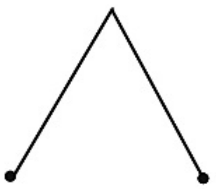
a) Calculate the measure of the angle marked x, to the nearest degree.

b) Calculate, to the nearest tenth of a km, the total distance travelled in the race.

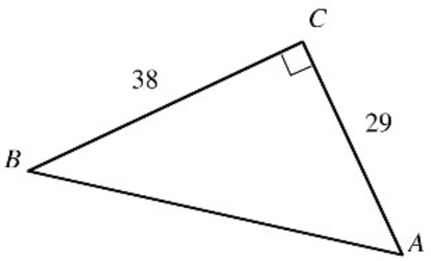
4. The diagonal of a rectangle is 35 cm long and makes an angle of 58° with the shorter side of the rectangle. Determine the length and width of the rectangle to the nearest tenth of a cm.

5. From the top of a building a surveyor determines the angle of depression of a parked car on the street below to be 33°. If the building is 28 m high, calculate the distance from the foot of the building to the parked car. Answer to the nearest metre.

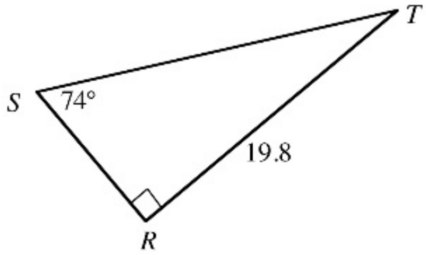
6. A pendulum 65 cm long swings through an angle of 40°. Calculate the distance, to the nearest 0.1 cm, between the two extreme positions of the pendulum bob.



7. Solve triangle *ABC* giving each measure correct to the nearest whole number.



8. Solve triangle *RST*. Give all sides correct to the nearest tenth and all angles correct to the nearest whole number.

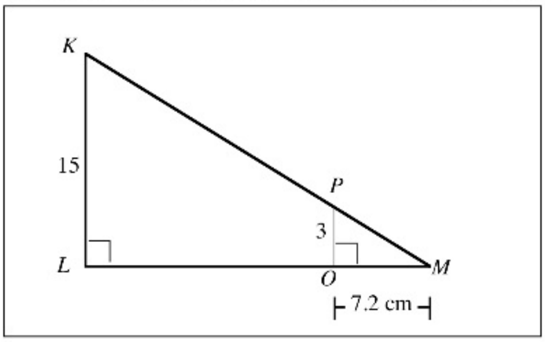


9. Solve triangle *DEF* in which angle $DEF = 90°$, angle $EDF = 50°$, and $DF = 173$ mm. Give all answers to the nearest whole number.

10. Why is it not possible to solve $\triangle PQR$ in which $\angle PQR = 90°$, $\angle PRQ = 67°$ and $\angle QPR = 23°$?

Matching

Use the following information for the matching question



Match each item in List 1 on the left with the equivalent item in List 2 on the right. Each item in List 2 may be used once, more than once, or not at all.

List 1

11. The length of side PM is

12. Angle MPO is

13. Angle LKM is

14. The length of side LM is

15. The length of side LO is

16. Angle PMO is

17. The length of side KP is

List 2

A. 67.4° **B.** 31.2 cm **C.** 36.0 cm

D. 22.6° **E.** 7.8 cm **F.** 15.8 cm

G. 29.3 cm **H.** 28.8 cm **I.** 39.0 cm

Answer Key

1.a) 62° **b)** 67.4 **c)** 49° **d)** 48.3 **e)** 188.7 **f)** 48° **2.** 130 m

3.a) 50° **b)** 18.8 km **4.** length = 29.7 cm, width = 18.5 cm **5.** 43 m **6.** 44.5 cm

7. angle $ABC = 37°$, angle $BAC = 53°$, angle $ACB = 90°$, $AB = 48$, $AC = 29$, $BC = 38$

8. angle $RST = 74°$, angle $SRT = 90°$, angle $RTS = 16°$, $RT = 19.8$, $RS = 5.7$, $ST = 20.6$

9. angle $DEF = 90°$, angle $EDF = 50°$, angle $DFE = 40°$, $DF = 173$ mm, $DE = 111$ mm, $EF = 133$ mm

10. No side length is given. **11.** E **12.** A **13.** A **14.** C **15.** H **16.** D **17.** B

Trigonometry Lesson #6:
Problem Solving Using Trigonometric Ratios

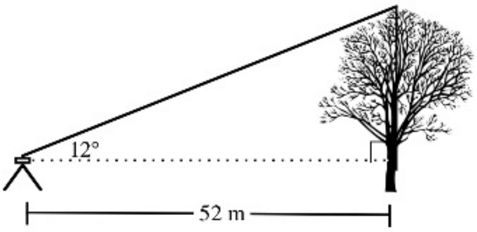
Overview

In this lesson we use our knowledge of trigonometry to solve problems including situations in which more than one calculation is required to determine the solution to a problem.

The illustrations in this lesson are not to scale and are for sketch purposes only.

Class Ex. #1

Karmen is a surveyor who is using a transit to determine the height of a tree. The transit is placed 52 m from the base of the tree and the angle to the top of the tree is measured to be 12°.



a) If the transit stands 1.8 m high, calculate the height of the tree to the nearest tenth of a metre.

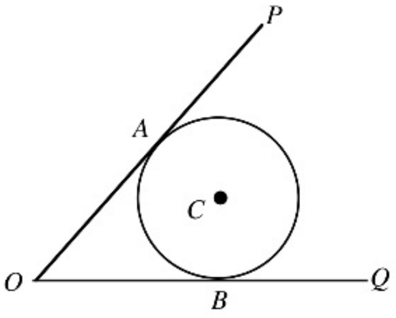
b) Determine the angle the base of the transit makes with the top of the tree. Answer to the nearest degree.

Class Ex. #2

A circle with centre C and radius 20 mm touches the arms of angle POQ at A and B.

Radii CA and CB are perpendicular to OP and OQ respectively and the diagram is symetrical about the line through O and C.

If angle $POQ = 50°$ determine the length of OC to the nearest mm.

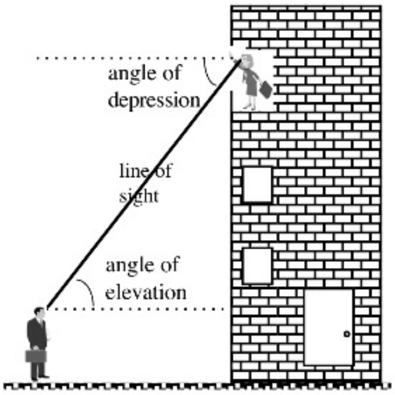


In many problems, the angle of elevation or depression is determined from the person's eye level and not from the ground.

Class Ex. #3

In the diagram the man on the ground is standing 12 feet from the wall.
The angle of elevation from his eye level to the eye level of his wife in the building is 62°.

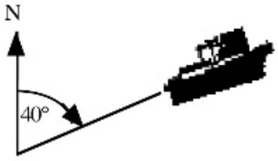
If the man's eye level is 6.0 feet above the ground and his wife's eye level is 5.2 feet from the top of the building, determine the height of the building to the nearest tenth of a foot.



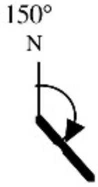
Bearings

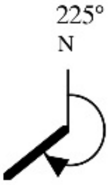
A bearing is an angle measured clockwise from the North direction.

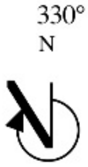
The boat is on a bearing of 40° (often written using three figures as 040°).



Here are three other bearings.



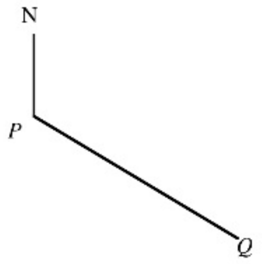




Class Ex. #4

A ship travels 6 km from a port, P, on a bearing of 120° to position Q as shown.

a) The ship changes direction at Q and travels north until it reaches position R which is due east of P.
How far east of P is the ship at position R? Answer to the nearest 0.1 km.



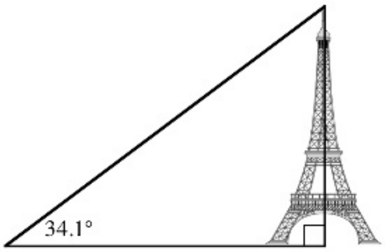
b) The ship continues travelling north for a further 12 km to position S.
Determine the three figure bearing of S from P.

c) If the ship maintains an average speed of 9 km/h throughout the journey, determine to the nearest tenth of an hour, the time taken for the journey.

Complete Assignment Questions #1 - #9

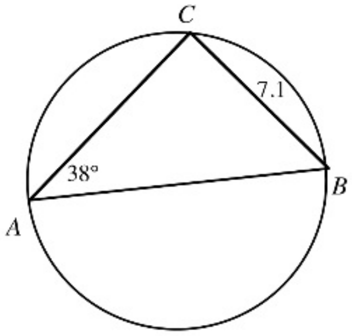
Assignment

1. On a sunny day, the Eiffel Tower standing 1063 ft high, casts a shadow on the ground. A tourist stands at the end of the shadow and measures the angle of elevation of the sun to be 34.1°. Determine, to the nearest foot, the length of the shadow.



2. In the diagram, AB is a diameter of a circle. $\angle CAB = 38°$, $CB = 7.1$ cm, and $\angle ACB$ is a right angle.

Determine the radius of the circle to the nearest tenth of a cm.



3. A boat is 300 m away from the foot of a cliff. The angle of elevation from the boat to the top of the cliff is 16°.

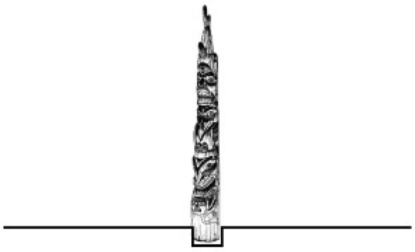
a) Show this information on a diagram and determine the height of the cliff to the nearest metre.

b) If the boat then sails 75 m closer to the cliff, determine, to the nearest degree, the new angle of elevation of the cliff top from the boat.

4. A totem pole is being placed in a 2 metre deep hole in the ground. Two ropes attached to the top of the totem pole are used to pull the totem pole upright. The ropes are anchored into the ground on opposite sides of the pole until the hole is filled.
Each rope is 19 m long and is anchored into the ground 12 m horizontally from the centre of the totem pole.

a) Complete the sketch to illustrate this situation.

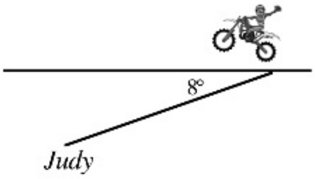
b) Calculate, to the nearest tenth of a degree, the angle of elevation of the top of the totem pole from the point where the rope is anchored into the ground.



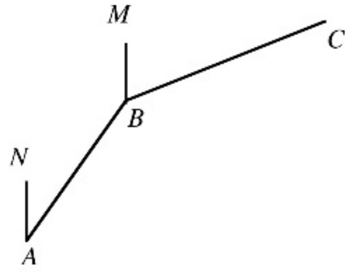
c) Determine, to the nearest 0.1 m, the length of the totem pole before it was placed in the ground.

5. Judy is 60 m south of a main east-west highway. She looks to her right and sees a motor bike rider approaching at a steady speed along the highway. The diagram shows their positions relative to each other.

Twenty seconds later the motor bike rider is directly in front of Judy.
Calculate the speed of the motor bike, to the nearest km/h.



6. A ship sails from a port A on a bearing of 030° for 22 miles to a marker B where it alters course and sails on a bearing of 075° for 27 miles to its destination C. AN and BM represent north lines.



a) On the diagram mark the sizes of $\angle BAN$ and $\angle CBM$.

b) Determine, to the nearest 0.1 mile, how far C is east of A.

c) Determine, to the nearest 0.1 mile, how far C is north of A.

d) Determine, to the nearest 0.1 mile, the direct distance from A to C.

e) Determine the three figure bearing of C from A.

Use the following information to answer questions #7 and #8.

A plane starts from a point P and flies 240 km on a bearing of 120° to Q.
At Q the plane makes a 90° turn and flies 100 km on a bearing of 210° to R.
At R the plane again changes direction and flies back to P.

Multiple Choice

7. The direct distance, in km, from R to P is

- **A.** 140
- **B.** 218
- **C.** 260
- **D.** 340

8. The three figure bearing the pilot uses to fly from R to P is

- **A.** 023°
- **B.** 037°
- **C.** 323°
- **D.** 337°

Numerical Response

9. A set of stairs has a vertical rise of 16 cm for every 27 cm horizontal run.
The angle, to the nearest tenth of a degree, between the stairs and the floor is _______.

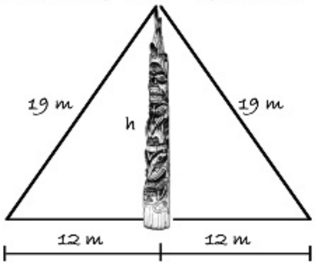
(Record your answer in the numerical response box from left to right) ☐☐☐☐

Answer Key

1. 1570 ft **2.** 5.8 cm

3. **a)** 86 m **b)** 21°

4. **a)** see diagram below **b)** 50.8° **c)** 16.7 m



5. 77 km/h

6. **a)** angle $BAN = 30°$, angle $CBM = 75°$. **b)** 37.1 miles
c) 26.0 miles **d)** 45.3 miles **e)** 055°

7. C **8.** C **9.** 3 | 0 | . | 7

Trigonometry Lesson #7:
More Problem Solving Using Trigonometric Ratios

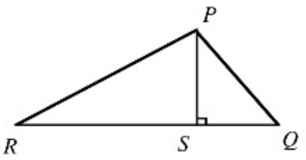
Overview

In this lesson we deal with problems involving two right triangles. In many cases we can determine the value of a quantity from one right triangle and then use that value in a second right triangle to determine the solution to a problem. When this situation arises, do not round intermediate answers. Only the final solution should be rounded. **The illustrations in this lesson are not to scale and are for sketch purposes only.**

Problems in Two Dimensions

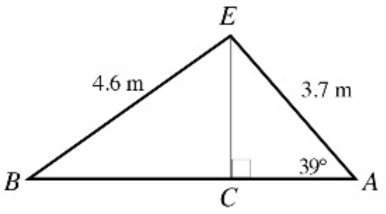
Class Ex. #1

In the diagram $QS = 32$ mm, angle $PQS = 50°$ and angle $RPS = 61°$.
Use this information to determine the length of QR to the nearest mm.



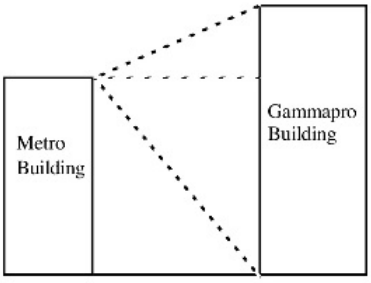
Class Ex. #2

Calculate the measure of $\angle BEC$ to the nearest degree.



Class Ex. #3

In January 2003, the tallest building in Rockyville was the Metro Building. Recently, a developer was commissioned by the Gammapro Oil Company to build a taller building next to the Metro Building. From the top of the Metro Building, the angle of elevation of the top of the Gammapro Building is 24° and the angle of depression to the foot of the Gammapro Building is 56°. If the buildings are 45 m apart, determine the height of each building to the nearest metre.



Complete Assignment Questions #1 - #6

Problems in Three Dimensions

As an aid to understanding a problem in three dimensions, we sketch a two dimensional representation of the triangle we are working with.

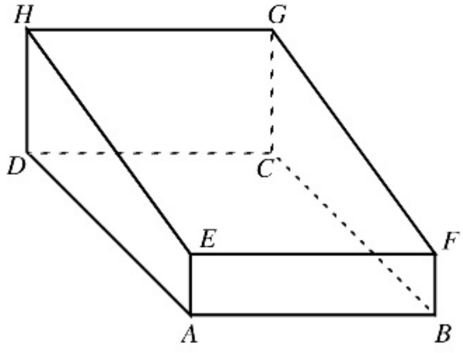
Class Ex. #4

The solid in the diagram was formed from a rectangular prism by removing a wedge.

$HD = GC = 5$ cm. $EA = FB = 2$ cm.
$AD = 6$ cm and $AB = 8$ cm.

a) Name four rectangles in the diagram.

b) Calculate the measure of $\angle HEA$ to the nearest degree.



c) Calculate the measure of angle HFE to the nearest degree.

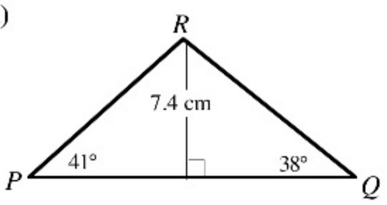
d) Calculate the measure of angle HBD to the nearest degree.

Complete Assignment Questions #7 - #10

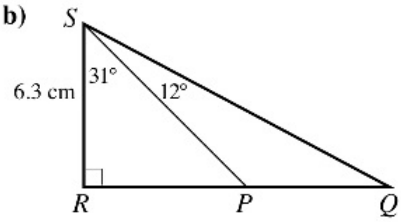
Assignment

1. Determine the length of PQ, to the nearest 0.1 cm.

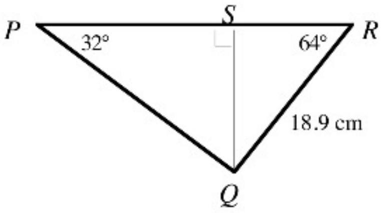
a)


R
7.4 cm
41°
38°
P
Q

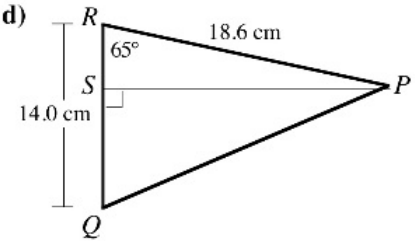
b)


S
6.3 cm
31°
12°
R
P
Q

c)

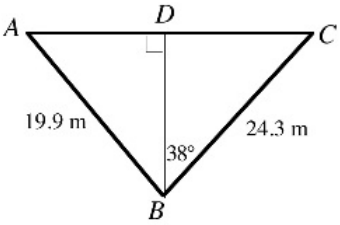

P
S
R
32°
64°
18.9 cm
Q

d)

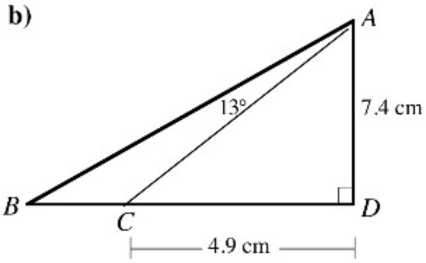

R
18.6 cm
65°
S
P
14.0 cm
Q

2. Determine the measure of angle ABC, to the nearest degree.

a)



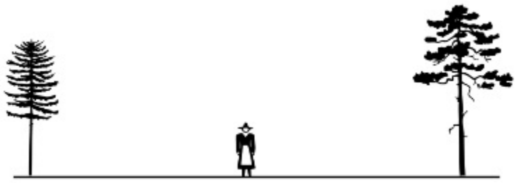
b)



3. Two trees in a park are 14.80 metres apart. An observer, whose eyeline is 1.80 m above the ground, is standing halfway between the trees. The angles of elevation of the tops of the trees from the observer's eyeline are 17° and 23°.

a) Show this information on the diagram.

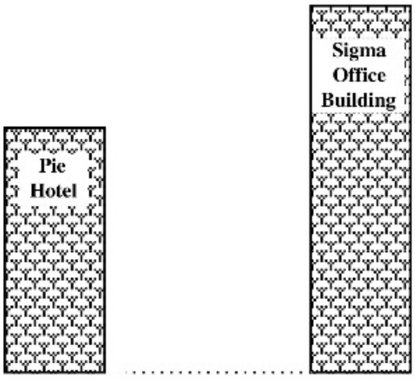
b) Determine the height of each tree to the nearest 0.01 m.



4. The angle of elevation from the top of the Pie Hotel to the top of the Sigma Office Building is 17°. The angle of depression from the top of the Pie Hotel to the foot of the Sigma Office Building is 60°. The height of the Pie Hotel is 150 m.

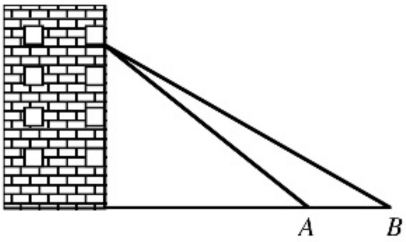
a) Complete the sketch to illustrate this situation.

b) Determine the height of the Sigma Office Building to the nearest tenth of a metre.



5. From the top of a cliff 110 m high, an observer sees two boats, one directly behind the other, heading for shore. The angle of depression from the observer to the boat furthest from the observer is 48° and the angle of depression to the nearest boat is 57°. Calculate the distance between the boats, to the nearest metre.

6. The diagram shows two marathon runners, A and B, heading towards the finish line of a race. From an apartment window 80 metres above the ground and 20 metres behind the finish line, Tony measures the angle of depression of the runners to be 28° and 24° respectively.



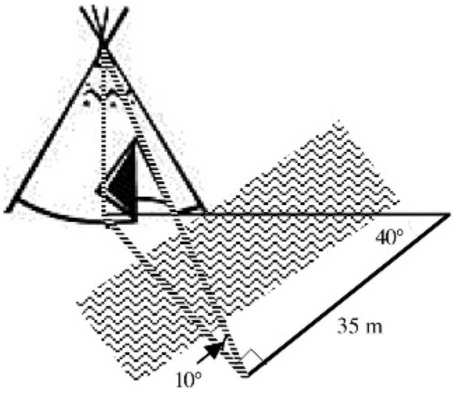
a) Calculate the distance between the runners to the nearest metre.

b) A is travelling at a constant speed of 4.5 m/s, while B is travelling at a constant speed of 5.1 m/s. Which runner will finish the race first?

Use the following information to answer the next question.

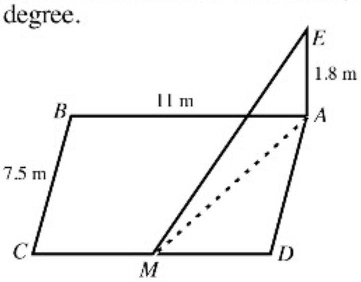
The Indigenous Plains people of what is now Canada often lived in tipis until the mid-1800s. Strategically, these tipis were often located near rivers or other water sources.

In order to find the height of a tipi on the opposite bank of a river, Dancing Eagle made the measurements shown in the diagram.

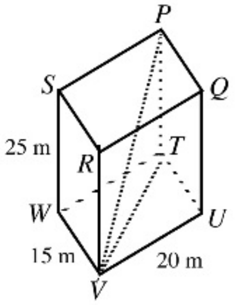


7. Calculate the height of the tipi to the nearest tenth of a metre.

8. In the diagram, $ABCD$ represents a rectangular sandbox where kindergarten children play. A teacher stands at the corner of the area to supervise the children. At a certain time of day, the tip of the shadow cast by the teacher on the play area is exactly at M, the midpoint of CD. If the teacher is 1.8 m tall, calculate the measure of angle EMA, to the nearest degree.



9. A rectangular prism has dimensions 15 m, 20 m, and 25 m as shown. The angle between diagonal VP and the plane $TUVW$ is defined to be angle PVT. The tangent ratio of this angle is



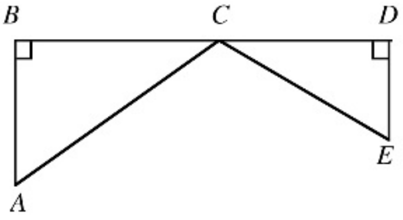
A. $\frac{3}{4}$

B. 1

C. $\frac{5}{4}$

D. $\frac{5}{3}$

Numerical Response **10.** In the diagram, $AB = 28$ m, $DE = 21$ m, $BD = 71$ m, and angle $BAC = 55°$. To the nearest degree, the measure of angle CED is _______°.



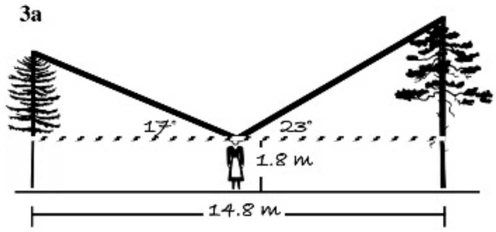
(Record your answer in the numerical response box from left to right)

Answer Key

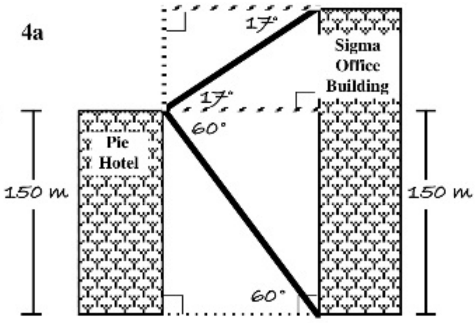
1. a) 18.0 cm **b)** 2.1 cm **c)** 32.1 cm **d)** 17.9 cm

2. a) 54° **b)** 43°
3. a) See the diagram below. **b)** 4.06 m and 4.94 m **4. a)** See below. **b)** 176.5 m

3a


17°
23°
1.8 m
14.8 m

4a


17°
Sigma Office Building
17°
60°
Pie Hotel
150 m
150 m
60°

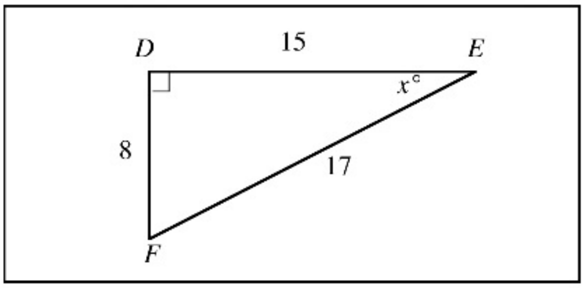
5. 28 m **6. a)** 29 m **b)** *A* takes 29 sec, *B* takes 31 sec, so *A* finishes first.

7. 5.2 m **8.** 11° **9.** B **10.**

5	6		

Trigonometry Lesson #8:
Practice Test

Use the following information to answer the first three questions.



1. The value of $\cos F$ is

A. $\frac{8}{15}$

B. $\frac{8}{17}$

C. $\frac{17}{8}$

D. 62

2. The value of x is

A. $\frac{15}{17}$

B. $\frac{8}{17}$

C. 28

D. 62

3. Which statement is false?

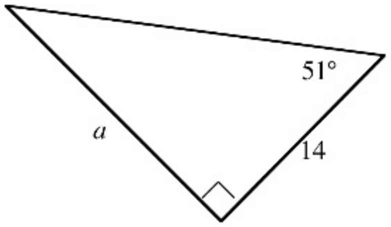
A. $\cos E = \sin F$

B. $\sin E = \cos F$

C. $\tan E = \tan F$

D. $\tan F = \frac{\sin F}{\cos F}$

4. The value of a is



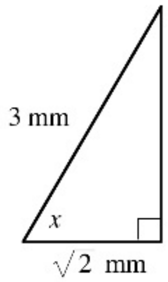
A. $14 \tan 51^\circ$

B. $\dfrac{14}{\tan 51^\circ}$

C. $14 \cos 51^\circ$

D. $\dfrac{14}{\cos 51^\circ}$

5. The value of x, to the nearest tenth of a degree, is



A. 35.0°

B. 61.9°

C. 72.3°

D. 29.3°

6. A guy wire to a pole makes an angle of 71.4° with the level ground and is 13.8 feet from the pole at ground level. How far above the ground is the guy wire attached to the pole?

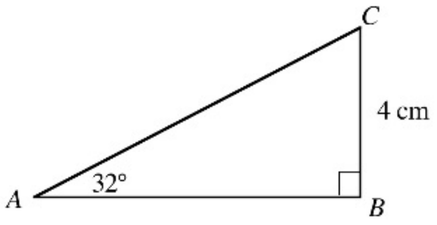
A. 4.6 feet

B. 13.1 feet

C. 14.6 feet

D. 41.0 feet

7. The value of length AC, to the nearest tenth, is



A. 0.1

B. 2.1

C. 7.2

D. 7.5

8. A corner flag in a World Cup soccer match is 5 feet high. At game time, the flag casts a shadow which is 3.2 feet long. To the nearest tenth of a degree, the angle of elevation of the sun is

A. 57.4°

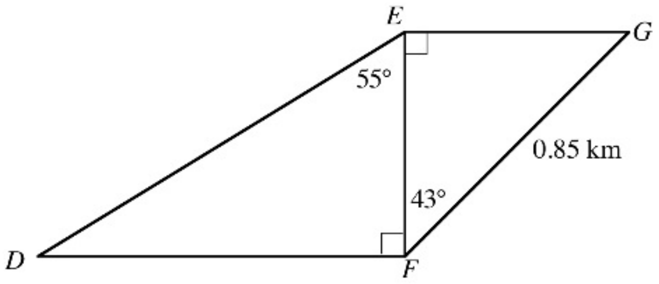
B. 50.2°

C. 39.8°

D. 32.6°

Use the following information to answer the next three questions.

The diagram shows how a horse breeder has used fencing to divide a parcel of land into two triangular paddocks for his horses. Each of the line segments represents fencing.



9. The length of the fencing represented by DF is

A. 0.89 km

B. 0.79 km

C. 0.62 km

D. 0.58 km

10. The value of cos G is

A. 0.68

B. 0.72

C. 0.80

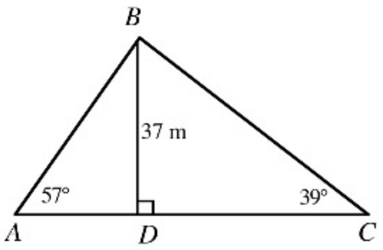
D. impossible to determine from the given information.

Numerical Response **1.** The total area of the two triangular paddocks, to the nearest $0.1\ \text{km}^2$, is ______.

(Record your answer in the numerical response box from left to right) ☐☐☐☐

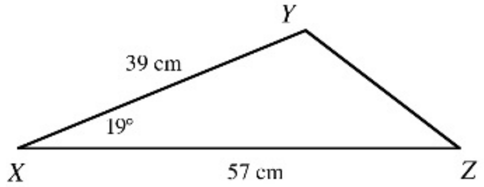
11. The length of AC, to the nearest metre, is

A. 54 m
B. 70 m
C. 87 m
D. 103 m



Numerical Response **2.** Determine the area of the following triangle to the nearest square centimetre.

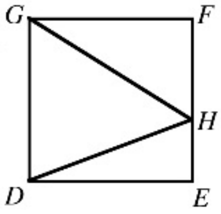
(Record your answer in the numerical response box from left to right)



12. $\triangle ABC$ is isosceles with $AB = CB$. The area of the triangle is 30 cm^2 and $AC =$ 12 cm. The measure of $\angle ABC$, to the nearest degree, is

A. 100°
B. 50°
C. 45°
D. 23°

Numerical Response **3.** In the diagram, $DEFG$ is a square of side 12 cm and $GH = 15$ cm. To the nearest degree, the difference between the measures of $\angle HGF$ and $\angle HDE$ is _____ .

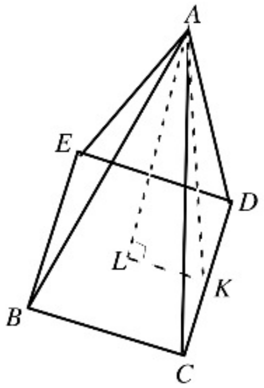


(Record your answer in the numerical response box from left to right)

☐☐☐☐

Numerical Response **4.** $ABCDE$ is a square based pyramid with vertical height 14 cm. K is the midpoint of CD and angle $AKL = 70°$.

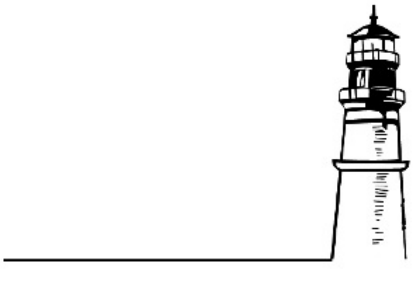
To the nearest degree, the measure of angle ACK is _____.



(Record your answer in the numerical response box from left to right)

☐☐☐☐

13. From a lighthouse window 30.0 m above sea level, the angles of depression of two buoys which are in a direct line from the lighthouse are 15° and 26° respectively. To the nearest tenth of a metre, the distance between the buoys is _____.



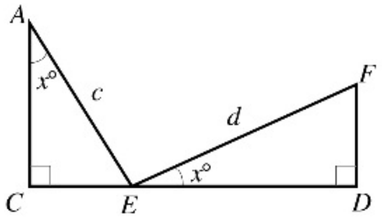
A. 173.5

B. 165.5

C. 50.5

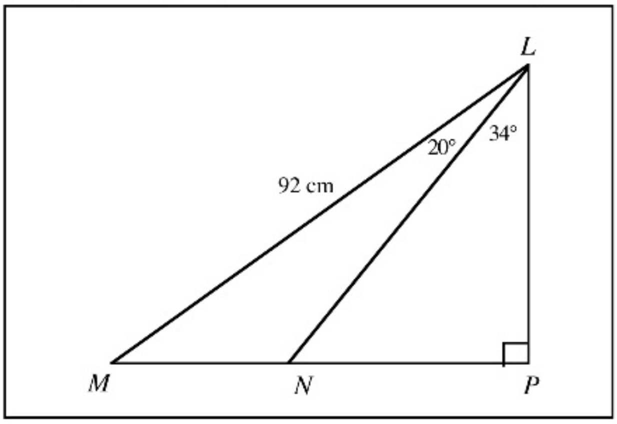
D. 24.4

Multiple Choice **14.** From the diagram, the length of CD is



A. $c \sin x° + d \cos x°$
B. $c \cos x° + d \sin x°$
C. $(c + d) \cos x°$
D. $(c + d) \sin x°$

Use the following diagram to answer the next two questions.



15. The length of LP, to the nearest centimetre, is

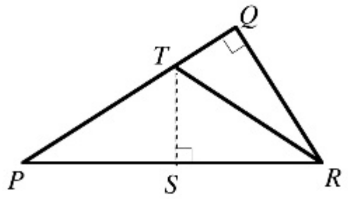
A. 51 cm
B. 54 cm
C. 74 cm
D. 76 cm

Numerical Response **5.** The length of MN, to the nearest centimetre, is

(Record your answer in the numerical response box from left to right) ☐☐☐☐

Written Response - 5 marks

1. In the diagram, $PQ = 16$ m, $QR = 10$ m, and $PT = TR$.



- Make a sketch of triangle PQR only and determine the exact length of PR.

- Explain why you cannot use only triangle QRT to determine the measure of $\angle QRT$.

- Calculate the measure of $\angle QPR$ and use it to determine the measure of $\angle QRT$ to the nearest degree.

- Calculate the length of PT to the nearest tenth of a metre.

Answer Key

1. B **2.** C **3.** C **4.** A **5.** B **6.** D **7.** D **8.** A

9. A **10.** A **11.** B **12.** A **13.** C **14.** A **15.** B

Numerical Response

1.

0	.	5	

2.

3	6	2	

3.

2	3		

4.

7	1		

5.

3	8		

Written Response

1.
- 20 m
- In triangle QRT, only two pieces of information are given, $\angle TQR$ and side QR. This is not enough information to calculate $\angle QRT$.
- $\angle QPR = 32°$, $\angle QRT = 26°$.
- 11.1 metres.

Polynomial Operations Lesson #1:
Review and Preview

Overview of Unit

In this unit we study algebraic expressions called polynomials. We review the classification of polynomials, addition and subtraction of polynomials, and multiplication by a monomial. We introduce the product of two binomials (concretely, pictorially, and symbolically) and extend this to the multiplication of polynomials. We also solve problems involving polynomial expressions.

Review

In algebra, a letter that represents one or more numbers is called a **variable**.

Expressions like $2a - b + 4$ or $\frac{5}{x} + 3$ are called **algebraic expressions**.

Certain algebraic expressions are called **polynomials** as explained below.

A **monomial** is a number or a variable or the product of numbers and variables. (Note that the exponent of any variable must be a positive integer in the numerator of the monomial.)

eg. $6,\ x,\ 6x,\ -\frac{1}{2}xy,\ 0.25x^3,\ abc,\ 2p^4q^2$ etc. are all monomials.

The number that multiplies the variable is called the **numerical coefficient**.

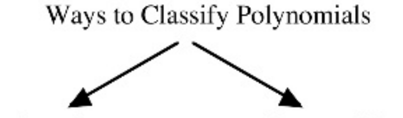
A **polynomial** is a monomial or a sum or difference of monomials.

- $6,\ x,\ 6 + x,\ 2y + 7z,\ x^2 - 5x - 9$ etc. are all examples of polynomials.

Explain why $\frac{5}{x} + 3$ is not a polynomial.

Classifying Polynomials

Polynomials may be classified in two different ways as shown below.


Ways to Classify Polynomials
number of terms in the polynomial
degree of the polynomial

Continued on the next two pages

Classifying Polynomials by The Number of Terms

A polynomial may be classified by the number of **terms** it contains.

- A term can be a number, a variable, or the product of a number and variable(s).
- When there is more than one term, the terms are connected by + or – signs.

A polynomial with 1 term is a **monomial** (eg. $4x$).

A polynomial with 2 terms is a **binomial** (eg. $x + 4$).

A polynomial with 3 terms is a **trinomial** (eg. $x^2 + x + 4$).

A polynomial with 4 or more terms is simply called a polynomial when classifying by the number of terms.

Consider the following algebraic expressions. In each case:

- State whether the expression represents a polynomial or not.
- If the expression does not represent a polynomial, explain why.
- If the expression does represent a polynomial, state whether the polynomial is a monomial, a binomial, or a trinomial.

a) $\frac{1}{4}xy - 10$ **b)** $3pq^{\frac{1}{2}}$ **c)** $\sqrt{7}\,x^4 - x^3 + 1$

d) $3x^2 + 9x - 4x^{0.2}$ **e)** $\frac{7}{a}$

Complete the following table.

Polynomial Expression	# Variables	# Terms	Classification by # Terms
$4x + 3yz$			
$2a - 4b + 7c$			
$x^2 + 3x + 4$			
$\sqrt{2}\,x$			
$2x^3 + 3x^2y + 3y^2 - 8$			

Classifying Polynomials by The Degree of The Polynomial

Polynomials can also be classified according to **degree** of the polynomial .

The **degree of a monomial** is the sum of the exponents of its variable(s).

eg. $2x^5$ has degree ____ $\quad -\frac{2}{3}ab^3c^2$ has degree ___

Recall from the lesson "Whole Number Exponents" on page 20 that a variable raised to the power zero is equal to 1.

For example, the monomial $7x^0$ can be written as $7(1)$ or 7.
Therefore the degree of a monomial with no variable present is 0.

The **degree of a polynomial** is given by the **term** or **monomial** with highest degree.

eg. $3x^2y^2 - 2x^4 + xy^4 - 2$ has degree ____

If a polynomial has a term with no variable present, this term is called a **constant term**.
In the polynomial $3x^2y^2 - 2x^4 + xy^4 - 2$, the constant term is ____.

State the degree of the following polynomials.

a) $3x^2 - 10x^4 - 9$ $\quad$ **b)** $7p^2q^3 - 8p^7q - 2q^7$

Give an example of
a) a binomial of degree 1 in one variable.
b) a trinomial in two variables with a constant term.
c) a monomial of degree 6 with a (numerical) coefficient of 9.
d) a binomial of degree 8 with each term containing two variables.

The following list classifies polynomials by using the degree of the polynomial.

- A **Constant** Polynomial has a degree of 0 eg. 8
- A **Linear** Polynomial has a degree of 1 eg. $x + 3$
- A **Quadratic** Polynomial has a degree of 2 eg. $x^2 - 2x + 5$
- A **Cubic** Polynomial has a degree of 3 eg. $x^3 - 8x^2 + x + 1$
- A **Quartic** Polynomial has a degree of 4 eg. $x^4 - 61x + 9$
- A **Quintic** Polynomial has a degree of 5 eg. $x^5 - 17$

There are names for polynomials of higher degree that are beyond the scope of this course.

Complete the following table.

Polynomial Expression	Degree	Classification by Degree	Constant Term
$4xy - 6$			
$9y^2 - 8y^3$			

Polynomials in a Single Variable

Polynomials in a single variable are usually arranged in **ascending** or **descending** order of the powers of the variable.

The **leading coefficient** of a polynomial in a single variable is the coefficient of the term with highest power of the variable.

Consider the polynomial expression $2x - 4x^3 - 7 + \frac{6x^2}{5}$.

a) Write the polynomial in descending powers of x.

b) Write the polynomial in ascending powers of x.

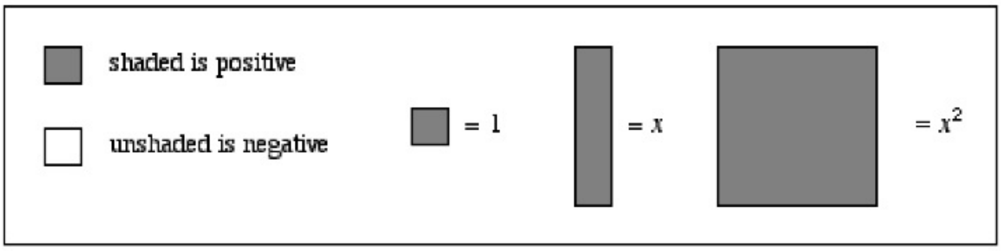
c) State the leading coefficient and the constant term.

d) State the numerical coefficient of the term in x^2 .

Complete Assignment Questions #1 - #9

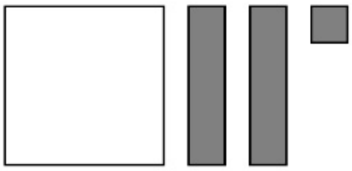
Representing Polynomials Using Algebra Tiles

The following legend will be used for algebra tiles in this workbook.



State the polynomial expression which describes each diagram.

a)



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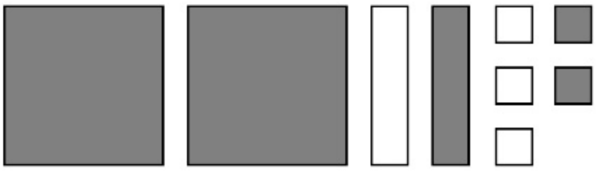


Addition and Subtraction Using Algebra Tiles

Class Ex. #9

Use algebra tiles to determine the result of the addition $(2x^2 + 1) + (x^2 - 2x - 3)$.

Note

Subtracting a polynomial is equivalent to adding the inverse polynomial,

eg. $(4x + 3) - (2x - 5)$ is equivalent to $(4x + 3) + (-2x + 5)$

Class Ex. #10

Use algebra tiles to determine the result of the subtraction $(-x^2 + 3x - 2) - (2x^2 - x - 1)$.

Addition and Subtraction of Polynomial Expressions

Like terms are terms with the same variable raised to the same exponent.

eg. $3a$, $7a$ and a are like terms. $2x^3$, $\frac{1}{5}x^3$ and $-4x^3$ are like terms.

Unlike terms have different variables or the same variable raised to different exponents.

eg. $2x^3$, $\frac{1}{5}x^2$ and $-4x$ are unlike terms. $4x$ and $4y$ are unlike terms.

Like terms can be added or subtracted to produce a single term.

Class Ex. #11

Simplify the following polynomials by collecting like terms.

a) $(3a - 4b + c) + (3b - 5c - 3a)$

b)
$$\begin{array}{r} (4x^2 - 9x + 6) \\ -\ \underline{(2x^2 - 3x - 1)} \end{array}$$

Class Ex. #12

Simplify

a) $4x - 2x^2 + 3 - 6x^2 + 5 - x$ **b)** $a^2b - ab^2 + 4a^3b - 7ab^2 + 5a^2b$

Complete Assignment Questions #10 - #20

Assignment

1. Identify as a monomial, a binomial, or a trinomial.

a) $x + 1$ **b)** $3x^3$ **c)** $2x^2 + 2x + 2$

2. State the degree of each monomial.

a) 5a **b)** $3x^3y$ **c)** 10 **d)** $-2a^2b^2$ **e)** $3xy^2z^3$

3. State whether or not the following are polynomial expressions.
If they are not polynomial expressions, explain why not.

a) $\frac{1}{2}x^2 - 3x$ **b)** $8m^{-2}$ **c)** $\sqrt{6}$

d) $\frac{7}{x^3}$ **e)** $\frac{8x^2}{3}$ **f)** $x^4 + 3x^{1.5}$

4. Complete the following table.

Polynomial Expression	# Variables	# Terms	Classification by Number of Terms	Degree
$2y^3 + y^4 - y + 13$				
$9ab - 4x + 11c$				
25				
$\frac{3}{5}x^3yz^5 + 3x^2yz^4$				

5. Complete the following table for the single variable polynomials.

Polynomial Expression	**Leading Coefficient**	**Constant Term**	**Degree**	**Classification by Degree**
$y^4 - y + 13$				
$0.2t^3 - 0.3t^2 + 0.4t - 0.5$				
$\sqrt{7} - x^5$				
$\pi x^2 - 7 - 3x$				
$-\frac{1}{10}$				
$9x + 12$				

6. Give an example of

a) a trinomial of degree 2 in one variable.

b) a binomial in four variables with a constant term of 6.

c) a monomial of degree 3 in two variables with a negative numerical coefficient.

d) a monomial with a degree of 0.

7. Arrange the following in descending powers of the variable.

a) $6w^2 - 9w + 5 + 2w^3$

b) $\frac{1}{4}a^2 - \frac{2}{3}a^3 - 1 - a$

c) $z - 3 - 4z^6 + z^3$

8. Arrange the following in ascending powers of the variable.

a) $6w^2 - 9w + 5 - 2w^3$

b) $3x^2 - 4x^5 - 2x^4 - 4x^3 + 9x - 7$

c) $8x^3 - 8x + 8$

9. State which of the following are true and which are false.

a) -54 is a polynomial.

b) The degree of the polynomial $3x^3y^3$ is 9.

c) The numerical coefficient of $\frac{6x}{5}$ is 6.

d) A polynomial may have 1000 terms.

e) $\frac{2}{a^3} - 1$ is a binomial.

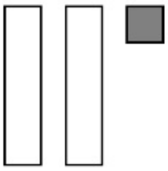
f) The degree of the polynomial 0 is 0.

g) The polynomial $x^3 + 2x^2 + 3x + 4$ is written in ascending powers of x.

h) The polynomials $3x^2 - 9x + 1$ and $1 - 9x + 3x^2$ are equivalent.

10. State the polynomial expression which describes each diagram.

a)



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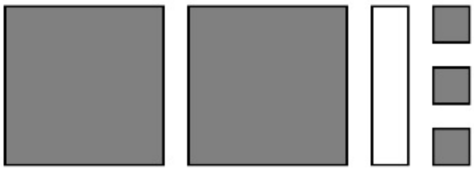
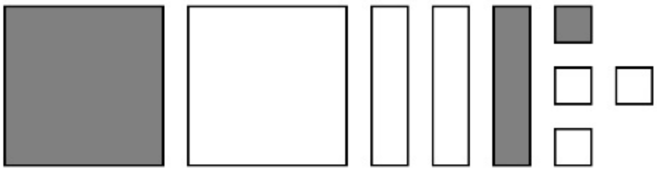


c)



11. Use algebra tiles to determine the result of the addition of :

a) $(x^2 - x - 3) + (x^2 - 2x - 3)$

b) $(3x + 1) + (2x^2 - 3x - 2)$

12. Use algebra tiles to determine the result of the subtraction of:

a) $(x^2 - 3) - (2x^2 + 4x + 1)$ **b)** $(2 - x - x^2) - (1 - 2x + x^2)$

13. Simplify

a) $6p - 7q - 3q - 2p$ **b)** $5x - 3x^2 + 2x - 8x^2$ **c)** $\frac{1}{2}x - 3 + \frac{3}{2}x + 18$

d) $4a^3 + 7a - 2a^2 - 6a - 4a^3 - a^2$ **e)** $3 - 2x + 7y + 4y - 2x + 8z - 9$

14. Simplify the following polynomial expressions by collecting like terms.

a) $(5a - 9b - 2c) + (c - 7b - 3a)$ **b)** $(3 - a - 2a^2) + (9 - 4a + 5a^2)$

c) $(2x^2 + 5x - 1) + (3x - 6 - 6x^2) + (4 - 5x + x^2)$ **d)** $(4a - 6b) - (5a - 2b)$

e) $\begin{array}{r} (5x^2 - 8x + 3) \\ -\ \underline{(3x^2 - 3x - 1)} \end{array}$ **f)** $\begin{array}{r} (7x^2 + 2x - 1) \\ -\ \underline{(-5x^2 - 3x - 1)} \end{array}$ **g)** $\begin{array}{r} (-4x^2 + 2x - 6) \\ -\ \underline{(3x + 6 - 2x^2)} \end{array}$

15. **a)** Subtract $3x^2 - 2x + 7$ from $6x^2 - 5x - 2$.

b) Subtract the sum of $2x^3 - 7x^2 - 6x + 1$ and $8 - 3x + 5x^2 - 4x^3$ from $2x^3 - 7x + 9$.

16. A triangle has a perimeter of $(6m + n)$ cm. One side measures $(2m - 3n)$ cm and another side measures $(3n + 2m)$ cm.

a) Write and simplify an expression for the length of the third side of the triangle.

b) Determine the measure of each side when $m = 4$ and $n = -1$.

Multiple Choice

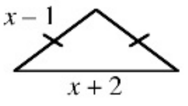
17. Which of the following is a polynomial expression of degree 4?

A. $4x^4 - 4x^7$
B. $5x^4 - 3x^3 + 2x^{-2} + x - 1$
C. $\dfrac{4x^4 - 3x}{x}$
D. $9 + 3x - \frac{1}{3}x^2 - x^3 + \frac{2}{5}x^4$

18. Which of the following polynomial expressions, when simplified, is equal to $5x$?

A. $(3x^2 - 3x) - (2x + 3x^2)$
B. $5x - (2x^2 - 2x) + (2x^2 + 2x)$
C. $8 + (4 - 2x) - (12 - 7x)$
D. $(2x^2 - 2x + 6) - (2x^2 - 2x) + (9x - 6)$

19. The perimeter of the isosceles triangle shown can be represented by



A. a monomial
B. a binomial
C. a trinomial
D. none of the above

Numerical Response **20.** If the polynomial $4 - 7x + 2x^2 - 5x^3$ has degree a, leading coefficient b, and constant term c, then the value of $3a - 2b - c$ is ______.

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** binomial **b)** monomial **c)** trinomial **2.** **a)** 1 **b)** 4 **c)** 0 **d)** 4 **e)** 6

3. **a)** yes **b)** no, negative exponent **c)** yes

d) no, $\frac{7}{x^3} = 7x^{-3}$, which is a negative exponent. **e)** yes

f) no, the exponent 1.5 is not a positive integer.

4.

Polynomial expression	*# variables*	*# terms*	*Classification by Number of Terms*	*degree*
$2y^3 + y^4 - y + 13$	1	4	polynomial	4
$9ab - 4x + 11c$	4	3	trinomial	2
25	0	1	monomial	0
$\frac{3}{5}x^3yz^5 + 3x^2yz^4$	3	2	binomial	9

5.

Polynomial expression	*leading coefficient*	*constant term*	*degree*	*Classification by Degree*
$y^4 - y + 13$	1	13	4	Quartic
$0.2t^3 - 0.3t^2 + 0.4t - 0.5$	0.2	-0.5	3	Cubic
$\sqrt{7} - x^5$	-1	$\sqrt{7}$	5	Quintic
$\pi x^2 - 7 - 3x$	π	-7	2	Quadratic
$-\frac{1}{10}$	$-\frac{1}{10}$	0	0	Constant
$9x + 12$	9	12	1	Linear

6. answers may vary **a)** $x^2 - x + 30$ **b)** $abcd + 6$ **c)** $-2xy^2$ **d)** 10

7. **a)** $2w^3 + 6w^2 - 9w + 5$ **b)** $-\frac{2}{3}a^3 + \frac{1}{4}a^2 - a - 1$ **c)** $-4z^6 + z^3 + z - 3$

8. **a)** $5 - 9w + 6w^2 - 2w^3$ **b)** $-7 + 9x + 3x^2 - 4x^3 - 2x^4 - 4x^5$ **c)** $8 - 8x + 8x^3$

9. **a)** true **b)** false **c)** false **d)** true **e)** false **f)** true **g)** false **h)** true

10. **a)** $-2x + 1$ **b)** $2x^2 - x + 3$ **c)** $-x - 2$ **11.** **a)** $2x^2 - 3x - 6$ **b)** $2x^2 - 1$

12. **a)** $-x^2 - 4x - 4$ **b)** $1 + x - 2x^2$

13. **a)** $4p - 10q$ **b)** $-11x^2 + 7x$ **c)** $2x + 15$ **d)** $-3a^2 + a$ **e)** $-4x + 11y + 8z - 6$

14. **a)** $2a - 16b - c$ **b)** $3a^2 - 5a + 12$ **c)** $-3x^2 + 3x - 3$ **d)** $-a - 4b$
e) $2x^2 - 5x + 4$ **f)** $12x^2 + 5x$ **g)** $-2x^2 - x - 12$

15. **a)** $3x^2 - 3x - 9$ **b)** $4x^3 + 2x^2 + 2x$

16. **a)** $(2m + n)$ cm **b)** 11 cm, 5cm, and 7 cm

17. D **18.** C **19.** A **20.**

1	5		

Polynomial Operations Lesson #2:
Multiplying a Polynomial by a Monomial

Using Algebra Tiles

In previous math courses, we learned how to multiply
i) two monomials, and **ii)** a monomial and a binomial or trinomial.

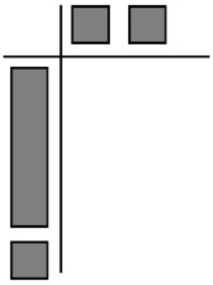
We can use algebra tiles to illustrate the process of multiplying a monomial by a polynomial.

Shaded tiles represent positive quantities and unshaded tiles represent negative quantities.

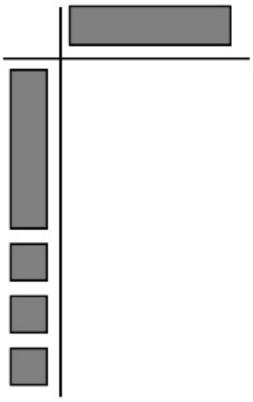
Class Ex. #1

Complete the diagram to determine the product.

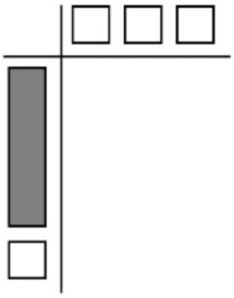
a) $2(x + 1) =$



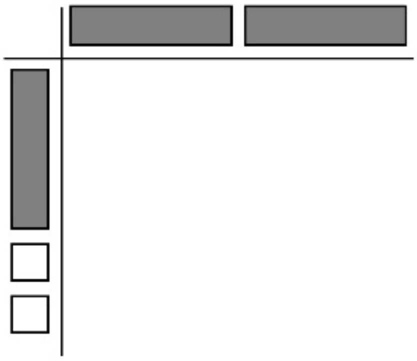
b) $x(x + 3) =$



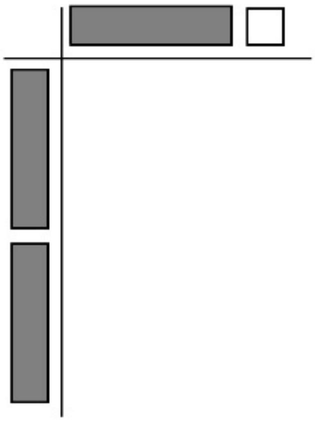
c) $-3(x - 1) =$



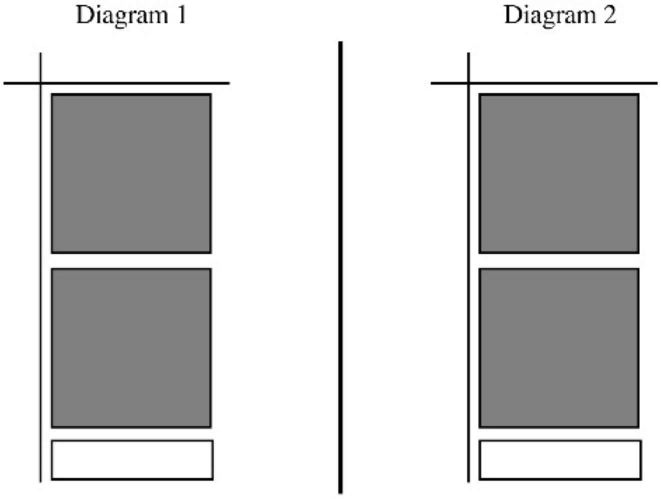
d) $2x(x - 2) =$



e) $(x - 1)(2x) =$



Each diagram below illustrates the result of the product of a monomial and a binomial.



a) State the polynomial represented in each of the diagrams.

b) Complete the left side and the top of Diagram 1 and write the polynomial product.

c) Complete Diagram 2 to illustrate and write a different polynomial product than in b).

d) Write each product as a sum or difference of terms.

e) Verify the polynomial products in d) when $x = 3$.

Complete Assignment Questions #1 - #3

The Distributive Property

In Class Example #1 we have shown that:

$2(x + 1) =$ ________ , $x(x + 3) =$ ________ , $2x(x - 2) =$ ________ ,

$-3(x - 1)$ ________ , and $(x - 1)(2x) =$ ________ .

These above are examples of the **distributive property**

$$a(b + c) = ab + ac \qquad \text{or} \qquad (b + c)(a) = ba + ca \;\Rightarrow\; ab + ac.$$

The distributive property can be extended to any number of terms.

Using Numerical Values to Verify the Distributive Property

Consider the expression $-2(3 - 5)$.

i) Evaluate $-2(3 - 5)$ by calculating the value inside the brackets first and then multiplying by -2.

ii) Evaluate $-2(3 - 5)$ by using the distributive property.

iii) Comment on your results from i) and ii).

Class Ex. #3

Use the distributive property to determine the following products.

a) $4(3x + 1)$

b) $-5(2x^2 + x - 6)$

c) $(x^3 - 2)x^2$

d) $-3x(7x - 2y + z)$

In the example above we have written a product of polynomials as a sum or difference of terms.

In this process we **expanded** the polynomial expressions by using the distributive property, $a(b + c) = ab + ac$ and the exponent rule, $x^a \times x^b = x^{a+b}$.

Expand and simplify.

a) $6 - 4(8x + 1)$

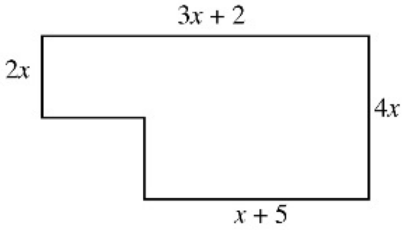
b) $4(2x - 3) - 2(x - 6)$

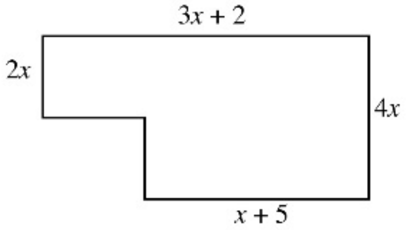
c) $5x(3x^2 - 7x + 1) - (4x + 3x^2)$

Determine a simplified expression for the area of the given shape by

i) adding the areas of two rectangles.

ii) subtracting the areas of two rectangles.



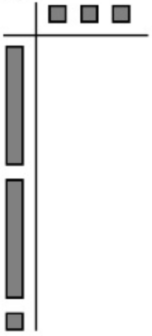


Complete Assignment Questions #4 - #11

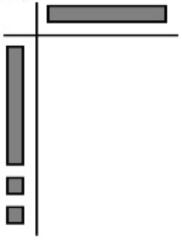
Assignment

1. In each case complete the diagram, state the polynomial product in x, and express the product as a sum or difference of terms.

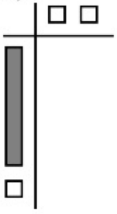
a)



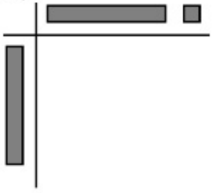
b)



c)



d)



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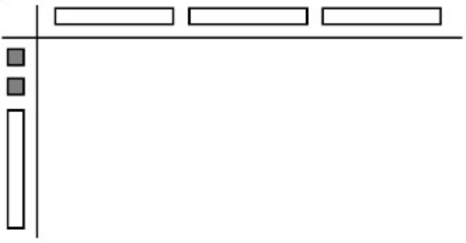
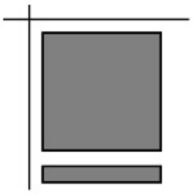


2. In each case state the polynomial product in x which is indicated by the algebra tile diagram. Express the product as a sum or difference of terms.

a)



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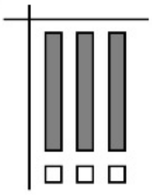
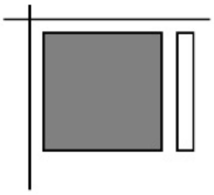
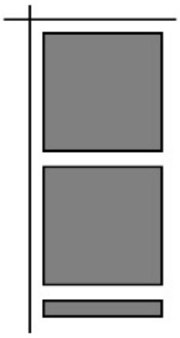


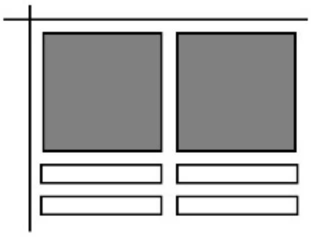
c)



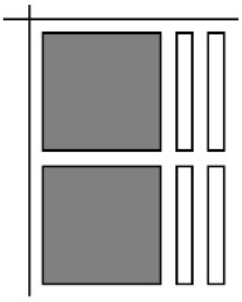
d)



e)



f)



3. For each of the following:

i) Draw an algebra tile diagram to model the product.
ii) Express the product as a sum or difference of terms.
iii) Verify the polynomial product when $x = 4$.

a) $2x(2x - 1)$ **b)** $-3x(2 - x)$

4. Expand.

a) $6(7x - 3)$ **b)** $-4(4x + 9)$ **c)** $4x(2y + 8z)$ **d)** $-x(x - 5y)$

e) $3(x - 2y + 3z)$ **f)** $-2a(b - c + 5d)$ **g)** $(x + 3)3x$

h) $2x(x - 5y + 4z)$ **i)** $x(x - 2x^2 + 3x^3)$ **j)** $(2x^2 + x - 6)(-4x)$

5. Expand and simplify.

a) $3(x+5)-7$

b) $8-2(5x+11)$

c) $6(x-2)+x$

d) $2(x+3)\ +4(2x-1)$

e) $2(x+3)-4(2x-1)$

f) $-2(x+1)\ +7(3x-2)$

g) $5(-x+12)\ +5(x-8)$

h) $(2-x)-2(2x-10)$

i) $6(-x+4)-(x-15)$

6. Identify the errors in the following and provide the correct simplification.

a) $3x(2x+y)=6x+3xy$

b) $x^2(x^3-2x+7)=x^6-2x^3+7x^2$

c) $4(x-2)-2(x-3)$
$=4x-8-2x-6$
$=2x-14$

d) $2(2t-3)-4(t+5)$
$=4t-3-4t-5$
$=-8$

e) $5(a+b)-(a+b)$
$=5a+5b-a+b$
$=4a+6b$

7. Expand and simplify.

a) $2a(a+3) - 4a(2a-1)$ **b)** $4(x^2+3) - (2x^2-1)$ **c)** $2(x+3) - x - 1$

d) $z(z^3+3) - (3z+7)$ **e)** $5(8x-3y) + 2(4y+x)$

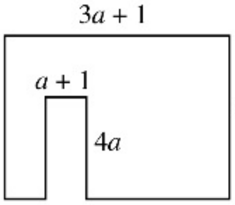
f) $-2x(x^4+3x^3) - 7x(2x^4-x^3)$ **g)** $3a(2a^2b - ab + b^2) - 6b(a^3 + 3ab - 5b^2)$

h) $3x(x-3) - 2x(x-1) + x(2x-2)$ **i)** $(p^2-3p)(4p) - (3+5p)(-2p^2)$

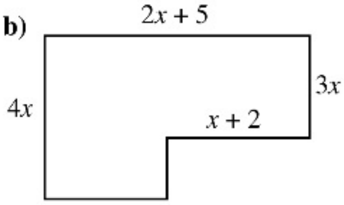
j) $a(b-c) + b(c-a) + c(a-b)$ **k)** $20x^3y^3 - 4x^3y^2(3x+5y-xy)$

8. Determine a simplified expression for the area of the given shape.

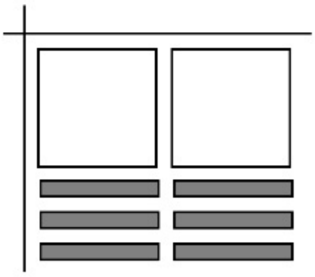
a)



b)



Multiple Choice **9.** The algebra tile diagram represents the expansion of:



A. $2x(x+3)$
B. $-2x(x+3)$
C. $2x(x-3)$
D. $-2x(x-3)$

10. Which of the following expansions is incorrect?

A. $-2x^2(3x+2) = -6x^3 - 4x^2$
B. $-4x(2-x) = -8x + 4x^2$
C. $-5x(x^2-3) = -5x^3 - 15x$
D. $7x^2(x^2+3) = 7x^4 + 21x^2$

Numerical Response **11.** The expression $2x(4-3x) + 5x(2x-1) - 3(4x+2)$ can be written in the form $ax^2 + bx + c$. The value of $a + b - c$ is ______.

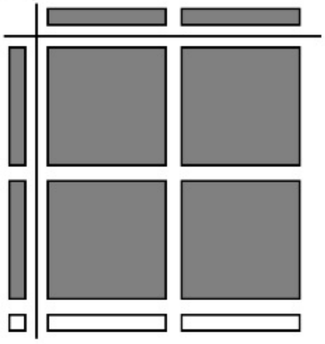
(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $3(2x + 1) = 6x + 3$ **b)** $x(x + 2) = x^2 + 2x$ **c)** $-2(x - 1) = -2x + 2$
d) $(x + 1)(x) = x^2 + x$ **e)** $-3x(2 - x) = -6x + 3x^2$

2. **a)** $x(x + 1) = x^2 + x$ **b)** $3(x - 1) = 3x - 3$ **c)** $(x - 1)(x) = x^2 - x$
d) $x(2x + 1) = 2x^2 + x$ **e)** $2x(x - 2) = 2x^2 - 4x$ **f)** $(x - 2)(2x) = 2x^2 - 4x$

3. **a)** **i)**

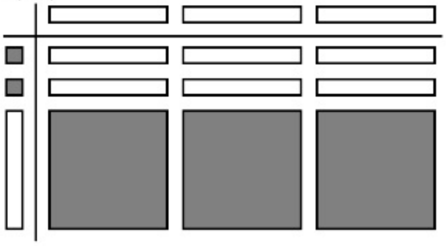


ii) $2x(2x - 1) = 4x^2 - 2x$

iii)

Left Side	Right Side
(2 x 4) ((2 x 4) – 1)	$4(4^2) - 2(4)$
= (8) ((7)	= 64 – 8
= 56	= 56

b) **i)**



ii) $-3x(2 - x) = -6x + 3x^2$

iii)

Left Side	Right Side
(–3 x 4) (2 – 4)	$(-6 \times 4) + 3(4^2)$
= (–12) ((–2)	= –24 + 48
= 24	= 24

4. **a)** $42x - 18$ **b)** $-16x - 36$ **c)** $8xy + 32xz$
d) $-x^2 + 5xy$ **e)** $3x - 6y + 9z$ **f)** $-2ab + 2ac - 10ad$
g) $3x^2 + 9x$ **h)** $2x^2 - 10xy + 8xz$ **i)** $x^2 - 2x^3 + 3x^4$ **j)** $-8x^3 - 4x^2 + 24x$

5. **a)** $3x + 8$ **b)** $-10x - 14$ **c)** $7x - 12$
d) $10x + 2$ **e)** $-6x + 10$ **f)** $19x - 16$
g) 20 **h)** $-5x + 22$ **i)** $-7x + 39$

6. **a)** $3x(2x) = 6x^2$, not $6x$. $3x(2x + y) = 6x^2 + 3xy$
b) $x^2(x^3) = x^5$ not x^6. $x^2(x^3 - 2x + 7) = x^5 - 2x^3 + 7x^2$
c) $-2(-3) = 6$, not -6. $4(x - 2) - 2(x - 3) = 4x - 8 - 2x + 6 = 2x - 2$
d) The monomials 2 and –4 multiply both terms in the binomials.
$2(2t - 3) - 4(t + 5) = 4t - 6 - 4t - 20 = -26$.
e) The negative multiplies both a and b. $5(a + b) - (a + b) = 5a + 5b - a - b = 4a + 4b$.

7. **a)** $-6a^2 + 10a$ **b)** $2x^2 + 13$ **c)** $x + 5$ **d)** $z^4 - 7$
e) $42x - 7y$ **f)** $-16x^5 + x^4$ **g)** $-3a^2b - 15ab^2 + 30b^3$ **h)** $3x^2 - 9x$
i) $14p^3 - 6p^2$ **j)** 0 **k)** $-12x^4y^2 + 4x^4y^3$

8. **a)** $14a^2 + 2a$ **b)** $7x^2 + 18x$ **9.** D **10.** C **11.**

1			

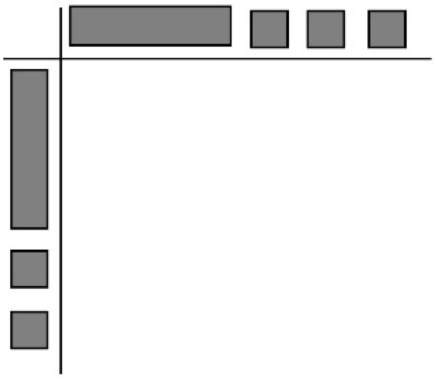
Polynomial Operations Lesson #3:
Multiplication of Two Binomials

Multiplying Two Binomials using Area Diagrams

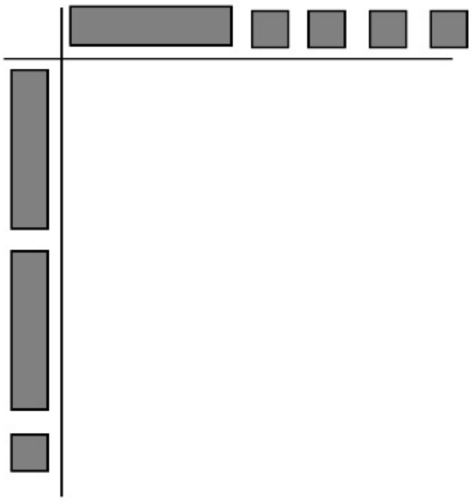
In the last lesson, we multiplied a monomial by a polynomial. In this lesson, we extend the process to the product of two binomials.

Complete the algebra tile diagrams and determine the binomial products.

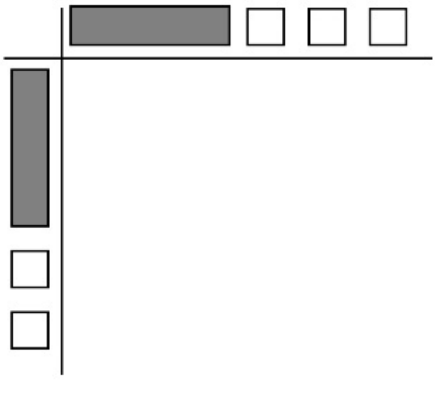
a) $(x+3)(x+2) =$



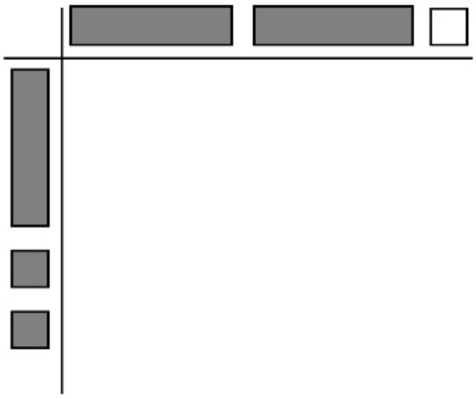
b) (   ) (   ) =



c) (   ) (   ) =



d) (   ) (   ) =



In class example 1a), we used an algebra tile diagram to show that the product $(x + 3)(x + 2)$ could be expressed in simplified expanded form as $x^2 + 5x + 6$.

The algebra tile diagram used to model $(x + 3)(x + 2)$ can be modified into the following area diagram which shows that the product of two binomials is equivalent to four monomial products.

	x	3
x	x^2	$3x$
2	$2x$	6

$(x + 3)(x + 2)$

$= x^2 + 5x + 6$

Class Ex. #2

Use an area diagram like the one above to determine the product of each of the following binomials.

a) $(5x - 6)(2x + 1)$

b) $(a^2 - 5)(a^2 - 8)$

c) $(3p + 2q)(p + 9q)$

d) $(a + b)(c + d)$

An area diagram can be used to show that the multiplication of two, two-digit numbers can be performed as four separate products.
For example the product 32×34 can be determined without a calculator, by long multiplication or by an area diagram as follows:

Long Multiplication

$$\begin{array}{r} 32 \\ \times\ 34 \\ \hline 128 \\ 96 \\ \hline 1088 \end{array}$$

Area Diagram

	30	2
30	900	60
4	120	8

32 x 34
= 900 + 120 + 60 + 8
= 1088

Class Ex. #3

Use an area diagram and no calculator to determine the following products.

a) 43×51

b) 76×82

Complete Assignment Questions #1 - #3

Multiplying Two Binomials using the Distributive Property

In the area diagram modelling $(x+3)(x+2)$, we noted that there were four separate monomial products involved in the expansion.
These products are simply the extension of the distributive property to binomial products.

Distributive property for binomials $(a+b)(c+d) = a(c+d)+b(c+d) = ac+ad+bc+bd$

Class Ex. #4

Use the distributive property to determine the following products.

a) $(x+3)(x+2)$ **b)** $(a-7)(2a-1)$ **c)** $(p-8)(q-8)$

d) $(x+4y)(x-5y)$ **e)** $(9a^2-1)(5a^3+6)$

The method used in the distributive property can be simplified by noticing that the four monomial products $(a+b)(c+d) = ac+ad+bc+bd$ can be memorized using the acronym FOIL .

F – first term in each bracket ie ac
O – outside terms ie ad
I – inside terms ie bc
L – last term in each bracket ie bd

Class Ex. #5

Use FOIL to determine each product.

a) $(x+6)(x+4)$ **b)** $(y-7)(y+2)$

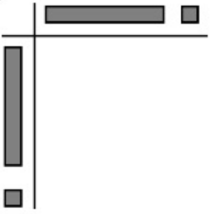
c) $(3x+1)(x-5)$ **d)** $(6a-5b)^2$

Complete Assignment Questions #4 - #9

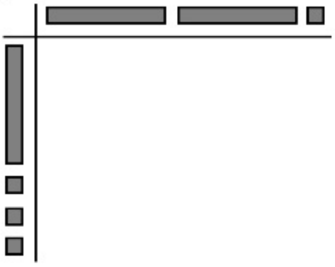
Assignment

1. Complete the algebra tile diagrams and determine the binomial products.

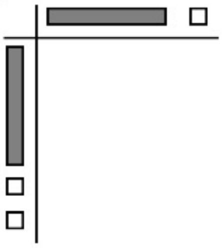
a)



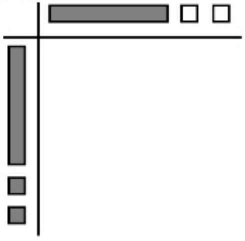
b)



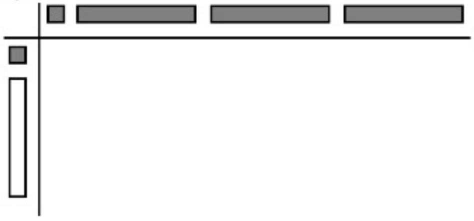
c)



d)



e)



2. Use an area diagram to determine the product of each of the following binomials.

a) $(x+6)(x-2)$

b) $(2x+3)(2x+7)$

c) $(y-3)(4y+1)$

d) $(3d-5)(6d-9)$

e) $(2x-y)(4x+y)$

f) $(3p-8q)(p-5q)$

g) $(a^2 + 8)(a^2 - 8)$ **h)** $(t^3 + 2s)(t^3 + 2s)$ **i)** $(a + b)(a + c)$

3. Without a calculator, use an area diagram to determine the following products.

a) 23×21 **b)** 34×12 **c)** 74×32

d) 65×73 **e)** 49×55 **f)** 86×86

4. Use the distributive property to determine the following products.

a) $(x + 4)(x + 7)$ **b)** $(a + 7)(3a - 5)$ **c)** $(p - 2)(p - 8)$

d) $(x + 6y)(x - 2y)$ **e)** $(4a + 9b)(2a + 3b)$ **f)** $(6 - y)(1 + 4y)$

g) $(2a - 1)(6b - 1)$ **h)** $(7x^2 - 3)(7x^2 - 3)$ **i)** $(2y^2 - 3)(5y^5 + 1)$

5. Use FOIL to determine each product.

a) $(x+3)(x+6)$ **b)** $(y+4)(y+9)$ **c)** $(x+1)(x-8)$ **d)** $(a-7)^2$

e) $(x+2)(5x+4)$ **f)** $(3y-5)(2y+9)$ **g)** $(6x+1)(x-6)$ **h)** $(6-5b)(6-5b)$

i) $(x+3\mathrm{y})(x+4\mathrm{y})$ **j)** $(a-7\mathrm{b})(3a+4\mathrm{b})$ **k)** $(5x+z)(5x-z)$ **l)** $(9-a^2)\,(5-a^2)$

6. A rectangle has length $(2a+5)$ cm and width $(a+4)$ cm.
Determine the area of the rectangle (in cm^2) by completing each of the following solutions.

Area = length × width = ( )( )

(i) *use a diagram*

(ii) *use the distributive property*
$(2a+5)(a+4)$
$= 2a(a+\ \ ) +$

(iii) *use FOIL*
$(2a+5)(a+4)$

7. Expand and simplify where possible.

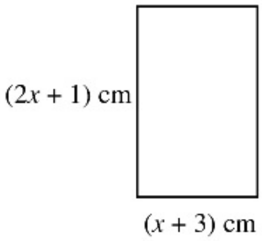
a) $(7x-2)(3x+5)$ **b)** $(2h-3)(2h-1)$ **c)** $(3z+4)(3z+5)$

d) $(4x-3)(3x-4)$ **e)** $(8x-3\mathrm{y})(2x+y)$ **f)** $(1+3b)^2$

g) $(x-2)(6y-1)$ **h)** $(1+3y^2)(1-3y^2)$ **i)** $(x^2+7y^2)(2x^2-5y^2)$

Numerical Response

8. The area of the rectangle shown can be written in the form $px^2 + qx + r$, where $p, q,$ and r are natural numbers.



Write the value of p in the first box.
Write the value of q in the second box.
Write the value of r in the third box.

(Record your answer in the numerical response box from left to right)

9. The expansion of $(3x - c)(x - 3)$, where c is a whole number, results in a polynomial in x with a leading coefficient of 3 and a constant term of 12. The value of c is ______.

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $(x + 1)(x + 1) = x^2 + 2x + 1$ **b)** $(2x + 1)(x + 3) = 2x^2 + 7x + 3$
c) $(x - 1)(x - 2) = x^2 - 3x + 2$ **d)** $(x - 2)(x + 2) = x^2 - 4$
e) $(1 + 3x)(1 - x) = 1 + 2x - 3x^2$

2. **a)** $x^2 + 4x - 12$ **b)** $4x^2 + 20x + 21$ **c)** $4y^2 - 11y - 3$
d) $18d^2 - 57d + 45$ **e)** $8x^2 - 2xy - y^2$ **f)** $3p^2 - 23pq + 40q^2$
g) $a^4 - 64$ **h)** $t^6 + 4st^3 + 4s^2$ **i)** $a^2 + ab + ac + bc$

3. **a)** 483 **b)** 408 **c)** 2368 **d)** 4745 **e)** 2695 **f)** 7396

4. **a)** $x^2 + 11x + 28$ **b)** $3a^2 + 16a - 35$ **c)** $p^2 - 10p + 16$
d) $x^2 + 4xy - 12y^2$ **e)** $8a^2 + 30ab + 27b^2$ **f)** $6 + 23y - 4y^2$
g) $12ab - 2a - 6b + 1$ **h)** $49x^4 - 42x^2 + 9$ **i)** $10y^7 - 15y^5 + 2y^2 - 3$

5. **a)** $x^2 + 9x + 18$ **b)** $y^2 + 13y + 36$ **c)** $x^2 - 7x - 8$
d) $a^2 - 14a + 49$ **e)** $5x^2 + 14x + 8$ **f)** $6y^2 + 17y - 45$
g) $6x^2 - 35x - 6$ **h)** $36 - 60b + 25b^2$ **i)** $x^2 + 7xy + 12y^2$
j) $3a^2 - 17ab - 28b^2$ **k)** $25x^2 - z^2$ **l)** $45 - 14a^2 + a^4$

6. Area = $(2a + 5)(a + 4) = 2a^2 + 13a + 20$

7. **a)** $21x^2 + 29x - 10$ **b)** $4h^2 - 8h + 3$ **c)** $9z^2 + 27z + 20$
d) $12x^2 - 25x + 12$ **e)** $16x^2 + 2xy - 3y^2$ **f)** $1 + 6b + 9b^2$
g) $6xy - x - 12y + 2$ **h)** $1 - 9y^4$ **i)** $2x^4 + 9x^2y^2 - 35y^4$

8.

2	7	3	

9.

4			

Polynomial Operations Lesson #4:
Multiplication of Polynomials - Part One

Three Important Products

Complete the following using the distributive property (FOIL).

i) $(a+b)^2 = (a+b)(a+b)$ **ii)** $(a-b)^2 = (a-b)(a-b)$ **iii)** $(a-b)(a+b)$

$=$ $=$ $=$

$=$ $=$ $=$

1. **The square of a binomial can be found by squaring the first term, doubling the product of the two terms and squaring the last term.**
2. **The product of the sum and difference of the same two monomials results in the difference of the squares of the monomials.**
   This important result will be considered in more detail in future lessons on factoring.

Expand each of the following.

a) $(x+7)^2$ **b)** $(3x-1)^2$ **c)** $(2m-3n)^2$

d) $(4-7x)(4+7x)$ **e)** $(5a+3b)(5a-3b)$ **e)** $(2p-9q)(2p-9q)$

Complete Assignment Question #1

Class Ex. #2

Expand and simplify.

a) $5(2x - 3)(x - 6)$

b) $-8(7p + 3)^2$

Class Ex. #3

Expand and simplify.

a) $(x + 5)(x - 5) - (x + 2)(x + 8)$

b) $(9a + 4)(4a - 9) - (6a - 5)^2$

Class Ex. #4

Expand and simplify.

a) $5x(3x^2 - 7x + 1) - (4x + 3x^2)(5x - 8)$

b) $4(2x - 7)(3x + 2) - 8(x - 1)(3x - 1)$

Class Ex. #5

Given that, for every value of x, the polynomial $x^2 + 20x + 50$ can be written in the form $(x + a)^2 + b$, determine the values of a and b.

Complete Assignment Questions #2 - #7

Assignment

1. Expand and simplify where possible.

a) $(x-8)^2$ **b)** $(x-9)(x+9)$ **c)** $(3x-y)^2$

d) $(5x+2y)^2$ **e)** $(3x-2)(3x+2)$ **f)** $(-2y+1)^2$

g) $(2p+7)^2$ **h)** $(4m+3n)(4m-3n)$ **i)** $(5a-6b)^2$

j) $(9-5x^2)(9+5x^2)$ **k)** $(6a-7b)(6a-7b)$ **l)** $(2a^3-7)(2a^3+7)$

2. Expand and simplify where possible.

a) $2(4x-3)(3x-4)$ **b)** $7(5x-2)(6x+1)$ **c)** $-3(a+8)(2a+9)$

d) $5(4x+1)^2$ **e)** $6(8x-3y)(2x+y)$ **f)** $-4(a+3b)(2a-5b)$

3. Expand and simplify where possible.

a) $(x-3)(x-6)+(x+2)(x+7)$ **b)** $(x-5)(x+4)-(x+1)(x-8)$

c) $(x-3)^2+(x+3)^2$ **d)** $(x-y)(x-4y)-(x+y)(x-y)$

4. Expand and simplify where possible.

a) $(3x-1)(x-3)-2x(x-1)$

b) $(4x+1)(2x+3)-(3x-7)(2x-5)$

c) $(9x-1)(x-4)-(3x+1)(3x-1)$

d) $8(5-3x)(2+5x)-3(1+x)^2$

e) $5(2x-3)(2x+5)+3(x+7)(x+2)$

f) $4(2p+3q)^2-(5p-q)(7p+11q)$

5. Expand and simplify where possible.

a) $(x+4)^2+(x+2)^2$

b) $(3a-b)^2-(2a+5b)^2$

c) $3(y-1)^2-2(2y-1)^2$

d) $9-2(x-1)(x+7)+(2x-5)(x-3)$

e) $3(1+3y)(4-y)-(3y-2)(3y-5)$

Use the following information to answer the next question .

A student provides the following expansions for four binomial products.

$(x + 3)^2 = x^2 + 9$ $(3x - y)(3x - y) = 9x^2 - 6xy - y^2$

$(2x + y)(2x - y) = 2x^2 - y^2$ $(5x + 7)^2 = 25x^2 + 35x + 49$

Multiple Choice **6.** How many of the student's expansions are incorrect?

A. One
B. Two
C. Three
D. Four

Numerical Response **7.** Given that for every value of x, $x^2 - 10x + 39 = (x - a)^2 + b$, then the value of b must be ______.

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $x^2 - 16x + 64$ **b)** $x^2 - 81$ **c)** $9x^2 - 6xy + y^2$
d) $25x^2 + 20xy + 4y^2$ **e)** $9x^2 - 4$ **f)** $4y^2 - 4y + 1$
g) $4p^2 + 28p + 49$ **h)** $16m^2 - 9n^2$ **i)** $25a^2 - 60ab + 36b^2$
j) $81 - 25x^4$ **k)** $36a^2 - 84ab + 49b^2$ **l)** $4a^6 - 49$

2. **a)** $24x^2 - 50x + 24$ **b)** $210x^2 - 49x - 14$ **c)** $-6a^2 - 75a - 216$
d) $80x^2 + 40x + 5$ **e)** $96x^2 + 12xy - 18y^2$ **f)** $-8a^2 - 4ab + 60b^2$

3. **a)** $2x^2 + 32$ **b)** $6x - 12$ **c)** $2x^2 + 18$ **d)** $-5xy + 5y^2$

4. **a)** $x^2 - 8x + 3$ **b)** $2x^2 + 43x - 32$ **c)** $-37x + 5$
d) $77 + 146x - 123x^2$ **e)** $23x^2 + 47x - 33$ **f)** $-19p^2 + 47q^2$

5. **a)** $2x^2 + 12x + 20$ **b)** $5a^2 - 26ab - 24b^2$ **c)** $-5y^2 + 2y + 1$
d) $-23x + 38$ **e)** $-18y^2 + 54y + 2$

6. D **7.**

1	4		

Polynomial Operations Lesson #5:
Multiplication of Polynomials - Part Two

In this lesson, we deal with more involved polynomial multiplication including multiplying a binomial by a trinomial and the product of three binomials.

Product of a Binomial and a Trinomial

A rectangle has length $(5x + 2)$ cm and width $(x^2 + 2x + 1)$ cm.
Determine the area of the rectangle (in cm^2) by completing each of the following solutions.

Area = length × width = (   )(   )

i) *use a diagram*

ii) *use the distributive property*

$$
\begin{aligned}
& (5x+2)(x^2+2x+1) \\
= \; & 5x(x^2+2x+1) + 2(x^2+2x+1) \\
= \; & \\
= \; &
\end{aligned}
$$

Expand and simplify.

a) $(x^2 - 4)(2x^3 + x - 5)$

b) $(2y^2 - 3y - 7)(y - 6)$

Expand and simplify $4(a - 4)(a^2 - 3a - 6) - (4a - 3)(4a + 3)$.

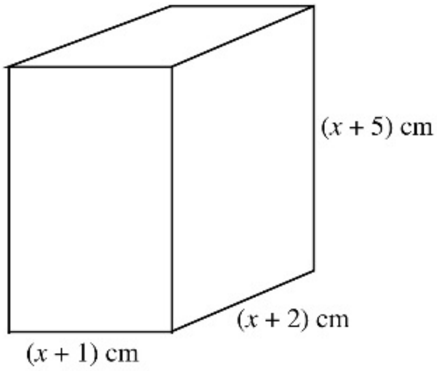
Complete Assignment Questions #1 - #3

Product of Three Binomials

In this section, we extend the multiplication of binomials to consider three factors. This leads to applications involving the volume of a rectangular prism.

Class Ex. #4

Consider the rectangular prism shown.



a) Write an expression which represents the volume of the prism in cm^3.

b) Expand the expression in a) to write the volume in the form $V = ax^3 + bx^2 + cx + d$.

Class Ex. #5

Expand and simplify.

a) $(x - 3)(x + 4)(2x - 1)$

b) $(2x - 1)(x - 3)(x + 4)$

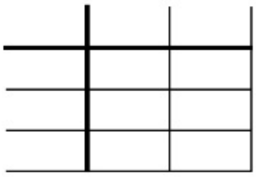
c) Comment on the results to a) and b).

Complete Assignment Questions #4 - #9

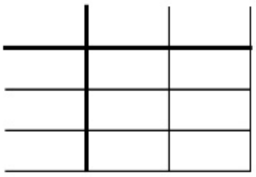
Assignment

1. Use a diagram to determine the expansion.

a) $(y-5)(y^2+2y+4)$



b) $(3m+7)(m^2-3m+6)$



2. Use the distributive law to determine the expansion.

a) $(x-4)(x^2-6x+3)$

b) $(2a+5)(a^2-7a-9)$

3. Expand and simplify.

a) $(x^2-7)(2x^3+4x-1)$

b) $(-m^2-m+1)(m+1)$

c) $(a-3b)(4a^2-3ab-2b^2)$

d) $2(5x+2)(3x^2+x-4)$

4. Expand and simplify.

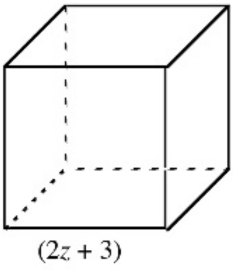
a) $(x+1)(x+2)(3x+5)$

b) $(h-4)(2h-3)(3h-1)$

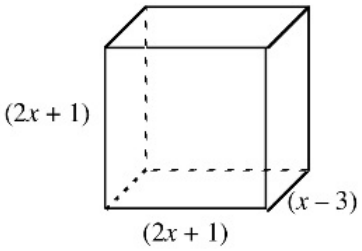
c) $(a+3b)(2a-5b)(2a+5b)$

d) $(3x+7y)(4x-3y)(x-4y)$

5. Calculate the volume of the cube shown below.



6. Calculate the volume of the rectangular prism illustrated.



7. Simplify

a) $-3(a^2 + 2)(3a^2 - a - 1)$

b) $(-2x^2 - 3x + 1)(x^2 - x - 3)$

c) $2(4x - 1)^2 - (3x - 2)^3$

Use the following information to answer the next question.

A student attempts to expand $(a + 2)^3$.
His work is shown below.

$(a + 2)^3$

$= (a + 2)(a + 2)(a + 2)$ Line 1

$= (a + 2)(a^2 + 4)$ Line 2

$= a^3 + 2a^2 + 4a + 8$ Line 3

Multiple Choice **8.** Which of the following statements is true?

A. The student made an error in Line 1.

B. The student made an error in Line 2.

C. The student made an error in Line 3.

D. The student's expansion is correct.

Numerical Response **9.** Subtracting the product of $(3x-1)$ and $(2x^2-4x+3)$ from the sum of $(2x^3-7x^2-6)$ and (x^2+6x-3) results in a polynomial of the form ax^3+bx^2+cx+d. The value of $b-2c$ is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $y^3-3y^2-6y-20$ **b)** $3m^3-2m^2-3m+42$

2. **a)** $x^3-10x^2+27x-12$ **b)** $2a^3-9a^2-53a-45$

3. **a)** $2x^5-10x^3-x^2-28x+7$ **b)** $-m^3-2m^2+1$
c) $4a^3-15a^2b+7ab^2+6b^3$ **d)** $30x^3+22x^2-36x-16$

4. **a)** $3x^3+14x^2+21x+10$ **b)** $6h^3-35h^2+47h-12$
c) $4a^3+12a^2b-25ab^2-75b^3$ **d)** $12x^3-29x^2y-97xy^2+84y^3$

5. $8z^3+36z^2+54z+27$

6. $4x^3-8x^2-11x-3$

7. **a)** $-9a^4+3a^3-15a^2+6a+6$ **b)** $-2x^4-x^3+10x^2+8x-3$
c) $-27x^3+86x^2-52x+10$

8. B **9.**

2	2		

Polynomial Operations Lesson #6:
Problem Solving with Polynomial Products

Bentwood boxes are containers traditionally constructed by the Northwest Coast Aboriginal peoples. An artist typically uses a single piece of wood that is bent to form a box in the shape of a rectangular prism using only steam and strategically placed grooves. Traditional uses of these boxes range from food and clothing storage to burials.

The images below are examples of a bentwood box and can be referenced at the following website https://www.rrncommunity.org/items/3442#

Front View

Back and Side View

Additional information on Bentwood art can be found at
https://en.wikipedia.org/wiki/Bentwood

A Haida artist constructs a bentwood box with the following dimensions:
length $(5x-2)$ cm, width $(3x-1)$ cm, and height $(3x+1)$ cm.

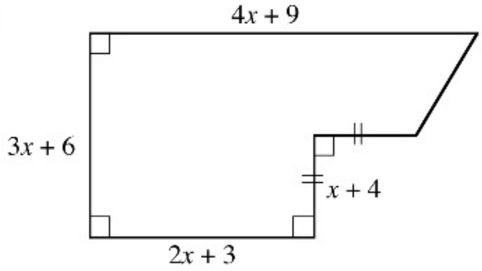
a) Write and simplify an expression for the volume of the bentwood box in cm^3.

b) Write and simplify an expression for the surface area of the bentwood box in cm^2.

c) If $x = 20$, calculate the volume and surface area of the bentwood box.

Class Ex. #2

a) The area of the given figure can be written as a trinomial in the form $ax^2 + bx + c$. Determine the values of a, b, and c.



b) Calculate the area if $x = 2.5$.

Class Ex. #3

The hypotenuse of a right triangle is $(5x + 5)$ cm long and the lengths of the other two sides are $(4x + 8)$ cm and $(3x - 5)$ cm.

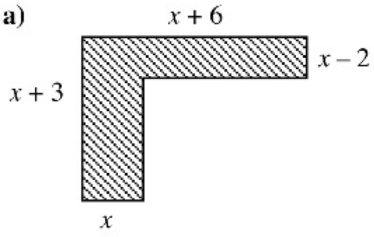
Form an equation and solve it to determine the lengths of the three sides of the triangle.

Complete Assignment Questions #1 - #15

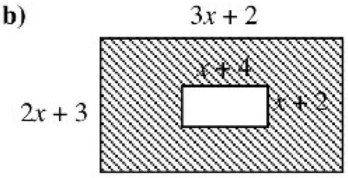
Assignment

1. In each case, the figures consist of a series of horizontal and vertical lines. The area of each figure can be written as a trinomial in the form $ax^2 + bx + c$. Determine the values of a, b, and c, and calculate the area when $x = 2.4$.

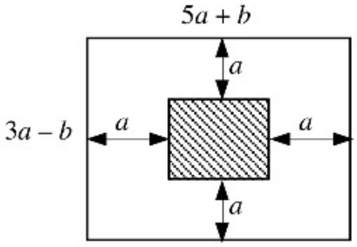
a)



b)



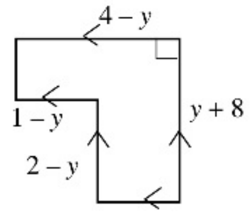
2. The figure consists of a rectangle within a rectangle.



a) Determine a simplified expression of the shaded area in terms of a and b.

b) Calculate the area when $a = 2.8$ and $b = -3.5$.

3. **a)** Determine the area of the figure in the form $ay^2 + by + c$.



b) Determine the area of the figure when $y = -2$.

4. A square metal plate of side 25 cm is heated so that each side increases in length by x cm.

a) Write and simplify an expression for the area of the heated plate.

b) Write and simplify an expression for the increase in area of the plate.

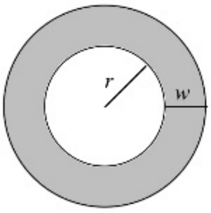
c) If $x = 0.2$, calculate the increase in area.

5. A square garden with a side length of $(3x + 1)$ m contains two square flower beds each with a side length of $(x + 1)$ m. The remainder of the garden is grass.

a) Draw a diagram to illustrate this information.

b) Write and simplify an expression for the area of grass in the garden.

6. A metal washer has internal radius r mm and width w mm as shown.



a) Write an expression for the outer radius of the washer.

b) Show that the area of the washer, A mm^2, is given by $A = 2\pi rw + \pi w^2$.

7. Solve the following equations where the variable is in the set of real numbers.

a) $(3x - 1)(x - 1) = 3x(x + 1)$

b) $(y + 2)^2 = y^2 + 2$

c) $t^2 - (t - 9)^2 = 9$

d) $2a^2 - (a - 3)^2 = (a + 2)(a - 1)$

8. The hypotenuse of a right triangle is $(5x - 6)$ cm long and the lengths of the other two sides are $(4x - 7)$ cm and $(3x - 1)$ cm. Form an equation and solve it to determine the value of x and the lengths of the three sides of the triangle.

9. Consider a set of rectangles with sides $(4a - 3)$ cm and $(2a + 7)$ cm.

a) Write and simplify an expression in a for the area of one of these rectangles.

b) If one of these rectangles has a perimeter of 50 cm, determine the length and width of this rectangle.

c) If another of these rectangles is a square, determine the length of each side.

10. A rectangle has length $(x^2 + 4x - 1)$ cm and width $(3x - 2)$ cm.

a) Write and simplify an expression for the area of the rectangle in cm^2.

b) If $x = 2.5$, calculate the area of the rectangle.

11. Dice for a children's board game are cubes with an edge length of $(3x - 2)$ mm.

a) Write and simplify an expression for the volume of a die in mm^3.

b) The manufacturer packages dice in cubic containers containing 64 dice. Determine the volume of the container in cm^3 if $x = 4$.

12. A rectangular garden with length $(8 - 3a)$ m and width $(a + 8)$ m contains three square flower beds, each with a side length of $(2a + 5)$ m. The remainder of the garden is grass.

a) Draw a diagram to illustrate this information.

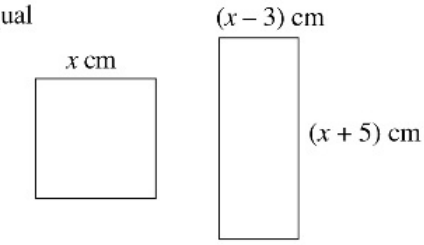
b) Write and simplify an expression for the area of grass in the garden.

c) Determine the area of grass if $a = -1.5$.

Multiple Choice **13.** A box is in the shape of a rectangular prism. The length of the box is y cm. The width is 2 cm less than the length, and the height is 2 cm more than the length. If the volume of the box can be written in the form $V = ay^3 + by^2 + cy + d$ where a, b, c, and d are integers, then how many of the parameters a, b, c, and d are equal to zero?

A. 0
B. 1
C. 2
D. 3

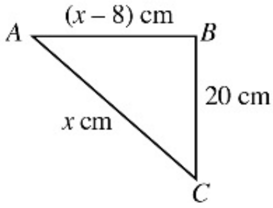
14. The square and the rectangle in the diagram are equal in area.
The value of x, to the nearest tenth, is ______.



(Record your answer in the numerical response box from left to right)

15. The diagram shows the lengths of the sides of right triangle ABC.

The perimeter (to the nearest tenth of a cm) of triangle ABC is ______.



(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $a = 1,\ b = 9,\ c = -12,$ area = 15.36 units^2 **b)** $a = 5,\ b = 7,\ c = -2,$ area = 43.6 units^2

2. a) $3a^2 - 2ab - b^2$ **b)** 30.87 units^2 **3. a)** $30 - y - 2y^2$ **b)** 24 units^2

4. a) $625 + 50x + x^2$ cm^2 **b)** $50x + x^2$ cm^2 **c)** 10.04 cm^2

5. b) $(3x+1)^2 - 2(x+1)^2 = 7x^2 + 2x - 1$ m^2 **6. a)** $(r + w)$ *mm*

7. a) $\frac{1}{7}$ **b)** $-\frac{1}{2}$ **c)** 5 **d)** $\frac{7}{5}$

8. $(5x-6)^2 = (4x-7)^2 + (3x-1)^2$; $x = 7$; 29 cm, 21 cm 20 cm

9. a) $(4a-3)(2a+7) = 8a^2 + 22a - 21$ cm^2 **b)** 11 cm by 14 cm **c)** 17 cm

10. a) $(x^2 + 4x - 1)(3x - 2) = 3x^3 + 10x^2 - 11x + 2$ cm^2 **b)** 83.875 cm^2

11. a) $(3x-2)^3 = 27x^3 - 54x^2 + 36x - 8$ mm^3 **b)** 64 cm^3

12. b) $-15a^2 - 76a - 11$ m^2 **c)** 69.25 m^2

c) volume = 2574 cm^3, surface area = 1150 cm^2

13. C

14.

7	.	5	

15.

7	0	.	0

Polynomial Operations Lesson #7:
Practice Test

1. Which of the following could not be classified as a monomial?

A. $9x^4$ **B.** $x + 1$ **C.** 7 **D.** $-6x$

2. The degree of the polynomial $a^3b + 2c^2$ is

A. 3 **B.** 4 **C.** 5 **D.** 6

3. Which of the following can be classified as quintic polynomials?

I $5x$ **II** $3x^5 - 2x^6$ **III** $2x^4 - 3x^{-2} + 5x^5$ **IV** $3x^2y^3$

A. **IV** only **B.** **III** and **IV** only **C.** **I**, **II**, **III**, and **IV**

D. some other combination of **I**, **II**, **III**, and **IV**

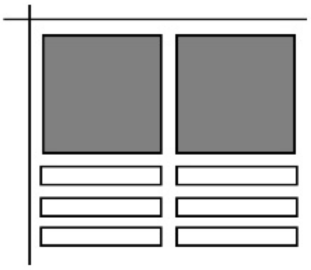
Numerical Response

1. The polynomial $2x^3 - 9 + 8x^4$ has leading coefficient P, degree Q, and constant term R. The value of $P - Q - R$ is ______.

(Record your answer in the numerical response box from left to right)

Use the following diagram to answer the next question.

Dark tiles represent positive quantities and light tiles represent negative quantities.



4. The algebra tile diagram represents the expansion of

A. $2x^2(x^2 - 3x)$

B. $x^2(x - 3)$

C. $2x(x - 3)$

D. $2x(x + 3)$

5. When $(x+2)(x-2)$ is expanded, what classification of the polynomial is the expanded form and how many terms are in the simplified product?

A. Linear with three terms

B. Quadratic with two terms

C. Quadratic with three terms

D. Quartic with two terms

6. The area (in mm^2) of a rectangle with length $6-a$ mm and width $8-a$ mm is

A. $48+a^2$

B. $48-14a-a^2$

C. $48-2a+a^2$

D. $48-14a+a^2$

7. Which of the following expansions is correct?

A. $-2x^2(3x^3+5) = -6x^6-10x^2$

B. $-7x(2-3x) = -14x-21x^2$

C. $-5x^3(4+x) = -20x^3-5x^3$

D. $-9x^2(1-x) = -9x^2+9x^3$

Numerical Response

2. The expression $6x(4-3x)-5x(x-4)-(9x+2)$ can be written in the form ax^2+bx+c. The value of abc is ______.

(Record your answer in the numerical response box from left to right)

8. Triangle PQR is isosceles and right angled at Q. Side PQ measures $2x+8$ cm. If the area of the triangle can be expressed in the form $ax^2+bx+c\ \text{cm}^2$, then the value of b is

A. 32

B. 16

C. 8

D. 6

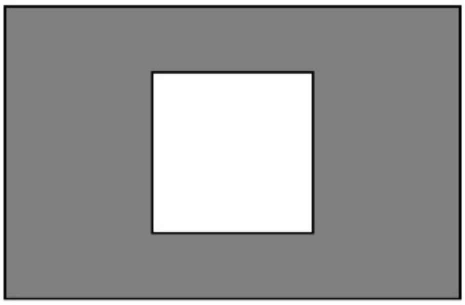
Numerical Response **3.** If $(x-a)^2+a = x^2-12x+c$, then the value of c is ______.

(Record your answer in the numerical response box from left to right) ☐☐☐☐

Use the following information to answer the next question.

The rectangle has length $4x+3$ mm and width $2x+4$ mm.

The square has side length $x+3$ mm.



9. The area, in mm^2, of the shaded region is

A. $7x^2+22x+21$
B. $7x^2+28x+21$
C. $7x^2+16x+3$
D. $7x^2+22x+3$

10. The expansion $(6t-1)(2t^2-7t-1)$ is

A. $12t^3-44t^2+t+1$
B. $12t^3-40t^2-13t+1$
C. $12t^3-44t^2-t+1$
D. $12t^3-44t^2-13t-1$

11. Subtracting the product of $(2x - 3)$ and $(3x + 2)$ from the sum of $(3x^2 + 3x - 2)$ and $(3x^2 + 2x - 4)$ results in a polynomial of the form $px^2 + qx + r$.
How many of the parameters p, q, r are equal to zero?

A. 0
B. 1
C. 2
D. 3

12. Expand $(a + 3)(a - 6)(2a + 1)$

A. $2a^3 - 5a^2 - 39a - 18$
B. $2a^3 + 7a^2 - 33a - 18$
C. $2a^3 + a^2 - 45a - 18$
D. $2a^3 - 18$

In questions #13 and #14 four responses are given.

Answer *A* if response 1 and response 2 only are correct

***B* if response 1 and response 3 only are correct**

***C* if response 2 and response 4 only are correct**

***D* if some other response or combination of the responses is correct.**

13. Which of the following are binomials of degree 6?

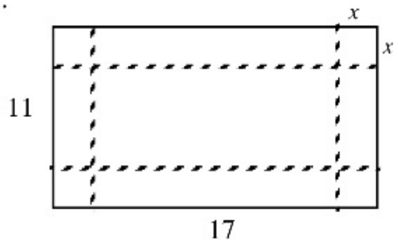
Response 1: $x^6 + x$
Response 2: $6x^3 + 6y^3$
Response 3: $4a^2b^4 + 2a^3$
Response 4: y^5z

14. Which of the following polynomial products contains a term of degree one?

Response 1 $(x - 3)(x - 2)$
Response 2 $(x - 3)(x - 3)$
Response 3 $(x - 3)(x + 3)$
Response 4 $(x - 3)(x - 6)(x + 2)$

Use the following information to answer questions #15 and Numerical Response #5.

A sheet of paper measures 17 cm by 11 cm. Squares of side x cm are cut out from each corner as shown. The paper is folded along the dotted lines to form a rectangular prism.



15. The length and width (in cm) of the rectangular prism are respectively

- **A.** $17 - x$ and $11 - x$
- **B.** $17 - 2x$ and $11 - 2x$
- **C.** $17 + x$ and $11 + x$
- **D.** $17 + 2x$ and $11 + 2x$

Numerical Response **4.** The volume of the rectangular prism can be written as the polynomial expression $ax^3 + bx^2 + cx$ where a, b, and c are integers. The value of $a + c$, to the nearest whole number, is _____.

(Record your answer in the numerical response box from left to right)

Numerical Response **5.** The trinomial $x^2 - 14x + 52$ can be expressed in the form $(x - p)^2 + q$.

Write the value of p in the first box.
Write the value of q in the second box.

(Record your answer in the numerical response box from left to right)

Written Response - 5 marks

Use the following information to answer the next question.

1. Three students, Wendy, Cooper and Sahil, are attempting to expand the polynomial $5(3y-1)^2$.
Their work is shown below.

	Wendy	Cooper	Sahil
	$5(3y-1)^2$	$5(3y-1)^2$	$5(3y-1)^2$
Line 1	$(15y-5)^2$	$5(6y^2-6y+1)$	$5(9y^2-3y+1)$
Line 2	$225y^2-150y+25$	$30y^2-30y+5$	$45y^2-15y+5$

- Each student's work contains one mathematical error.
  Describe in detail the error each student made.

- Determine the correct expansion of $5(3y-1)^2$.

- Verify the expansion by substituting $y=-4$ in the original expression and in your expansion and confirming the same numerical value.

- Classify the expanded form of the polynomial using

  i) the number of terms **ii)** the degree

Answer Key

1. B	**2.** B	**3.** A	**4.** C	**5.** B
6. D	**7.** D	**8.** B	**9.** C	**10.** A
11. C	**12.** A	**13.** B	**14.** A	**15.** B

Numerical Response

1.	1	3		
2.	1	6	1	0
3.	4	2		
4.	1	9	1	
5.	7	3		

Written Response

1.
- In line 1 Wendy distributed the 5 before squaring. BEDMAS tells us to do exponents before multiplication.
  In line 1 Cooper expanded $(3y - 1)^2$ incorrectly. The first term should be $(3y)(3y) = 9y^2$.
  In line 1 Sahil also expanded $(3y - 1)^2$ incorrectly. The middle term should be $(3y)(-1)(2) = -6y$.

- $5(3y - 1)^2 = 5(9y^2 - 6y + 1) = 45y^2 - 30y + 5$

- $LS = 5[3(-4) - 1]^2 = 5[-12 - 1]^2 = 5(-13)^2 = 5(169) = 845$

  $RS = 45(-4)^2 - 30(-4) + 5 = 45(16) + 120 + 5 = 720 + 120 + 5 = 845$

  $LS = RS$ so the expansion is verified.

- i) Trinomial ii) Cubic

Factoring Polynomial Expressions Lesson #1:
Common Factors

Overview of Unit

In this unit, we introduce the process of factoring. This includes factoring by removing a common factor, factoring a trinomial, and factoring a difference of squares. These techniques are illustrated concretely, pictorially, and symbolically. We express a polynomial as a product of its factors and include, for enrichment, polynomial equation solving.

Expanding and Factoring

In the previous unit, we were concerned with multiplying polynomial expressions.
In particular we multiplied

i) a monomial by a polynomial e.g. $2x(x+5) =$ ___________

ii) a binomial by a binomial to form a trinomial e.g. $(x+1)(x+3) =$ ___________

iii) a binomial by a binomial to form a binomial e.g. $(x-5)(x+5) =$ ___________

In these examples, we have **expanded** a product of polynomials to form a sum or difference of monomials.

In this unit, we are concerned with the opposite process. We want to write a sum or difference of monomials as a product of polynomials. This process is called **factoring**.

We will be studying the following three major types of factoring.

Complete the following using the results obtained above.

i) factoring by removing a common factor e.g. $2x^2 + 10x =$ ___________

ii) factoring a trinomial. e.g. $x^2 + 4x + 3 =$ ___________

iii) factoring a difference of squares e.g. $x^2 - 25 =$ ___________

Greatest Common Factor

In the lesson "Applications of Prime Factors" page 9, we met the concept of the greatest common factor of whole numbers.

The GCF of 48 and 72 was found by using prime factorization.

$48 = 2 \times 2 \times 2 \times 2 \times 3$ and $72 = 2 \times 2 \times 2 \times 3 \times 3$

To determine the greatest common factor of 48 and 72, we found the product of each prime factor (including repeats) which is common to each prime factorization.

GCF of 48 and 72 is $2 \times 2 \times 2 \times 3 =$ _____.

The same process can be used to determine the greatest common factor of two monomials like $6a^3$ and $9a^2b$.

$6a^3 = 2 \times 3 \times a \times a \times a$ and $9a^2b = 3 \times 3 \times a \times a \times b$

GCF of $6a^3$ and $9a^2b$ is $\times \quad \times \quad =$ _____.

Class Ex. #1

Write the prime factorization of $8x^2y^2$ and $20xy^3$ and determine the greatest common factor of $8x^2y^2$ and $20xy^3$.

- The greatest common factor of two simple monomials can be determined by inspection, by taking the GCF of any numerical coefficients and multiplying by each common variable to the lowest common exponent.
  The greatest common factor of $10p^3q$ and $15p^2q^2$ is determined by multiplying 5 by p^2 by q, i.e. $5p^2q$.
- If all the monomials are negative, the GCF is usually considered to be negative (see example d) below).

Class Ex. #2

In each case, state the greatest common factor of the following sets of monomials.

a) $12ab,\ 15a^2b^3$

b) $18x^4y^2,\ -24x^3y^5$

c) $a^3bc^2,\ 2ac^7$

d) $-40a^3b,\ -20a^2b^3,\ -10a^2b^2$

Complete Assignment Question #1 - #3

Factoring a Polynomial by Removing the Greatest Common Factor

Factoring is a process in which a sum or difference of terms is expressed as a product of factors.

A polynomial like $8x^2y^2 + 20xy^3$ can be factored by removing (or taking out, or dividing out) the greatest common factor from each term.

We know that

$4xy^2(2x + 5y)$ can be expanded to give $8x^2y^2 + 20xy^3$.

It follows that

$8x^2y^2 + 20xy^3$ can be factored to give $4xy^2(2x + 5y)$.

In this case, the greatest common factor $4xy^2$ has been removed from each term.

Class Ex. #3

In each case, complete the factoring.

a) $21x + 14y =$ ____ $(3x + 2y)$

b) $5x^4 + 15x^3 + 5x^2 =$ ____ $(x^2 + 3x + 1)$

Class Ex. #4

In each case, the greatest common factor has been removed. Complete the factoring.

a) $5a^2 + 25a = 5a(a + \quad)$

b) $18p - 16q = 2(\quad - 8q)$

c) $-4mn - 6m^2 = -2m($

d) $18x^2y^2 - 45xy + 9x = 9x($

Class Ex. #5

Factor each polynomial by removing the greatest common factor.

a) $20x - 6$

b) $16x^4 + 4x^2$

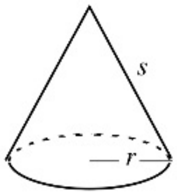
c) $10a^3b^2 + 8ab^3 + 2ab^4$

d) $12p^3 - 6p^2 + 15p$

e) $25xy^2z^3 - 20x^2y^4z^2 + 30x^4y^2z^5$

Class Ex. #6

The surface area of a cone is given by the formula $A = \pi r^2 + \pi rs$, where r is the radius of the base of the cone and s is the slant height.



i) Determine the surface area of a cone, to the nearest $0.01\ \text{cm}^2$, which has slant height 7.40 cm and base radius 2.60 cm.

ii) Write the formula for A in factored form.

iii) Calculate the surface area of the cone, to the nearest $0.01\ \text{cm}^2$, using the factored form of A.

iv) Which method i) or iii) is simpler to use?

Complete Assignment Questions #4 - #13

Assignment

1. Write the prime factorization of $12a^3$ and $30a^2$ and determine the greatest common factor of $12a^3$ and $30a^2$.

2. Write the prime factorization of $10xy^4$ and $25x^2y^3$ and determine the greatest common factor of $10xy^4$ and $25x^2y^3$.

3. In each case, state the greatest common factor of the following sets of monomials.

a) $7m,\ 14m$

b) $6x^2,\ 9x$

c) $bc^2,\ bc^7$

d) $ab,\ a^2b^2$

e) $4x^4,\ 8x^3$

f) $3xyz,\ 9rst,\ 12def$

g) $-8pq^3,\ 18p^2q$ **h)** $-10x^5z^6,\ -15x^5z^4$ **i)** $8ab^2,\ 9ab,\ 6a^2b$

j) $10xy,\ 16xz,\ 20xyz$ **k)** $-2x^3y,\ -4x^3y^4,\ -4x^2y^4$ **l)** $-28pqr^3,\ -56p^2q,\ -64q^2r$

4. Complete the factoring in each case.

a) $12a + 24b = \_\_\_\ (a + 2b)$ **b)** $4p^2 - 7p = \_\_\_\ (4p - 7)$

c) $2xy + 3xz = \_\_\_\ (2y + 3z)$ **d)** $5x^2 + 10x + 15 = \_\_\_(x^2 + 2x + 3)$

e) $6cde - 4cd = \_\_\_\_(3e - 2)$ **f)** $3y^3 - 9y = \_\_\_(y^2 - 3)$

5. In each case, the greatest common factor has been removed. Complete the factoring.

a) $3a^2 + 15a = 3a(a +$ **b)** $20p - 10q = 10(\quad - q)$

c) $6x^3 - 9x^2 = 3x^2($ **d)** $4a^2b + 8a^3b^2 = 4a^2b($

e) $-15x^2y - 10x^2y^2 = -5x^2y($ **f)** $16xm^2n^3 - 12mn^2 - 4mn = 4mn($

6. Factor the following polynomials by removing the greatest common factor.

a) $6m + 6n$ **b)** $7xy^2 + 49$ **c)** $15pq - 5$ **d)** $8c + 12d$

e) $xy + y$ **f)** $6x^2 - 9x$ **g)** $9ab - 12ac$ **h)** $48y^2 - 72y^5$

7. Factor the following polynomials

a) $12x - 8y + 16z$ **b)** $9pq + 6pr - 15p$ **c)** $t^3 + t^2 + t$

d) $5x^2 - 10xy - 20xz$ **e)** $4abc - 2abd + 8abe$ **f)** $14a^2b^2 + 21a^3b^2 - 35a^2b^3$

8. In each of the following:
i) simplify the expression by combining like terms.
ii) factor the resulting polynomial.

a) $5x^2 - 2x + 7 - 2x^2 + 8x - 7$

b) $6 - 2y + 5y^2 - 10y + 3y^2 - 12$

c) $xy^3 - 2x^3y + 6x^2y^2 - 5xy^3 + 8x^3y$

d) $2(x^3 - 3x) - 4x(x - 6) + 5x^2(x - 2) - 4x$

9. Factor the following polynomials. Expand your answer to verify the factoring.

a) $24x^3 - 60x^2$

b) $-8p^3 - 32p^2 - 8p$

10. An archer standing on the ground fires an arrow vertically upward into the air at a speed of 30 m/s.
The height (h metres) of the arrow above the ground after t seconds can be approximated by the formula $h = 30t - 5t^2$.

a) Write h in factored form.

b) Use the factored form of h to calculate the height for each of the times in the table. Record your answer in the table.

Time (t seconds)	0	1	2	3	4	5	6
Height (h metres)							

c) Explain why the height of the arrow after two seconds is the same as the height of the arrow after four seconds.

d) Calculate h when $t = 7$. Explain why this has no meaning in the context of the question.

Multiple Choice **11.** One factor of $9x^4 - 6x^3 + 3x^2$ is

A. $9x^4$
B. $3x^2 - 2x$
C. $3x^2 - 6x + 3$
D. $3x^2 - 2x + 1$

Numerical Response **12.** When $x^4y^3 - x^2y^3 + x^6y$ is factored, the greatest common factor has degree A and the remaining trinomial factor has degree B. The value of $A + 2B$ is _____.

(Record your answer in the numerical response box from left to right)

13. When the greatest common factor is removed from the binomial $98x^2 - 28x$, the binomial can be written in the form $ax(bx + c)$. The value of $a + b + c$ is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. $12a^3 = 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot a$ $30a^2 = 2 \cdot 3 \cdot 5 \cdot a \cdot a$ $GCF = 2 \cdot 3 \cdot a \cdot a = 6a^2$

2. $10xy^4 = 2 \cdot 5 \cdot x \cdot y \cdot y \cdot y \cdot y$ $25x^2y^3 = 5 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot y$ $GCF = 5 \cdot x \cdot y \cdot y \cdot y = 5xy^3$

3. **a)** $7m$ **b)** $3x$ **c)** bc^2 **d)** ab **e)** $4x^3$ **f)** 3 **g)** $2pq$ **h)** $-5x^5z^4$ **i)** ab **j)** $2x$ **k)** $-2x^2y$ **l)** $-4q$

4. **a)** 12 **b)** p **c)** x **d)** 5 **e)** $2cd$ **f)** $3y$

5. **a)** $a+5$ **b)** $2p-q$ **c)** $2x-3$ **d)** $1+2ab$ **e)** $3+2y$ **f)** $4xmn^2-3n-1$

6. **a)** $6(m+n)$ **b)** $7(xy^2+7)$ **c)** $5(3pq-1)$ **d)** $4(2c+3d)$
e) $y(x+1)$ **f)** $3x(2x-3)$ **g)** $3a(3b-4c)$ **h)** $24y^2(2-3y^3)$

7. **a)** $4(3x-2y+4z)$ **b)** $3p(3q+2r-5)$ **c)** $t(t^2+t+1)$ **d)** $5x(x-2y-4z)$
e) $2ab(2c-d+4e)$ **f)** $7a^2b^2(2+3a-5b)$

8. **a)** $3x^2+6x=3x(x+2)$ **b)** $8y^2-12y-6=2(4y^2-6y-3)$
c) $6x^3y+6x^2y^2-4xy^3=2xy(3x^2+3xy-2y^2)$ **d)** $7x^3-14x^2+14x=7x(x^2-2x+2)$

9. **a)** $12x^2(2x-5)$ **b)** $-8p(p^2+4p+1)$

10. **a)** $h=5t(6-t)$ **b)** 0, 25, 40, 45, 40, 25, 0
c) At 2 sec. the arrow is on the way up and at 4 sec. the arrow is on the way down.
d) $h = -35$. The arrow has already hit the ground at $t=6$. It does not travel 35m below the ground.

11. D

12.

1	1		

13.

1	9		

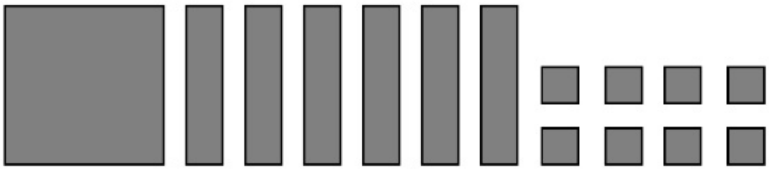
≤

Factoring Polynomial Expressions Lesson #2:

Factoring Trinomials of the Form $x^2 + bx + c$ - Part One

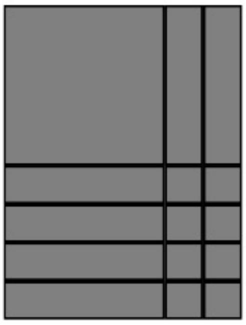
Factoring Trinomials using Algebra Tiles

Consider the algebra tile diagram shown.



- Write the polynomial expression which is represented by the algebra tiles.

- The algebra tiles can be rearranged into a rectangular form as shown below.



i) Write an expression for the length of the rectangle.

ii) Write an expression for the width of the rectangle.

iii) Write the area of the rectangle as a product of two binomials.

iv) Write the area of the rectangle in expanded form.

- The work above provides a method for factoring the trinomial $x^2 + 6x + 8$ into the product of two binomials $(x + 2)(x + 4)$: i.e. $x^2 + 6x + 8 = (x + 2)(x + 4)$.

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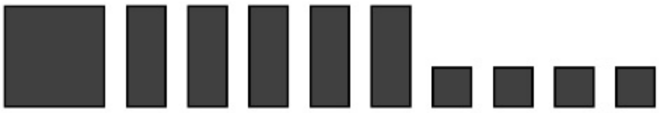
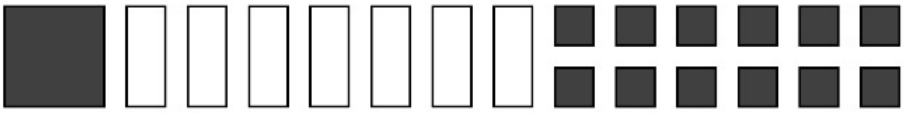
a) Write the polynomial expression which is represented by the algebra tiles.



b) Arrange the algebra tiles into a rectangle and write an expression for the length and width of the rectangle.

c) Use the results above to express the polynomial in factored form.

a) Write the polynomial expression which is represented by the algebra tiles.



b) Arrange the algebra tiles into a rectangle and express the polynomial in factored form.

Complete Assignment Questions #1 - #3

Investigation: Factoring Trinomials by Inspection

- Expand the following binomials as shown.

$(x + 2)(x + 4) = x^2 + 4x + 2x + 8 = x^2 + 6x + 8$

$(x + 3)(x + 3) =$

$(x + 1)(x + 7) =$

$(x + 5)(x + 2) =$

$(x - 5)(x - 2) =$

$(x + 8)(x - 6) =$

- Consider the expansion $(x + p)(x + q) = x^2 + bx + c$.

  In each of the examples above what is the connection between

  i) the value of b and the values of p and q? $b =$ _______

  ii) the value of c and the values of p and q? $c =$ _______

Class Ex. #3

Use FOIL to show that $(x + p)(x + q)$ can be written in the form $x^2 + (p + q)x + pq$.

Factoring $x^2 + bx + c$ *by Inspection*

In order to factor $x^2 + bx + c$ by inspection we need to find two integers which have a product equal to c and a sum equal to b. If no two such integers exist, then the polynomial cannot be factored.

In order to factor $x^2 + 8x + 12$ we need to find two numbers which multiply to ___ and add to ____ .

In order to factor $x^2 - 13x + 12$ we need to find two numbers which multiply to ___ and add to ____ .

The next example practices this skill.

Class Ex. #4

Complete the tables to find two numbers with the given sum and the given product.

Sum	Product	Integers
12	20	2, 10
9	20	
4	4	
–9	18	

Sum	Product	Integers
–15	14	
–1	–6	
2	–15	
–26	48	

Notice that:

- if the product is **positive**, then the two integers must be either **both positive** or **both negative.**
- if the product is **negative**, then one integer is **positive** and the other is **negative**.

For the remainder of this lesson, we will only deal with examples where the product is positive. In the next lesson we will include examples where the product is negative.

Class Ex. #5

Factor the following trinomials where possible.

a) $x^2 + 8x + 12$ **b)** $x^2 + 13x + 12$ **c)** $x^2 - 13x + 12$

d) $a^2 - 11a + 10$ **e)** $y^2 + 3y + 4$ **f)** $x^2 + 27x + 50$

Class Ex. #6

Factor the polynomial expressions by first removing a common factor.

a) $4x^2 - 32x + 48$ **b)** $3x^3 + 21x^2 + 30x$

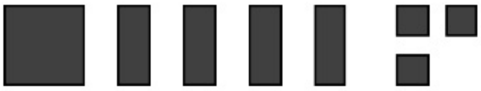
Note

In this example there were two steps in the factoring process - a common factor followed by a trinomial. If we are asked to factor a polynomial expression, it is understood this means to continue factoring until no further factoring is possible. This is sometimes written as "factor completely ...". The operation "factor" means "factor completely".

Complete Assignment Questions #4 - #15

Assignment

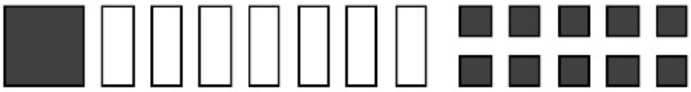
1. a) Write the polynomial expression which is represented by the algebra tiles.



b) Arrange the algebra tiles into a rectangle and write an expression for the length and width of the rectangle.

c) Use the results above to express the polynomial in factored form.

2. a) Write a polynomial expression for the group of algebra tiles shown.



b) Arrange the algebra tiles into a rectangle.

c) State the length and width of the rectangle and hence express the polynomial in factored form.

3. Use algebra tiles to factor the following trinomials.

a) $x^2 + 5x + 6$ **b)** $x^2 + 6x + 5$ **c)** $x^2 - 6x + 8$

4. Complete the tables to find two numbers with the given sum and the given product.

	Sum	Product	Integers
a)	5	6	
b)	8	7	
c)	11	30	
d)	−11	30	

	Sum	Product	Integers
e)	11	10	
f)	−8	15	
g)	−15	56	
h)	−18	56	

5. Complete the following.

a) $x^2 + 7x + 12 = (x + 3)(x + \quad)$ **b)** $x^2 + 9x + 8 = (x + \quad)(x + \quad)$

c) $x^2 - 7x + 10 = (x - 2)(x - \quad)$ **d)** $t^2 - 14t + 24 = (t - \quad)(t - \quad)$

e) $z^2 + 8z + 15 = (z + 5)(\qquad)$ **f)** $b^2 - 12b + 20 = (b - 2)(\qquad)$

6. Factor the following.

a) $x^2 + 3x + 2$ **b)** $x^2 - 3x + 2$ **c)** $x^2 + 9x + 18$

d) $x^2 + 8x + 12$ **e)** $x^2 - 10x + 21$ **f)** $x^2 - 11x + 24$

7. Factor where possible.

a) $x^2 + 11x + 10$ **b)** $x^2 + 10x + 11$ **c)** $n^2 + 12n + 32$

d) $y^2 - 11y + 28$ **e)** $y^2 + 17y + 42$ **f)** $f^2 - 10f + 21$

g) $p^2 - 16p + 28$ **h)** $x^2 + 24x + 80$ **i)** $c^2 - 32c + 60$

j) $a^2 - 12a + 24$ **k)** $d^2 + 18d + 45$ **l)** $p^2 - 29p + 100$

m) $m^2 + 22m + 121$ **n)** $n^2 - 23n + 102$ **o)** $q^2 - 28q + 115$

8. a) The expression $x^2 + bx + 12$ can be factored over the integers. Determine all possible values of b.

b) If the expression $x^2 + 6x + c$, where $c > 0$, can be factored over the integers, determine all possible values of c.

9. A volleyball court has an area of $x^2 + 15x + 36$ square metres.

a) Factor $x^2 + 15x + 36$ to find binomials that represent the length and width of the court.

b) If $x = 3$, determine the length and width of the court.

10. Factor.

a) $2x^2 + 6x + 4$ **b)** $4x^2 - 48x + 128$ **c)** $-2a^2 - 30a - 108$

d) $5x^2 - 20x + 15$ **e)** $ax^2 - 14ax + 45a$ **f)** $-10a^4 + 100a^3 - 240a^2$

11. Consider the following in which each letter represents a whole number.

$x^2 + 5x + 6 = (x + A)(x + B)$ $x^2 + 10x + 21 = (x + B)(x + G)$

$x^2 - 9x + 20 = (x - T)(x - L)$ $2x^2 - 16x + 32 = 2(x - T)^2$

$x^3 + 10x^2 + 9x = x(x + S)(x + E)$ $6x^2 - 54x + 48 = 6(x - I)(x - S)$

Determine the value of each letter and hence name the famous person represented by the following code.

(3) (8) (5) (5) (7) (2) (4) (9) (1)
_ _ _ _ _ _ _ _ _

Multiple Choice

12. Which of the following is **not** a factor of $3m^2 - 27m + 54$?

A. $m - 3$
B. $m - 6$
C. $m - 9$
D. 3

13. For which of the following trinomials is $a + 5$ **not** a factor?

A. $a^2 + 6a + 5$
B. $a^2 + 11a + 30$
C. $a^2 + 10a + 50$
D. $a^2 + 10a + 25$

14. The expression $t^2 + kt + 12$ **cannot** be factored if k has the value

A. -13 **B.** -8 **C.** 7 **D.** 11

Numerical Response

15. The largest value of b for which $x^2 + bx + 32$ can be factored over the integers is ____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $x^2 + 4x + 3$ **b)** $x + 3$, $x + 1$, **c)** $x^2 + 4x + 3 = (x + 3)(x + 1)$
2. a) $x^2 - 7x + 10$ **c)** $x - 2, x - 5,$ $x^2 - 7x + 10 = (x - 2)(x - 5)$
3. a) $(x + 2)(x + 3)$ **b)** $(x + 1)(x + 5)$ **c)** $(x - 4)(x - 2)$
4. a) 2, 3 **b)** 1, 7 **c)** 5, 6 **d)** –5, –6
e) 1, 10 **f)** –3, –5 **g)** –7, –8 **h)** –4, –14
5. a) $(x + 3)(x + 4)$ **b)** $(x + 1)(x + 8)$ **c)** $(x - 2)(x - 5)$
d) $(t - 2)(t - 12)$ **e)** $(z + 5)(z + 3)$ **f)** $(b - 2)(b - 10)$

6. a) $(x + 1)(x + 2)$ **b)** $(x - 1)(x - 2)$ **c)** $(x + 3)(x + 6)$
d) $(x + 2)(x + 6)$ **e)** $(x - 3)(x - 7)$ **f)** $(x - 3)(x - 8)$

7. a) $(x + 1)(x + 10)$ **b)** not possible **c)** $(n + 4)(n + 8)$
d) $(y - 4)(y - 7)$ **e)** $(y + 3)(y + 14)$ **f)** $(f - 3)(f - 7)$
g) $(p - 2)(p - 14)$ **h)** $(x + 4)(x + 20)$ **i)** $(c - 2)(c - 30)$
j) not possible **k)** $(d + 3)(d + 15)$ **l)** $(p - 4)(p - 25)$
m) $(m + 11)(m + 11)$ **n)** $(n - 6)(n - 17)$ **o)** $(q - 5)(q - 23)$
OR $(m + 11)^2$
8. a) 7, 8, 13,–7, –8, –13 **b)** 5, 8, 9 **9. a)** $(x + 12)(x + 3)$ **b)** 15m, 6m

10. a) $2(x + 1)(x + 2)$ **b)** $4(x - 4)(x - 8)$ **c)** $-2(a + 6)(a + 9)$
d) $5(x - 1)(x - 3)$ **e)** $a(x - 5)(x - 9)$ **f)** $-10a^2(a - 4)(a - 6)$
11. BILL GATES **12.** C **13.** C **14.** D **15.** 3 3

Factoring Polynomial Expressions Lesson #3:

Factoring Trinomials of the Form $x^2 + bx + c$ - Part Two

Review of Factoring By Inspection

In order to factor $x^2 + bx + c$ by inspection, we need to find two integers which have a product equal to c and a sum equal to b. If no two such integers exist, then the polynomial cannot be factored.

In order to factor $x^2 + 6x + 9$, we need to find two numbers whose product is ___ and whose sum is ____ .

In order to factor $x^2 + x - 12$, we need to find two numbers whose product is ___ and whose sum is ____ .

Recall the following points from the previous lesson.

- If the product is **positive**, then the two integers must be either **both positive** or **both negative.**
- If the product is **negative**, then one integer is **positive** and the other is **negative**.

Class Ex. #1

Factor the following trinomials by inspection.

a) $x^2 - x - 12$ **b)** $x^2 + 3x - 18$ **c)** $a^2 - 7a - 8$

Class Ex. #2

Factor where possible.

a) $-a^2 - 6a + 27$

b) $2t^2 - 14t + 20$

c) $x^2 - 3x - 6$

d) $4x^4 - 16x^3 - 20x^2$

Complete Assignment Questions #1 - #5

Factoring Trinomials of the form $x^2 + bxy + cy^2$

Complete the following statements:

i) $(x+2)(x+4)$ can be expanded to $x^2 + 6x + 8$,
so $x^2 + 6x + 8$ can be factored to ( )( ) .

ii) $(x+2y)(x+4y)$ can be expanded to ________________ ,

so ________________ can be factored to ( )( ) .

Factor.

a) $x^2 + 13xy + 30y^2$ **b)** $x^2 + 71xy - 72y^2$ **c)** $3a^2 - 15ab - 252b^2$

Complete Assignment Questions #6 - #11

Assignment

1. Complete the table to find two numbers with the given sum and the given product.

	Sum	Product	Integers
a)	8	–20	
b)	–8	–20	
c)	–1	–20	

	Sum	Product	Integers
d)	3	–70	
e)	–11	28	
f)	0	–16	

2. Factor the following trinomials.

a) $x^2 - 2x - 15$ **b)** $x^2 - 2x - 24$ **c)** $x^2 + 2x - 24$

d) $x^2 + 2x - 3$ **e)** $x^2 + x - 30$ **f)** $x^2 - 3x - 10$

3. Factor where possible.

a) $x^2 + 10x + 16$ **b)** $x^2 - 11x + 18$ **c)** $x^2 - 2x - 8$ **d)** $x^2 + 3x - 18$

e) $x^2 - 4x + 12$ **f)** $x^2 - 4x - 12$ **g)** $x^2 - 10x + 25$ **h)** $x^2 + x - 20$

i) $m^2 + 21m + 38$ **j)** $a^2 - 17a + 42$ **k)** $p^2 - 10p - 9$ **l)** $p^2 - 9p - 10$

4. Factor.

a) $-x^2 - 7x - 12$ **b)** $4x^2 - 28x - 32$ **c)** $5x^2 - 20x + 15$

d) $-2a^2 + 2a + 220$ **e)** $b^2x^2 - 4b^2x - 45b^2$ **f)** $2x^3 + 2x^2 - 40x$

5. Consider the following in which the each letter represents a whole number.

$x^2 + 4x - 5 = (x + A)(x - O)$ $x^2 - 3x - 54 = (x - E)(x + I)$

$x^3 + 2x^2 - 8x = x(x - Y)(x + P)$ $3x^2 - 48x + 192 = T(x - R)^2$

$-5x^2 + 20x + 105 = -5(x + T)(x - H)$

Determine the value of each letter and hence name the fictional character represented by the following code.

(7) (5) (8) (8) (2) (4) (1) (3) (3) (9) (8)

_ _ _ _ _ _ _ _ _ _ _

6. Factor.

a) $x^2 + 18xy + 45y^2$ **b)** $x^2 + 10xy - 24y^2$ **c)** $a^2 - 12ab + 36b^2$

d) $p^2 - 12pq + 11q^2$ **e)** $x^2 + xy - 72y^2$ **f)** $x^2 - 54xy - 112y^2$

7. Factor completely.

a) $4x^2 - 80xy + 144y^2$ **b)** $3b^2 - 15bv - 72v^2$ **c)** $2c^2 + 66cd - 140d^2$

Multiple Choice

8. When factored, the trinomials $x^2 - 10x + 21$ and $x^2 - 4x - 21$ have one binomial factor in common. This factor is

A. $x - 7$ **B.** $x + 7$
C. $x - 3$ **D.** $x + 3$

9. One factor of $-m^3 - m^2 + 6m$ is

A. $m - 2$ **B.** $m + 2$
C. $m - 3$ **D.** $m - 6$

10. One factor of $3x^2 - 6xy - 9y^2$ is

A. $3x$ **B.** $x + 2y$
C. $x + 3y$ **D.** $x + y$

11. The expression $x^2 - 4x + c$ **cannot** be factored if c has the value

A. -5
B. 0
C. 4
D. 5

Answer Key

1. a) $-2, 10$ **b)** $-10, 2$ **c)** $-5, 4$ **d)** $-7, 10$ **e)** $-4, -7$ **f)** $-4, 4$
2. a) $(x - 5)(x + 3)$ **b)** $(x - 6)(x + 4)$ **c)** $(x + 6)(x - 4)$ **d)** $(x + 3)(x - 1)$
e) $(x + 6)(x - 5)$ **f)** $(x - 5)(x + 2)$
3. a) $(x + 8)(x + 2)$ **b)** $(x - 9)(x - 2)$ **c)** $(x + 2)(x - 4)$ **d)** $(x + 6)(x - 3)$
e) not possible **f)** $(x - 6)(x + 2)$ **g)** $(x - 5)^2$ **h)** $(x + 5)(x - 4)$
i) $(m + 2)(m + 19)$ **j)** $(a - 14)(a - 3)$ **k)** not possible **l)** $(p - 10)(p + 1)$
4. a) $-(x + 3)(x + 4)$ **b)** $4(x - 8)(x + 1)$ **c)** $5(x - 3)(x - 1)$ **d)** $-2(a - 11)(a + 10)$
e) $b^2(x - 9)(x + 5)$ **f)** $2x(x + 5)(x - 4)$ **5.** HARRY POTTER
6. a) $(x + 15y)(x + 3y)$ **b)** $(x - 2y)(x + 12y)$ **c)** $(a - 6b)^2$ **d)** $(p - q)(p - 11q)$
e) $(x - 8y)(x + 9y)$ **f)** $(x + 2y)(x - 56y)$
7. a) $4(x - 18y)(x - 2y)$ **b)** $3(b - 8v)(b + 3v)$ **c)** $2(c + 35d)(c - 2d)$
8. A **9.** A **10.** D **11.** D

Factoring Polynomial Expressions Lesson #4:
Difference of Squares

Investigation

a) Complete the following using the trinomial factoring method from the previous lessons.

	Sum	Product	Integers	Polynomial	Factored Form
i)	–6	–16		$x^2 - 6x - 16$	
ii)	–15	–16			
iii)	0	–16		$x^2 + 0x - 16 = x^2 - 16$	
iv)	0	–64			
v)	0	–25			

b) The third row in a) shows that the factored form of $x^2 - 16$ is $(x - 4)(x + 4)$. Use the pattern from the last three rows to factor the following.

i) $x^2 - 9 =$

ii) $x^2 - 49 =$

iii) $x^2 - 36 =$

iv) $x^2 - 1 =$

v) $a^2 - 100 =$

c) Extend the procedure from above to factor $m^2 - n^2$.
Verify your answer by expanding the factored form.

d) Consider the expansion $(x - y)(x + y) = x^2 + bx + c$.

i) Explain why the value of b is zero.

ii) Express c in terms of y.

Difference of Squares

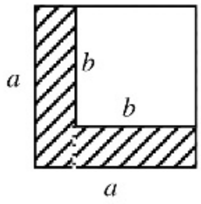
The examples on the previous page are trinomials of the form $x^2 + bx + c$, where $b = 0$ and c is the negative of a square number.

This results in a **difference of squares** such as $x^2 - 25$, $x^2 - 100$, etc.

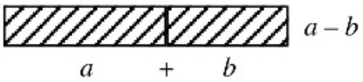
To factor a difference of squares we can use the identity:

$$\boxed{a^2 - b^2 = (a - b)(a + b)}$$

The identity $a^2 - b^2 = (a - b)(a + b)$ can be illustrated in the following diagram.



Shaded area = $a^2 - b^2$



Shaded area = $(a - b)(a + b)$

The shaded area on the left is cut along the dotted line and rearranged to form the diagram on the right.
The shaded area on the left is represented by $a^2 - b^2$ *and the shaded area on the right is represented by* $(a - b)(a + b)$.

Class Ex. #1

Factor the following polynomials using the difference of squares method.

a) $a^2 - 4$

b) $t^2 - 144$

c) $x^2 - y^2$

d) $p^2 - 7^2$

Note

Note that it is not possible to factor a sum of squares like $x^2 + 4$, i.e. $x^2 + 0x + 4$.
It is not possible to find two integers whose product is positive and whose sum is zero.

In the identity $a^2 - b^2 = (a - b)(a + b)$ we can replace a and/or b by numbers, variables, monomials and even polynomials.

For example, $4x^2 - 25$ can be written as $(2x)^2 - (5)^2$ and can be factored using the above identity with $a = 2x$ and $b = 5$.

$$4x^2 - 25 = (\qquad)(\qquad)$$

$9m^2 - 4n^2$ can be written as $(3m)^2 - (2n)^2$, and can be factored using the above identity with $a = 3m$ and $b = 2n$.

$$9m^2 - 4n^2 = (\qquad)(\qquad)$$

The factoring above can be verified by expanding the product of the factors.

Factor, if possible, using the difference of squares method.

a) $16t^2 - 49$ **b)** $81a^2 - 1$ **c)** $100 - y^2$

d) $36p^2 - 25q^2$ **e)** $4x^2 + 25$ **f)** $64 - 9a^2b^2$

The floor of an international doubles squash court is rectangular with an area of $25a^2 - b^2$ square feet.

a) Write expressions for the length and width of the floor.

b) The perimeter of the floor is 140 feet. Determine the length and width of the floor if the length is 1.8 times the width.

Difference of Squares involving a Common Factor

The first step in factoring any polynomial expression should be to determine if we can remove a common factor.

Factor the following polynomials by first removing the greatest common factor.

Class Ex. #4

a) $2a^2 - 50$ **b)** $3x^2 - 12y^2$ **c)** $144p^2q^2 - 4$ **d)** $3x^3 - 27x$

Complete Assignment Questions #1 - #14

Assignment

1. Complete the following by determining the missing factor.

a) $x^2 - 36 = (x - 6)(\quad)$ **b)** $c^2 - 121 = (c + 11)(\quad)$ **c)** $j^2 - k^2 = (j - k)(\quad)$

2. Factor the following polynomials using a difference of squares.

a) $x^2 - 49$ **b)** $x^2 - 1$ **c)** $x^2 - 15^2$ **d)** $x^2 - 400$

3. Explain how factoring a difference of squares in one variable can be regarded as a special case of factoring trinomials by inspection.

4. Factor where possible.

a) $m^2 - n^2$ **b)** $c^2 - 7^2$ **c)** $1 - k^2$ **d)** $g^2 - 64h^2$

e) $25x^2 - 144$ **f)** $16a^2 - 9b^2$ **g)** $4x^2 + z^2$ **h)** $121a^2 - 36b^2$

i) $49 - 4h$ **j)** $100 - 81b^2$ **k)** $1 - 25z^2$ **l)** $225a^2 - b^2$

m) $169z^2 - 4q^2$ **n)** $256 - y^2$ **o)** $t^2 + 36z^2$ **p)** $49a^2 - 400$

5. The floor of a classroom is rectangular with an area of $81m^2 - 4n^2$ square metres.

a) Write expressions in m and n for the length and width of the floor.

b) If the perimeter of the floor is 72 metres, form an equation in m and n and solve for m.

c) Determine the length and width of the floor if the length is 25% greater the width.

6. Factor.

a) $8x^2 - 32$ **b)** $4a^2 - 100y^2$ **c)** $3t^2 + 27s^2$

d) $7x^2 - 7y^2$ **e)** $9a^2b^2 - 36$ **f)** $8 - 50p^2q^2$

g) $xy^2 - x^3$ **h)** $20a^2b^2 - 5a^4b^4$

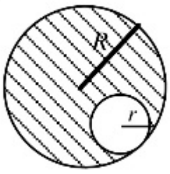
7. Factor.

a) $a^2b^2 - 9$ **b)** $c^2 - d^2e^2$ **c)** $100x^2 - y^2z^2$

d) $p^2q^2 - r^2s^2$ **e)** $25x^2y^2 - 1$ **f)** $c^2d^2 - 4f^2$

g) $4x^2a^2 - 49z^2t^2$ **h)** $16a^2c^2 - 225b^2d^2$

8. The diagram shows a circle of radius R with a circle of radius r removed.



a) Write an expression for the shaded area.

b) Write the expression in a) in factored form.

c) Determine the shaded area (as a multiple of π) if $R = 8.5$ and $r = 1.5$.
Do not use a calculator.

9. The expression $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$ occurs in physics.

a) Write the expression in factored form.

b) Determine the value of the expression when $m = 10$, $v = 75$, and $u = 25$.
Do not use a calculator.

10. Consider the following in which each letter represents a whole number.

$64x^2 - y^2 = (Hx - y)(Hx + y)$

$16x^2 - 4 = C(Ix + 1)(Ix - 1)$

$7x^2 - 252y^2 = P(x - Ey)(x + Ey)$

$Lx^2 - Ny^2 = (3x - 5y)(Sx + Ay)$

Determine the value of each letter and hence name the country represented by the following code.

(4) (8) (2) (9) (6)
_ _ _ _ _

11. Susan was showing Rose how the difference of squares method can be used to multiply certain numbers without using a calculator. She showed Rose the following:

$$38 \times 42$$
$$= (40-2)(40+2) \quad = (40^2 - 2^2) \quad = (1600 - 4) \quad = 1596$$

a) Use the above process to evaluate:

i) 27×33

ii) 61×59

b) Explain why this process is more difficult to determine the product 66×72.

c) Make up your own multiplication question which can be answered using this process.

Multiple Choice

12. One factor of $16 - 4m^2$ is

A. $4 - m$
B. $8 - 2m$
C. $4 + m$
D. $2 + m$

13. Given that $x^2 - y^2 = 45$ and $x + y = 9$, the value of x is

A. 2
B. 5
C. 7
D. impossible to determine

Numerical Response

14. $3x + 2y$ is a factor of the binomial $a^2x^2 - b^2y^2$.
The value of $a^2 + b^2$ is _____ .
(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $(x + 6)$ **b)** $(c - 11)$ **c)** $(j + k)$

2. **a)** $(x - 7)(x + 7)$ **b)** $(x - 1)(x + 1)$ **c)** $(x - 15)(x + 15)$ **d)** $(x - 20)(x + 20)$

3. A difference of squares can be regarded as a trinomial of the form $x^2 + bx + c$ in which $b = 0$ and c is negative. We need to find two numbers which multiply to c and add to zero.

4. **a)** $(m - n)(m + n)$ **b)** $(c - 7)(c + 7)$ **c)** $(1 - k)(1 + k)$
d) $(g - 8h)(g + 8h)$ **e)** $(5x - 12)(5x + 12)$ **f)** $(4a - 3b)(4a + 3b)$
g) not factorable **h)** $(11a - 6b)(11a + 6b)$ **i)** not factorable using whole number exponent.
j) $(10 - 9b)(10 + 9b)$ **k)** $(1 + 5z)(1 - 5z)$ **l)** $(15a + b)(15a - b)$
m) $(13z - 2q)(13z + 2q)$ **n)** $(16 - y)(16 + y)$ **o)** not factorable **p)** $(7a + 20)(7a - 20)$

5. **a)** $(9m + 2n)$ metres, $(9m - 2n)$ metres **b)** $2(9m + 2n) + 2(9m - 2n) = 72$, $m = 2$
c) Length = 20 metres, Width = 16 metres.

6. **a)** $8(x - 2)(x + 2)$ **b)** $4(a - 5y)(a + 5y)$ **c)** $3(t^2 + 9s^2)$ **d)** $7(x - y)(x + y)$
e) $9(ab - 2)(ab + 2)$ **f)** $2(2 - 5pq)(2 + 5pq)$ **g)** $x(y - x)(y + x)$ **h)** $5a^2b^2(2 - ab)(2 + ab)$

7. **a)** $(ab - 3)(ab + 3)$ **b)** $(c - de)(c + de)$ **c)** $(10x - yz)(10x + yz)$
d) $(pq - rs)(pq + rs)$ **e)** $(5xy - 1)(5xy + 1)$ **f)** $(cd - 2f)(cd + 2f)$
g) $(2xa - 7zt)(2xa + 7zt)$ **h)** $(4ac - 15bd)(4ac + 15bd)$

8. **a)** $A = \pi R^2 - \pi r^2$ **b)** $\pi(R - r)(R + r)$ **c)** 70π

9. **a)** $\frac{1}{2}m(v - u)(v + u)$ **b)** 25 000 **10.** CHILE

11. **a)** **i)** 891 **ii)** 3599
b) 66×72 expressed as a difference of squares (69^2 - 3^2) cannot easily be evaluated without a calculator or long multiplication.

12. D **13.** C **14.**

1	3		

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Factoring Polynomial Expressions Lesson #5:
Factoring Review

Guidelines for Factoring a Polynomial Expression

If we are asked to factor a polynomial expression, the following guidelines should help us to determine the best method.

1. Look for a common factor. If there is one, take out the common factor and look for further factoring.
2. If there is a binomial expression, look for a difference of squares.
3. If there is a trinomial expression of the form $x^2 + bx + c$, look for factoring by inspection.
4. After factoring, check to see if further factoring is possible.

Polynomial expressions of the form $ax^2 + bx + c$ will be discussed in the next math course.

Factor the following.

a) $9x^2 - 36$ **b)** $x^2 - 16x - 36$ **c)** $-x^2 + 26x + 27$ **d)** $x^2 - 3x - 5x + 15$

Complete Assignment Questions #1 - #9

Assignment

1. Factor.

a) $x^2 - 49$ **b)** $x^2 - 8x + 15$ **c)** $8x^2 + 32$

d) $-a^2 + 64$ **e)** $e^2 - 3e + 4$ **f)** $v^2 + 7v + 10$

g) $a^2 + 2ab - 35b^2$ **h)** $4 - 25t^2$ **i)** $x^2 + 16$

2. Factor.

a) $a^2 - 64b^2$ **b)** $108 - 3z^2$ **c)** $-x^2 - 5x - 4$

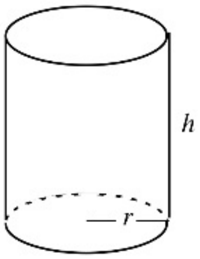
d) $625p^2 - 1$ **e)** $-3x^2 - 3x + 36$ **f)** $8v^2 - 32v - 96$

3. Factor.

a) $b^2 - 16 - 6b + 24$ **b)** $x^3 - 81x$ **c)** $-256 + t^2$

d) $12 - 4x - x^2$ **e)** $x^2 - 8xy - 33y^2$

4. The surface area of a cylinder is given by the formula $A = 2\pi r^2 + 2\pi rh$, where r is the radius of the base and h is the height of the cylinder.



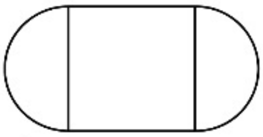
a) Calculate the surface area, to the nearest $0.01\ \text{cm}^2$, of a cylinder which has vertical height 14.5 cm and base diameter 11 cm.

b) Write the formula for A in factored form.

c) Calculate, using the factored form of A, the surface area of the cylinder to the nearest $0.01\ \text{cm}^2$.

d) Which method a) or c) is simpler to use?

5. A square of side $2r$ cm has semicircles drawn externally on each of two opposite sides.



Find expressions in factored form for

a) the external perimeter of the shape

b) the area of the shape

Multiple Choice

Use the following information to answer the next two questions.

In questions #6 -#7 one or more of the four responses may be correct. Answer

A. if only 1 and 2 are correct
B. if only 1, 2, and 3 are correct
C. if only 3 and 4 are correct
D. if some other response or combination of responses is correct

6. The set of factors of $5x^2 - 10x - 15$ contains

1. $x - 1$ **2.** $x + 3$ **3.** $x + 1$ **4.** $x - 3$

7. $x + 4$ is a factor of
1. $-x^2 - 6x - 8$ **2.** $48 - 3x^2$ **3.** $3x^2 + 12x$ **4.** $x^2 + 16$

8. $\pi r^3 + 3\pi r$ is equivalent to

A. $3\pi^2 r^4$
B. $3\pi(r^2 + r)$
C. $\pi r(2r + 3)$
D. $\pi r(r^2 + 3)$

Numerical Response

9. Triangle PQR is right angled at P. The area of the triangle is $\frac{1}{2}x^2 + 10x + 18$ cm^2, where x is a positive integer.

If the length of PQ is 10 cm, then the length of PR, is _____ cm.
(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $(x-7)(x+7)$ **b)** $(x-5)(x-3)$ **c)** $8(x^2+4)$
d) $-(a+8)(x-8)$ **e)** not factorable **f)** $(v+5)(v+2)$
g) $(a+7b)(a-5b)$ **h)** $(2-5t)(2+5t)$ **i)** not factorable

2. **a)** $(a-8b)(a+8b)$ **b)** $3(6-z)(6+z)$ **c)** $-(x+4)(x+1)$
d) $(25p-1)(25p+1)$ **e)** $-3(x-3)(x+4)$ **f)** $8(v+2)(v-6)$

3. **a)** $(b-2)(b-4)$ **b)** $x(x-3)(x+3)$ **c)** $(t-4)(t+4)$
d) $-(x-2)(x+6)$ **e)** $(x-11y)(x+3y)$

4. **a)** 691.15 cm^2 **b)** $A = 2\pi r(r+h)$ **c)** 691.15 cm^2 **d)** c) is simpler

5. **a)** $2r(\pi+2)$ cm. **b)** $r^2(\pi+4)$ cm^2.

6. C **7.** B **8.** D **9.**

2	6		

Factoring Polynomial Expressions Lesson #6
Enrichment Lesson - Solving Polynomial Equations

Investigation

The **Zero Product Property** states the following:

If $ab = 0$, then either $a = 0$ or $b = 0$

Complete the following to investigate the use of the Zero Product Property (also referred to as the **Zero Product Rule**) in solving polynomial equations.

- The statement $x - 3 = 0$ is true only if $x =$ ______.
- The statement $x + 1 = 0$ is true only if $x =$ ______.
- The statement $(x - 3)(x + 1) = 0$ is true if $x =$ ______ or if $x =$ _____.
- The statement $4(x - 3)(x + 1) = 0$ is true if ______________________.

All the above statements are polynomial equations in which the left side is a polynomial expression and the right side equals zero.

The **solution** to a polynomial equation is given by stating the value(s) of the variable which make(s) the left side and the right side equal. These values are said to **satisfy** the equation.

Solving Polynomial Equations

Consider the equation $x^2 - 2x - 3 = 0$. Factoring the left side leads to $(x - 3)(x + 1) = 0$. This is true if $x = 3$ or if $x = -1$. Since the equation is satisfied by both $x = 3$ and $x = -1$, the solutions to the equation are $x = 3$ **and** $x = -1$, sometimes written as $x = -1, 3$.

Class Ex. #1

Complete the solution to the equation $x^2 - 9x + 20 = 0$.

$x^2 - 9x + 20 = 0$

$(x - \quad)(x - \quad) = 0$

$x - \quad = 0$ or $x - \quad = 0$ The solutions are $x =$ ____ and $x =$ ______

$x =$ ____ or $x =$ _____ or $x =$ ___ , ___

Class Ex. #2

Solve the equation.

a) $x^2 - 81 = 0$ **b)** $4x^2 - 9 = 0$ **c)** $10x^2 - 90x = 0$ **d)** $10x^2 - 90 = 0$.

Class Ex. #3

Solve the following equations.

a) $(3x + 2)(x - 5) = 0$ **b)** $5x^2 + 30x = -25$

Complete Assignment Questions #1 - #3

Problem Solving with Polynomial Equations

Some problems in mathematics can be solved by the following procedure:

i) Introduce a variable to represent an unknown value.

ii) Form a polynomial equation from the given information.

iii) Solve the polynomial equation using the methods in this lesson.

iv) State the solution to the problem.

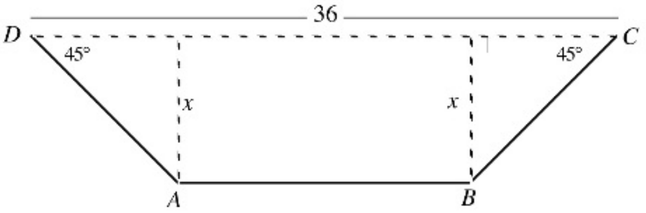
In this section we will consider fairly routine problems. This topic will be extended in a higher level math course.

Class Ex. #4

The area of a rectangular sheet of paper is $300\ \text{cm}^2$. The length is 5 cm more than the width. Form a polynomial equation and solve it to determine the perimeter of the rectangular sheet.

Class Ex. #5

The diagram shows the cross-section of a water trough whose sloping sides AD and BC make an angle of $45°$ with the horizontal. The length $DC = 36$ cm.



a) Show that the area of the cross-section is $x(36 - x)$ cm^2.

b) If the area of the cross-section is 260 cm^2, determine the value of x.

Complete Assignment Questions #4 - #8

Assignment

1. Solve the equation.

a) $(x - 2)(x + 7) = 0$

b) $(3x - 2)(2x + 5) = 0$

c) $5x(10 - x) = 0$

d) $x^2 + 2x = 0$

e) $x^2 - 121 = 0$

f) $9x^2 - 100 = 0$

g) $36x^2 = 25$

h) $9x - 4x^2 = 0$

i) $4(49 - x^2) = 0$

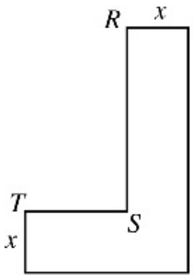
2. Solve the equation.

a) $x^2 - 3x + 2 = 0$ **b)** $x^2 + 13x + 30 = 0$ **c)** $x^2 + 2x - 15 = 0$

3. Solve the equation.

a) $x(x + 4) = 32$ **b)** $(2x - 3)^2 = 1$ **c)** $(x + 1)(x - 1) = 5(x + 1)$

4. The diagram shows a piece of wood of uniform width x cm.
$RS = 10$ cm and $ST = 7$ cm.



a) Find the area of the piece of wood in terms of x.

b) Find the value of x if the area is 60 cm^2.

5. The sum of the first n even numbers, starting with 0, is given by the formula $S = n(n - 1)$.

a) Determine the sum of the first 25 even numbers, starting with 0.

b) How many consecutive even numbers, starting with 0, add up to 870?

6. The height of a triangle is 8 mm more than the base. The area is 172.5 mm^2.

a) Write a polynomial equation to model this information.

b) Determine the height of the triangle.

Multiple Choice

7. The complete solution to the equation $x(x-1) = 2$ is

A. $x = 0$ and $x = 1$
B. $x = 2$ and $x = 3$
C. $x = -1$ and $x = 2$
D. $x = -2$ and $x = 1$

Numerical Response

8. The sum of the first n natural numbers is given by the formula $S = \frac{1}{2}n(n+1)$.
If the first k natural numbers have a sum of 496, the value of k is _____.

(Record your answer in the numerical response box from left to right) ☐☐☐☐

Answer Key

1. **a)** $2, -7$ **b)** $\frac{2}{3}, -\frac{5}{2}$ **c)** $0, 10$ **d)** $0, -2$

e) ± 11 **f)** $\pm\frac{10}{3}$ **g)** $\pm\frac{5}{6}$ **h)** $0, \frac{9}{4}$ **i)** ± 7

2. **a)** $1, 2$ **b)** $-10, -3$ **c)** $-5, 3$

3. **a)** $-8, 4$ **b)** $1, 2$ **c)** $-1, 6$

4. **a)** $x^2 + 17x$ cm^2 **b)** 3 **5.** **a)** 600 **b)** 30

6. **a)** $x^2 + 8x - 345 = 0$ **b)** 23 mm **7.** C **8.**

3	1		

Factoring Polynomial Expressions Lesson #7:
Practice Test

1. One factor of $9x^4 - 6x^3 + 3x^2$ is

- **A.** $9x^4$
- **B.** $3x^2 - 2x$
- **C.** $3x^2 - 6x + 3$
- **D.** $3x^2 - 2x + 1$

2. When fully factored, the expression $x^3y^2 - x^2y^3$ is written

- **A.** $xy^2(x - xy)$
- **B.** $x^2y(xy - y)$
- **C.** $x^2y^2(x - y)$
- **D.** $x^3y^2(1 - xy)$

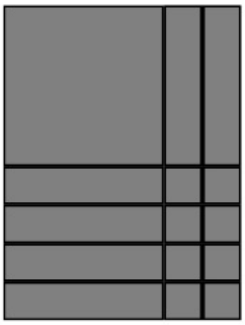
Numerical Response **1.** When the greatest common factor is removed from the binomial $75x - 50x^2$, the binomial can be written in the form $ax(b + cx)$. The value of $a - b - c$ is _____ .

(Record your answer in the numerical response box from left to right)

3. The expression $3ab^2 - 6a^3b + 3ab$, when fully factored, is written

- **A.** $ab(3b - 2a^2 + 3)$
- **B.** $3a(b^2 - 2a^2b + b)$
- **C.** $3ab(b - 2a^2)$
- **D.** none of these

4. The algebra tile diagram represents the factored form



- **A.** $(x^2 + 2x)(x^2 + 4x)$
- **B.** $8(x + 2)(x + 4)$
- **C.** $(x + 2)(x + 4)$
- **D.** $(x + 1)(x + 8)$

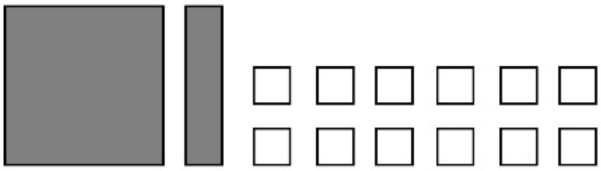
5. One factor of $a^2 - 10a - 24$ is

- **A.** $a - 2$
- **B.** $a - 4$
- **C.** $a - 6$
- **D.** $a - 12$

Use the following information to answer the next question.

An algebraic expression is represented by the algebra tiles shown.

Shaded tiles are positive.

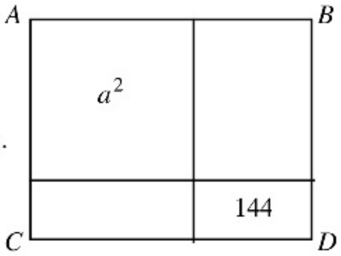


6. The factored form of the algebraic expression represented by the algebra tiles is

A. $(x-3)(x+4)$
B. $(x-3)(x-4)$
C. $(x-4)(x+3)$
D. x^2+x-12

Use the following information to answer the next question.

Rectangle *ABCD* has been subdivided into four regions. The areas of two of these regions are a^2 and 144 as indicated. The combined area of the other two regions is $25a$.



7. The perimeter of rectangle *ABCD* is

A. $a^2+25a+144$
B. $4a+48$
C. $4a+50$
D. unable to be determined from the given information

8. For which of the following trinomials is $b+3$ **not** a factor?
A. b^2+3b
B. b^3-9b
C. $b^2+2b-15$
D. $b^2-6b-27$

9. The expression $x^2 + px + 20$ **cannot** be factored over the integers if p has the value

A. -9
B. -12
C. 21
D. 20

Numerical Response **2.** The largest value of w for which $x^2 - wx + 48$ can be factored over the integers is ____ .

(Record your answer in the numerical response box from left to right)

10. One factor of $7a^2 - 28a^4$ is

A. $1 - 2a$
B. $28a^4$
C. $1 - 4a$
D. $1 - a$

11. Which of the following is a factor of $y^2 - y - 42$?

A. $y - 6$
B. $y + 7$
C. $y - 2$
D. $y + 6$

Numerical Response **3.** The polynomial expression $4x^2 + 40x + 100$ can be written in the form $A(x + B)^2$. The value for the product AB is _____.

(Record your answer in the numerical response box from left to right)

Use the following information to answer the next three questions.

In each of questions #12 - 14 four responses are given.
Answer
- **A.** if response 1 and response 2 only are correct
- **B.** if response 1 and response 3 only are correct
- **C.** if response 2 and response 4 only are correct
- **D.** if no response or some other response or combination of the responses is correct

12. Which of the following are factors of $2t^2 + 4t - 30$?

Response 1: $t + 3$
Response 2: $t + 5$
Response 3: $t - 5$
Response 4: $t - 3$

13. $x^2 + 25y^2$ has as a factor

Response 1 $x - 5y$
Response 2 $x - y$
Response 3 $x + 5y$
Response 4 $x + y$

14. The trinomial $x^2 - 12x + c$ can be factored over the integers if

Response 1 $c = 20$
Response 2 $c = -28$
Response 3 $c = -32$
Response 4 $c = -27$

Numerical Response **4.** A farmers field has an area of $a^2 + 22a + c$ m^2.
If the field is square, then the value of c must be _______ .
(Record your answer in the numerical response box from left to right)

15. When fully factored, the expression $16a^4 - a^2$ is written

A. $a^2(16a^2 - 1)$
B. $(4a^2 - a)(4a^2 + a)$
C. $(4a^2 - 1)(4a^2 + 1)$
D. $a^2(4a - 1)(4a + 1)$

Numerical Response **5.** Three algebraic expressions have been partially factored.

$x^2 - 5x - 14 = (x - 7)(x + A)$

$x^4 - 9x^2 = x^2(x - B)(x + B)$

$5x^2 - 40x + 80 = C(x - D)^2$

Write the value of A in the first box.
Write the value of B in the second box.
Write the value of C in the third box.
Write the value of D in the fourth box.
(Record your answer in the numerical response box from left to right)

Written Response - 5 marks

1. Students are investigating polynomials of the form $x^2 + bx + c$, where b and c are integers.

- State any polynomial with $c = 16$ which can be factored over the integers.
- State any polynomial with $c = 16$ which cannot be factored over the integers. **Explain** why factoring is not possible in this case.

- If $c = 16$, determine how many polynomials of this type can be factored over the integers.

- State two polynomials of the form $x^2 + bx + c$ which can be factored over the integers and in which $b + c = 20$.

Answer Key

1. D	**2.** C	**3.** D	**4.** C	**5.** D	**6.** A	**7.** C	**8.** C
9. D	**10.** A	**11.** D	**12.** C	**13.** D	**14.** A	**15.** D	

Numerical Response

1. 2 4

2. 4 9

3. 2 0

4. 1 2 1

5. 2 3 5 4

Written Response

1. • $x^2 + 10x + 16$ or any answer in bullet 3.

- Many answers are possible e.g. $x^2 + 3x + 16$ is not able to be factored because it is not possible to find two integers which multiply to 16 and add to 3.

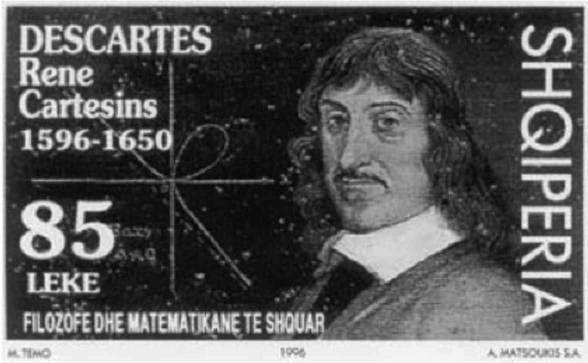
- polynomials are possible $x^2 + 10x + 16$ $x^2 + 8x + 16$ $x^2 + 17x + 16$
  $x^2 - 10x + 16$ $x^2 - 8x + 16$ $x^2 - 17x + 16$

- For example $x^2 + 10x + 9$ $x^2 + 7x + 12$

Relations Lesson #1:
Review and Preview

This lesson reviews some important concepts that will appear throughout the next four units.

The Cartesian Coordinate System



In mathematics, "Cartesian"refers to the French mathematician **René Descartes**. In 1637 he introduced the idea of specifying the position of a point on a surface using two intersecting axes as measuring guides.

The modern Cartesian coordinate system, in two dimensions, is defined by two axes at right angles to each other forming a plane (called the *xy* plane). The horizontal axis is labelled *x*, and the vertical axis is labelled *y*. All the points in a Cartesian coordinate system form a **Cartesian plane**.

The point of intersection of the two axes is called the **origin**, usually labelled O. On each axis, a unit length is chosen and units are marked off to form a grid. To specify a particular point on the grid, we use a unique **ordered pair** of numbers called **coordinates**. The first number in the ordered pair, called the ***x*-coordinate**, identifies the position with regard to the *x*-axis, while the second number, called the ***y*-coordinate**, identifies the position with regard to the *y*-axis. The point $P(4, 3)$ is shown on the grid below. The intersection of the *x*-axis and *y*-axis creates four **quadrants**, numbered counterclockwise starting from the north-east quadrant.

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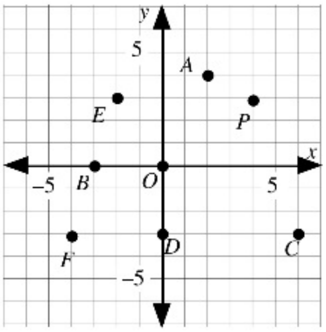
a) Complete the following by writing the coordinates of the points represented by the letters on the grid.

A(*B*(*C*(

D(*O*(



b) Write the coordinates of the point in the second quadrant.

c) Write the coordinates of the point in quadrant III.

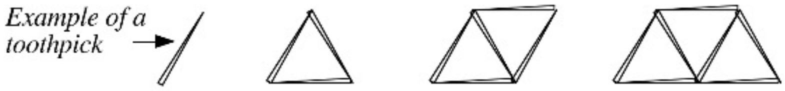
d) Complete the following table using "positive" or "negative.

Quadrant	*x*-coordinate	*y*-coordinate
I		
II		
III		
IV		

Describing a Pictorial Pattern Using a Linear Relation

Use the following information to answer this Class Example.

Three toothpicks are used to form a triangle. A second triangle is formed by adding two more toothpicks. A third triangle is formed by adding another two more toothpicks, and the pattern continues.

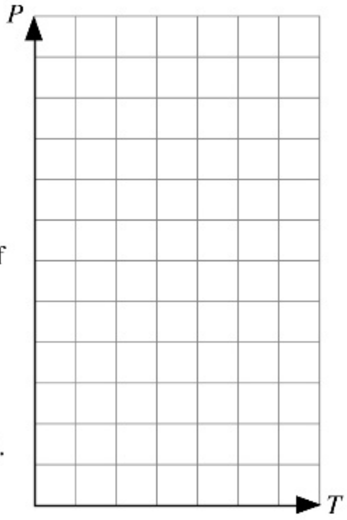


a) Draw the next two diagrams in the pattern.

b) Complete the table relating the number of toothpicks, P, to the number of triangles, T.

Number of Triangles, T	1	2	3	4	5
Number of Toothpicks, P	3				

c) Represent the data from the table of values on the grid.



d) Explain why it does not make sense to join the points in a straight line.

e) Describe in words the relationship between the number of toothpicks and the number of triangles.

f) Write an equation that can be used to determine the number of toothpicks if we know the number of triangles.

g) Use the equation to determine the number of toothpicks if there are 27 triangles.

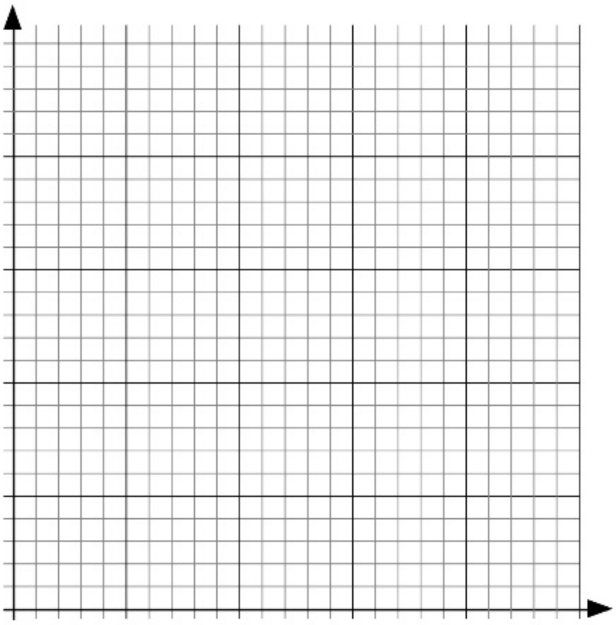
h) Use the equation to determine the number of triangles if there are 83 toothpicks.

Students in a Physics classroom determined the resistance in an electrical circuit by measuring the voltage (V volts) when a current (I amps) was passed through the circuit. The table below shows the results.

Current, (I) in amps	0.5	1.5	3	3.5	4
Voltage, (V) in volts	6	18	36	42	48

a) Represent the data from the table on the grid.

b) Does it make sense to join the points together with a straight line? Explain.



c) Use the graph to determine the voltage for a current of 2.5 amps.

d) Write an equation expressing V in terms of I.

e) Use the equation to determine the voltage when the current is 6.2 amps.

f) The resistance (R ohms) is calculated from the equation $V = IR$. State the resistance in this electrical circuit.

Discrete and Continuous Variables

In Class Example #3, the variables I and V are examples of **continuous variables** since they can take on every value within a particular interval, i.e. a variable for which it is possible to find an intermediate value between any two values. For the graph in this example, the current can take on any value between 0 and 5.

In Class Example #2, the variables P and T can only take on limited values (in this case whole number values) and are therefore NOT continuous variables.
Such variables are called **discrete variables**.

A graph relating two discrete variables consists of a series of unconnected points, whereas in the graph of two continuous variables the points would be connected.

Classify each of the following variables as discrete or continuous.

a) time taken to complete a 100 m sprint **b)** number of students who pass Math 10

c) height of students **d)** shoe size

Complete Assignment Questions #1 - #12

Assignment

1. Complete the following statements.

a) The first coordinate of an ordered pair is called the _____ coordinate and the second coordinate is called the _____ coordinate.

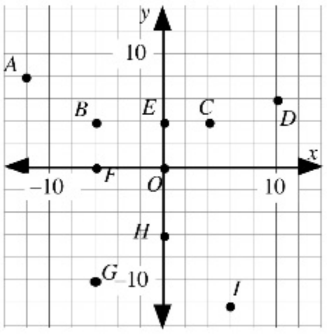
b) _____________ _____________ are used to locate, or plot, points on a Cartesian Plane.

c) The numbers of an ordered pair are called the ______________ of a point on the grid.

d) The x-axis and the y-axis intersect at the ____________ .

e) The Cartesian plane is divided into four ____________ .

2. The following questions refer to the grid on the right.



a) Name the points represented by the following coordinates.
i) (–6, 4) **ii)** (6, –12) **iii)** (0, 4)

b) List the coordinates of each point.
i) *C* **ii)** *G* **iii)** *F* **iv)** *O* **v)** *H*

c) Which points are in
i) quadrant 1 **ii)** quadrant 2

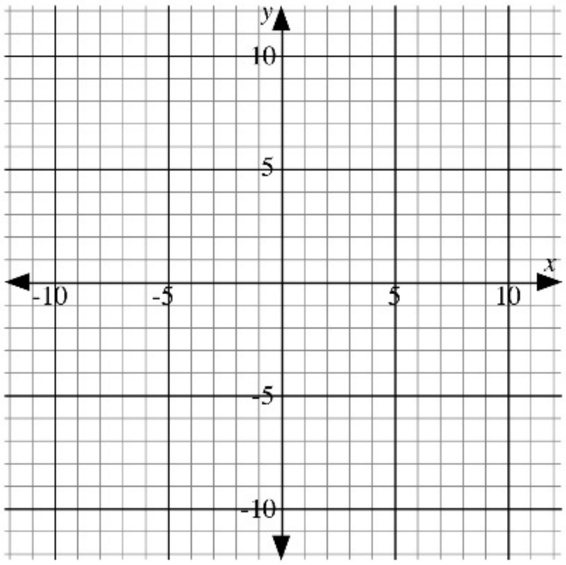
iii) quadrant 3 **iv)** quadrant 4

d) Which points are in between quadrants?

e) Which points have the same x-coordinate? What visual check can be used?

f) Which points have the same y-coordinate? What visual check can be used?

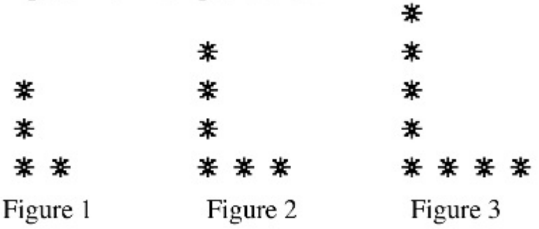
3. The following is a scrambled message using ordered pairs. Plot the ordered pairs on the grid provided. Unscramble the letters, and find the message. The symbol • represents the beginning of a new letter.



- Join (5, –9) to (5, –5). Join (7, –9) to (5, –7) to (7, –5).
- Join (–11, –2) to (–10, 0) to (–9, 2) to (–8, 0) to (–7, –2). Join (–8, 0) to (–10, 0).
- Join (0, –10) to (0, –6) to (2, –6) to (2, –8) to (0, –8) to (2, –10).
- Join (2, 0) to (0, 0) to (0, –2) to (2, –2) to (2, –4) to (0, –4).
- Join (8, 5) to (8, 9). Join (10, 5) to (10, 9). Join (8, 7) to (10, 7).
- Join (–3, 2) to (–2, 6) to (–1, 4) to (0, 6) to (1, 2).
- Join (3, 8) to (5, 8). Join (4, 8) to (4, 4).
- Join (6, 1) to (4, 1) to (4, –3) to (6, –3).
- Join (–6, –6) to (–6, –10) to (–4, –10) to (–4, –6) to (–6, –6).

Use the following information to answer the next question.

A pattern of L-shapes is shown

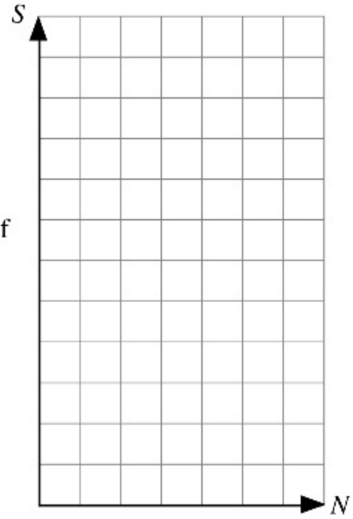


4.a) Draw the next two diagrams in the pattern.

b) Complete the table relating the number of stars, S, to the figure number, N.

Figure Number, N	1	2	3	4	5
Number of Stars, S					

c) Represent the data from the table of values on the grid.



d) Does it make sense to join the points in a straight line? Explain.

e) Describe in words the relationship between the number of stars and the figure number.

f) Write an equation that can be used to determine the number of stars, if we know the figure number.

g) Use the equation to determine the number of stars in figure 43.

h) Use the equation to determine the figure number, if there are 140 stars.

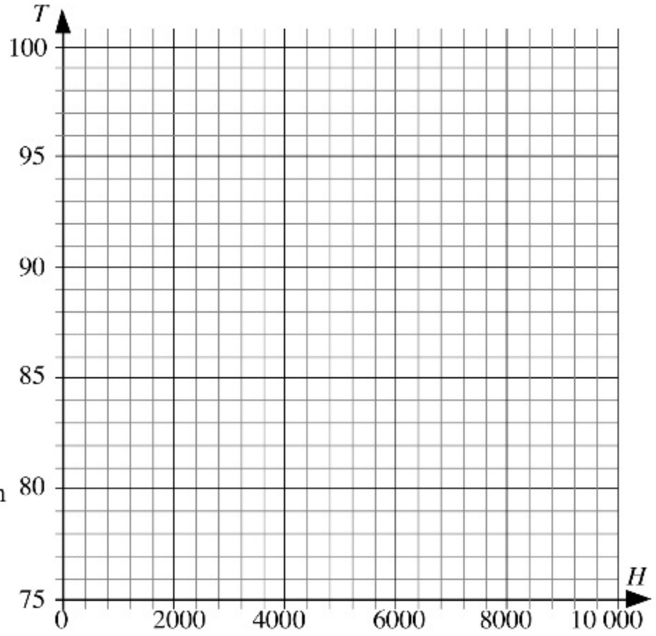
Use the following information to answer the next question.

The boiling point of water varies according to the atmospheric pressure and the height above sea level. The table below shows the approximate boiling temperature of water in $°C$, relating to the height in metres above sea level.

Height Above Sea Level, H	0	2000	4000	6000	8000
Boiling Temperature, T	100	94	88	82	76

5.a) Represent the data from the table on the grid.

b) Does it make sense to join the points together with a straight line? Explain.



c) Use the graph to estimate the boiling temperature of water at a height of 3000 m above sea level.

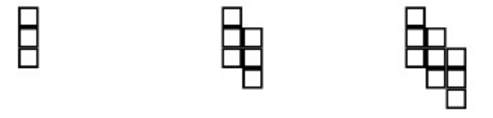
d) Use the graph to estimate the height above sea level at which the boiling temperature of water is 85°C.

e) The equation relating T and H is $T = 100 - 0.003H$. Use the equation to check the accuracy of your estimates in **c)** and **d)**.

f) In the context of the question, what does the value 100 represent in the equation $T = 100 - 0.003H$?

Use the following information to answer the next question.

A pattern of squares is shown. The length of each side of a square is 1 cm.

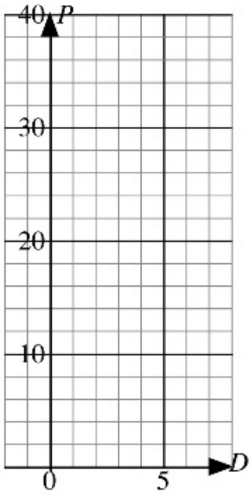


6.a) Draw the next two diagrams in the pattern.

b) Complete the table relating the perimeter of each shape, P, to the diagram number, D.

Diagram Number, *D*	1	2	3	4	5
Perimeter in cm, *P*	8				

c) Represent the data from the table of values on the grid.



d) Does it make sense to join the points in a straight line? Explain.

e) Describe in words the relationship between the perimeter and the diagram number.

f) Write an equation that can be used to determine the perimeter if we know the diagram number.

g) Use the equation to determine the perimeter of diagram 8.

h) Use the equation to determine the diagram number if the perimeter is 56 cm.

7. Describe whether data points should or should not be connected on the graph of a relation in a given situation.

Multiple Choice

8. Which of the following variables is discrete?

A. temperature
B. weight
C. altitude
D. number of goals

9. Which of the following variables is continuous?

A. number of correct answers on a test
B. number of letters in the alphabet
C. number of grams of sugar in a pear
D. number of students in a class

10. Consider the following variables:

i) The age of a truck.
ii) The weight of a truck.
iii) The number of wheels on a truck.
iv) The number of litres of gas in the gas tank of a truck.

Which of the variables above is/are continuous?

A. ii) only
B. i) and ii) only
C. i), ii), and iv) only
D. some other combination of i), ii), iii), and iv)

11. On a Cartesian Plane, the line segment joining the points $(-3, -2)$ and $(5, -5)$

A. intersects both the x-axis and y-axis.
B. intersects the y-axis but not the x-axis.
C. intersects the x-axis but not the y-axis.
D. does not intersect the x-axis or the y-axis.

Numerical Response

12. The relationship between degrees Celsius, C, and degrees Fahrenheit, F, is described by the equation $C = \frac{5}{9}(F - 32)$. The ignition temperature of paper is 451°F.
To the nearest degree, the ignition temperature of paper in degrees Celsius is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** x y **b)** ordered pairs **c)** coordinates **d)** origin **e)** quadrants

2. **a)** **i)** B **ii)** I **iii)** E
b) **i)** (4, 4) **ii)** (–6, –10) **iii)** (–6, 0) **iv)** (0, 0) **v)** (0, –6)
c) **i)** C, D **ii)** A, B **iii)** G **iv)** I
d) E, F, H, O
e) B, F, G, E, O, H They lie on the same vertical line.
f) B, E, C F, O, They lie on the same horizontal line.

3. MATH ROCKS

4. **a)**

```
                  *
    *             *
    *             *
    *             *
    *             *
    *             *
    * * * * *     * * * * * *
    Figure 4      Figure 5
```

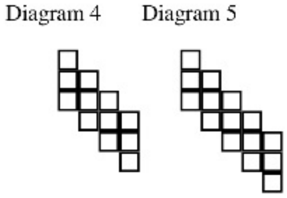
b)

Figure Number, N	1	2	3	4	5
Number of Stars, S	**4**	**6**	**8**	**10**	**12**

d) No, the variables are discrete.
Intermediate values have no meaning.
e) The number of stars is twice the figure number plus two.
f) $S = 2N + 2$ **g)** 88 **h)** 69

5. **b)** Yes. The data is continuous. Intermediate values have meaning.
c) 91 °C **d)** 5 000 m **e)** 91 °C, 5 000 m
f) 100°C is the boiling temperature of water at sea level.

6. **a)**



b)

Diagram Number, D	1	2	3	4	5
Perimeter in cm, P	**8**	**12**	**16**	**20**	**24**

d) No, the variables are discrete.
Intermediate values have no meaning.
e) The perimeter is 4 more than 4 times the diagram number.
f) $P = 4D + 4$ **g)** 36 cm **h)** Diagram 13

7. If the data is continous, i.e. all intermediate values have meaning, then the data points should be connected. If the data is discrete, ie not all intermediate points have meaning, then the data points should not be connected.

8. D **9.** C **10.** C **11.** B **12.**

2	3	3	

Relations Lesson #2:
Relationships Between Two Quantities

Relations

Much of mathematics involves the search for patterns and relationships between sets of data. Many real life applications of mathematics investigate the relationship between two quantities.

For example:

- the value of a computer is related to its age
- the price of a watermelon is related to its weight
- the time taken for a person to walk to school is related to the walking distance.

In mathematics, a comparison between two sets of elements is called a **relation**.

Class Ex. #1

List one more example of a relation.

Representing the Relationship Between Two Quantities

In the next two units we will consider seven ways in which the relationship between two quantities can be represented.

- in words
- a table of values
- a set of ordered pairs
- a mapping (or arrow) diagram
- an equation
- a graph
- function notation (some relations can be represented in this way as in the next unit)

We will use the relation below as an example.

Investigating a Relation

Consider the following relation:

> **" The cost, C (cents per km), of driving a car is related to the speed, s (km/h), at which it is driven."**

We will use this relation to introduce some ideas which will be developed throughout the course of this unit. Our task is to represent this relation in some form.

The example illustrates a relationship between two **variables**, C and s.
In the statement of the relation, the cost depends on the speed.

C is called the **dependent variable** and s is called the **independent variable**.

When representing a relation, we often regard the values of the independent variable as the **input** and the values of the dependent variable as the **output**.

Before considering how to represent this relation we need some data: we need input values and output values.

The input values make up the **domain** of the relation, and the output values make up the **range** of the relation. These concepts will be discussed in more detail later.

Obviously we would not attempt to collect data for every possible input value (i.e. for every possible speed at which the car could be driven). Suppose that we choose as input values speeds of 20, 30, 40, ...120 km/h. and that the output values are as given in the diagrams below.

Table of Values

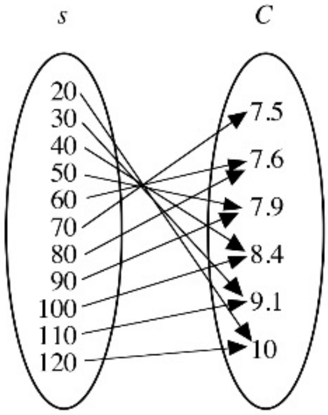
Input (s)	Output (C)
20	10
30	9.1
40	8.4
50	7.9
60	7.6
70	7.5
80	7.6
90	7.9
100	8.4
110	9.1
120	10

Ordered Pairs

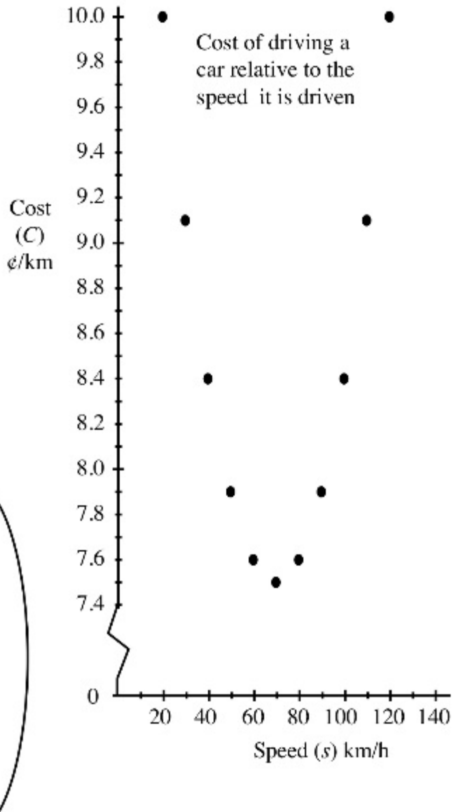
(s, C)

(20, 10)
(30, 9.1)
(40, 8.4)
(50, 7.9)
(60, 7.6)
(70, 7.5)
(80, 7.6)
(90, 7.9)
(100, 8.4)
(110, 9.1)
(120, 10)

Mapping Diagram



Graph



The diagrams above also illustrate how the information collected can be represented

- as ordered pairs in a table of values
- as a mapping diagram
- graphically

We have only chosen some of the possible input values, however it is obvious that an output value could have been determined for any input value greater than zero and up to the maximum speed of the car. It makes sense then to connect the points on the graph in some way.

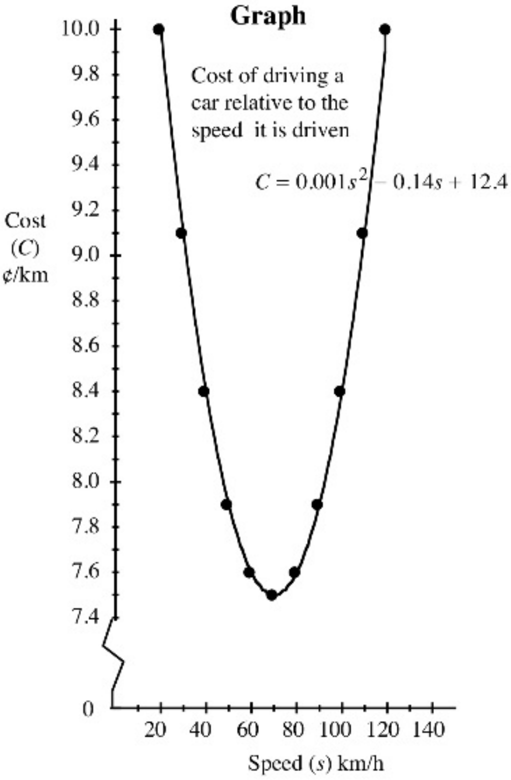
Later we will learn how this can be done and how an equation can be determined that best represents the data.

The graph and equation for the relation are given below. Note that the equation is only valid for certain input values which make up the domain of the relation. For example, the equation would not be valid for $s = 5000$!

Equation

$$C = 0.001s^2 - 0.14s + 12.4$$

Graph



Independent and Dependent Variables in a Relation

The values of the independent variable represent the inputs, and the corresponding values of the dependent variable are the outputs.

- In an ordered pair, the values of the first coordinate are those of the independent variable and the values of the second coordinate are values of the dependent variable.
- In a table of values, the independent variable is usually given first - either to the left or above the values of the dependent variable.
- In a mapping diagram, the arrows point from the independent variable to the dependent variable.
- On a graph, the independent variable is on the horizontal axis, often the x-axis, and the dependent variable is on the vertical axis, often the y-axis.
- In an equation, we usually try to isolate the dependent variable to the left side.

The illustration below uses the equation $y = 3x - 5$ as an example to illustrate the independent and dependent variables of an equation.

$$y = 3x - 5$$

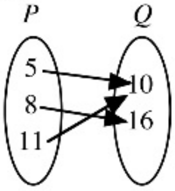
• The **dependent variable**.	• The **independent variable.**
• Values of the dependent variable represent the outputs of the relation.	• Values of the independent variable represent the inputs of the relation.
• Values of the dependent variable are represented by the **second** coordinate of an ordered pair and are on the **vertical axis**.	• Values of the independent variable are represented by the **first** coordinate of an ordered pair and are on the **horizontal axis**.

The diagrams show relations expressed in different ways. In each case

i) state the independent variable **ii)** state the dependent variable

iii) list the inputs **iv)** list the outputs

a)



b)

V	A
4	15
10	12
25	15

c)

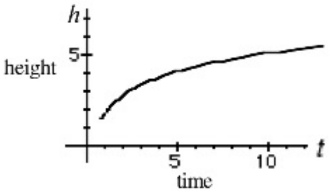
(B, c): $(3, 7)$, $(4, 11)$, $(5, 15)$, $(6, 19)$

Relations are expressed in different ways below. In each case

i) state the independent variable **ii)** state the dependent variable

a) $C = 2\pi r$

b)



c) The amount of sap, s, obtained from a maple tree is dependent on the time, t, a container is left attached to the maple tree.

Complete Assignment Questions #1 - #4

Investigating Relationships by Plotting Ordered Pairs

In this section we will consider relations defined by an equation, and sketch a graph by plotting ordered pairs using the following steps:

- Make a table of inputs by choosing replacements for the independent variable.
- For each of the input values, calculate the corresponding value (the output) of the dependent variable.
- Plot the ordered pairs on a Cartesian plane.

Consider the relation described by the equation $y = 2x - 5$.

a) Complete the first five rows of the following table of values which shows some of the possible input values.

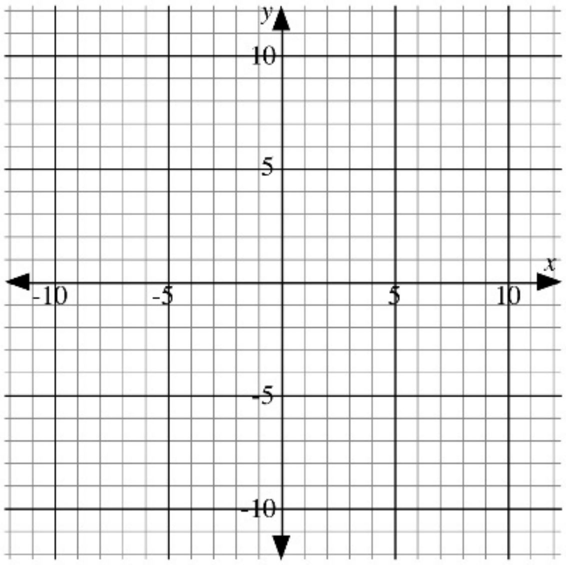
Input (x)	Output (y)	Ordered pair (x, y)
−2		
−1		
0		
1		
2		

b) Plot the ordered pairs in **a)** on the grid provided.

c) Connect the points on the grid, and extend the line in both directions with arrows at both ends.

d) Use the graph to determine the value of y when $x = 6$.

e) Use the equation to determine the value of y when $x = 6$, and verify the answer in **d)**.



f) Write the value of y when $x = 6$ in the table of values using the first blank space in **a)**.

g) Use the graph to determine the value of x when $y = 3$. Put this information in the last row in **a)**.

h) Complete the following statement:

This relation is called a _______________ relation because the graph of the relation is a straight line.

Consider the relation described by the equation $y = x^2 - 6$.

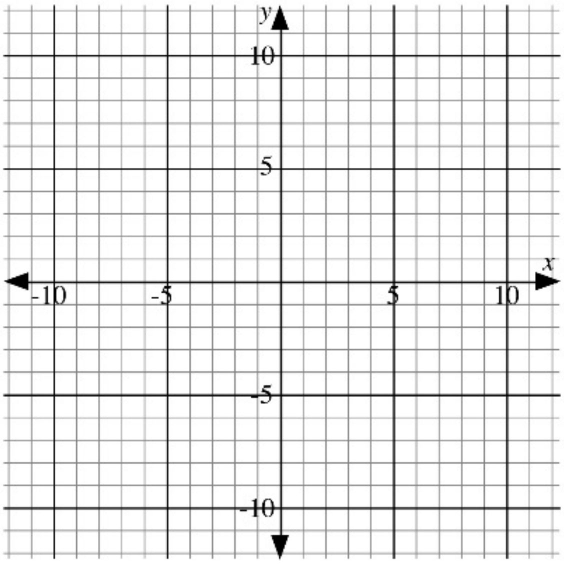
a) Complete the table of values to the right which shows some of the possible input values.

Input (x)	Output (y)	Ordered pair (x, y)
4		
3		
2		
1		
0		
–1		
–2		
–3		

b) Plot the ordered pairs in **a)** on the grid provided.

c) Use the symmetry of the graph or table to predict the value of y when $x = -4$.

d) Use the equation to determine the value of y when $x = -4$, and verify the answer in **c)**.



e) Write the value of y when $x = -4$ in the table of values using the first blank space in **a)**.

f) Connect the points on the grid with a smooth curve.

g) Why do you think this type of relation is called a **nonlinear relation**?

Complete Assignment Questions #5 - #9

Assignment

1. Complete the following.

a) The mathematical relationship between two quantities is called a ____________ .

b) The variable used for inputs in a relation is known as the _____________ variable.

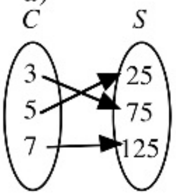
c) The variable used for outputs in a relation is known as the _____________ variable.

d) In the equation $A = \pi r^2$, the independent variable is _____, and the dependent variable is ____.

2. The diagrams show relations expressed in different ways. In each case:

i) state the independent and dependent variables **ii)** list the inputs and outputs.

a)



b)

C	n
8	22
20	19
50	35

c)

(f, e): $(2, 3)$, $(-2, 19)$, $(8, 17)$, $(0, 2)$

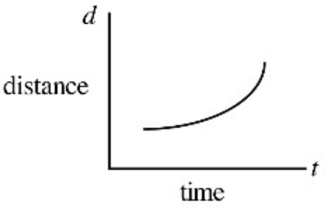
3. For each of the following relations, state

i) the independent variable **ii)** the dependent variable

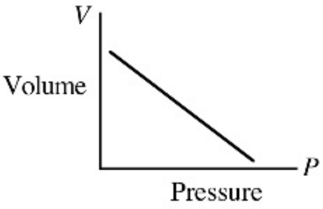
a) $V = \frac{4}{3}\pi r^3$

b) $C = \frac{5}{9}(F - 32)$

c)



d)



e) A truck's value, v, depends on its age, a.

f) The cost, C, of producing business cards is dependent on the number of cards, n, produced.

4. List the different ways a relation may be represented.

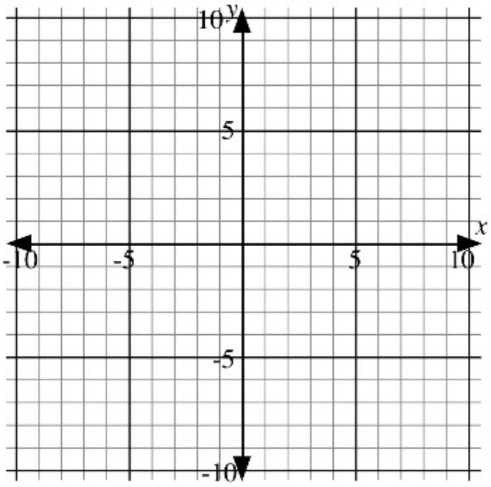
5. Consider the relation described by the equation $y = -x - 2$.

a) Identify the independent and dependent variables.

b) Complete the following table of values.

Input (x)	Output (y)	Ordered pair (x, y)
–3		
–1		
0		
1		

c) Plot the ordered pairs in **b)** on the grid provided.



d) Connect the points on the grid, and then extend the line in both directions with arrows at both ends.

e) Use the graph to determine the value of y when $x = 5$.

f) Use the equation to determine the value of y when $x = 5$, and verify the answer in **e)**.

g) Write the value of y when $x = 5$ in the table of values in **b)**.

h) Use the graph to determine the value of x when $y = 0$. Include this ordered pair in the table of values.

i) Use the graph to determine the value of x when $y = 4$. Include this ordered pair in the table of values.

j) Verify the answer in i) using the equation.

k) Is this a linear or a nonlinear relation?

6. Consider the relation described by the equation $y = -0.5x^2 + 8$.

a) Identify the independent and dependent variables.

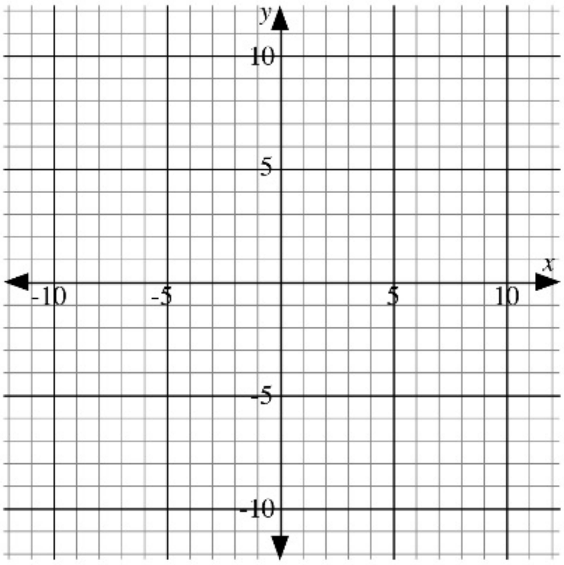
b) Complete the following table of values.

Input (x)	Output (y)	Ordered pair (x, y)
–6		
–4		
–2		
0		
2		
4		

c) Plot the ordered pairs in **b)** on the grid provided.

d) Use the plotted points and table to predict the value of y when $x = 6$. Plot this point on the grid.

e) Use the equation to determine the value of y when $x = 6$, and verify the answer in **d)**.



f) Connect the points on the grid with a smooth curve.

g) Is this a linear or a nonlinear relation?

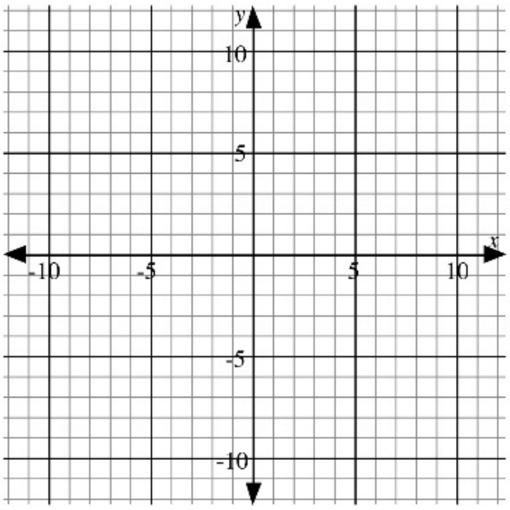
h) Use the graph to determine the values of x when $y = 3.5$. Verify your answer by using the equation.

7. For the following relations:

i) Complete the table of values choosing your own input values where necessary.
ii) Plot the ordered pairs on the grid, and sketch the graph of the relation.
iii) State whether the relation is linear or nonlinear.

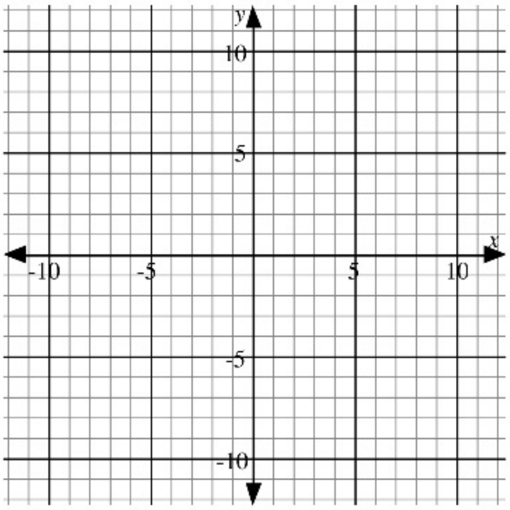
a) $y = -2x + 3$

Input (x)	Output (y)	Ordered pair (x, y)
0		
3		



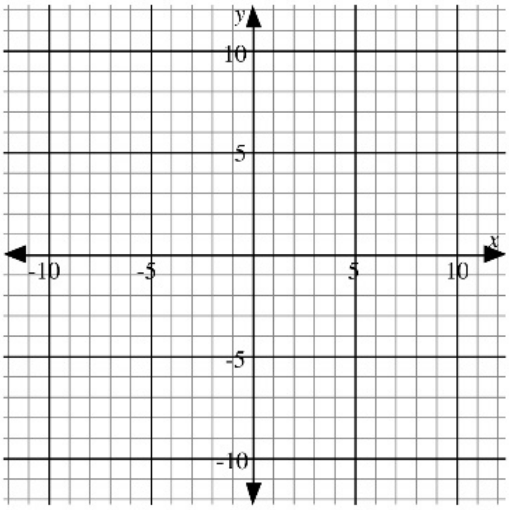
b) $y = 0.5x - 8$

Input (x)	Output (y)	Ordered pair (x, y)
–8		
0		
6		



c) $y = -x^2 + 5$

Input (x)	Output (y)	Ordered pair (x, y)
4		
3		
2		
1		
0		
–1		
–2		
–3		
–4		



Multiple Choice **8.** Which of the following statements is false?

A. The dependent variable is represented on the vertical axis of a Cartesian Plane.
B. The independent variable is represented by the first coordinate of an ordered pair.
C. The outputs of a relation are shown on the horizontal axis of a Cartesian Plane.
D. The independent variable is usually shown on the right side of an equation.

Numerical Response **9.** Consider the relation described by the equation $y = 1.5^{x-2}$. If the input is 4, then the output is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** relation **b)** independent **c)** dependent **d)** r, A

2. **a)** **i)** independent - C
dependent - S
ii) input - 3, 5, 7
output - 25, 75, 125

b) **i)** independent - C
dependent - n
ii) input - 8, 20, 50
output - 22, 19, 35

c) **i)** independent - f
dependent - e
ii) input - 2, –2, 8, 0
output - 3, 19, 17, 2

3. **a)** **i)** independent - r
ii) dependent - V

b) **i)** independent - F
ii) dependent - C

c) **i)** independent - time
ii) dependent - distance

d) **i)** independent - pressure
ii) dependent - volume

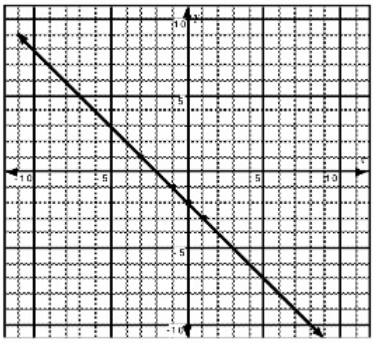
e) **i)** independent - a
ii) dependent - v

f) **i)** independent - n
ii) dependent - C

4. words, table of values, set of ordered pairs, mapping, equation, graph, function notation

5. **a)** Independent → x, Dependent → y
b) See table below **c), d)** See graph below **e)** –7 **f)** –7

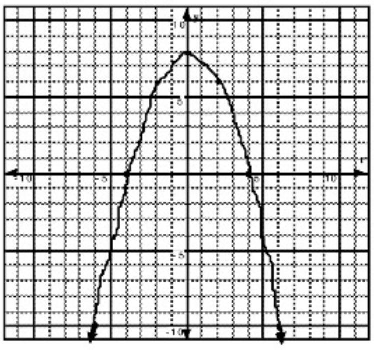
	Input (x)	Output (y)	Ordered pair (x, y)
	–3	**1**	**(–3, 1)**
	–1	**–1**	**(–1, –1)**
	0	**–2**	**(0, –2)**
	1	**–3**	**(1, –3)**
g)	5	**–7**	**(5, –7)**
h)	**–2**	**0**	**(–2, 0)**
i)	**–6**	**4**	**(–6, 4)**



k) linear

6. **a)** Independent → x Dependent → y
b) See table below. **c), f)** See graph below. **d)** –10 **e)** $y = -0.5(6)^2 + 8\,y = -10$ **g)** non-linear

Input (x)	Output (y)	Ordered pair (x, y)
–6	**–10**	**(–6, –10)**
–4	**0**	**(–4, 0)**
–2	**6**	**(–2, 6)**
0	**8**	**(0, 8)**
2	**6**	**(2, 6)**
4	**0**	**(4, 0)**

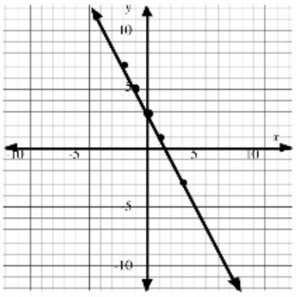


h) $x = \pm 3$

7. a) i) See table below. Inputs may vary.

Input (x)	Output (y)	Ordered pair (x, y)
–2	**7**	**(–2, 7)**
–1	**5**	**(–1, 5)**
0	**3**	**(0, 3)**
1	**1**	**(1, 1)**
3	**–3**	**(3, –3)**

ii) See grid below.

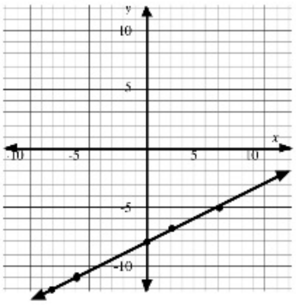


iii) linear

b) i) See table below. Inputs may vary.

Input (x)	Output (y)	Ordered pair (x, y)
–8	**–12**	**(–8, –12)**
–6	**–11**	**(–6, –11)**
0	**–8**	**(0, –8)**
2	–7	**(2, –7)**
6	**–5**	**(6, –5)**

ii) See grid below.

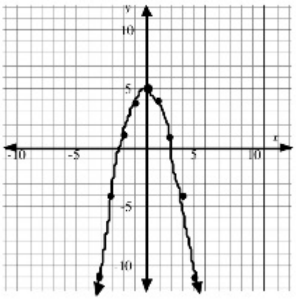


iii) linear

c) i) See table below.

Input (x)	Output (y)	Ordered pair (x, y)
4	**–11**	**(4, –11)**
3	**–4**	**(3, –4)**
2	**1**	**(2, 1)**
1	**4**	**(1, 4)**
0	**5**	**(0, 5)**
–1	**4**	**(–1, 4)**
–2	**1**	**(–2, 1)**
–3	**–4**	**(–3, –4)**
–4	**–11**	**(–4, –11)**

ii) See grid below.



iii) non-linear

8. C

9.

2	.	2	5

Relations Lesson #3:
x- and y-intercepts and Interpreting Relations

Review

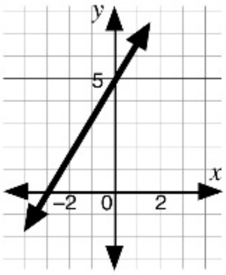
a) A **relation** is a connection between two quantities. A relation can be represented graphically by a set of ______________ ___________ .

b) The first component of a set of ordered pairs is the ___ coordinate, also known as the input. Values of the input are values of the________________ variable.

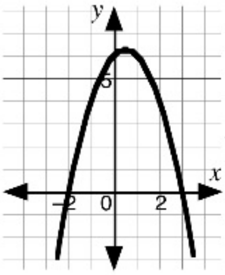
c) The second component of a set of ordered pairs is the ___ coordinate, also known as the output. Values of the output are values of the________________ variable.

Exploring x- and y-intercepts

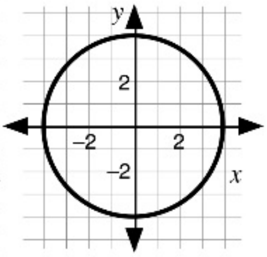
Consider the following graphs.



Graph 1



Graph 2



Graph 3

a) List the coordinates of the point(s) where each graph crosses the x-axis.

- Graph 1 crosses the x-axis at (,).
- Graph 2 crosses the x-axis at (,) and (,).
- Graph 3 crosses the x-axis at (,) and (,).

b) What do all the points in a) have in common?

c) List the coordinates of the point(s) where each graph crosses the y-axis.

- Graph 1 crosses the y-axis at (,).
- Graph 2 crosses the y-axis at (,).
- Graph 3 crosses the y-axis at (,) and (,).

d) What do all the points in c) have in common?

x- and y- intercepts of a Graph

The **x-intercept** of a graph is the x-coordinate of the ordered pair where the graph intersects the x-axis. An x-intercept occurs at a point on the graph where the y-coordinate is zero. The x-intercept can be given as a value or as an ordered pair.

The **y-intercept** of a graph is the y-coordinate of the ordered pair where the graph intersects the y-axis. A y-intercept occurs at a point on the graph where the x-coordinate is zero. The y-intercept can be given as a value or as an ordered pair.

Note

1. Given the equation of the graph of a relation:
- to determine the x-intercept, set $y = 0$ and solve for x.
- to determine the y-intercept, set $x = 0$ and solve for y.

2. The equation of a graph can be written in different forms, all of which are equivalent. The equation of Graph 1 on the previous page is $y = \frac{5}{3}x + 5$, which can be written as $3y = 5x + 15$ or $5x - 3y + 15 = 0$. Equivalent forms of an equation will be studied in detail in a later unit. For the time being, use the instruction in note 1 to find the x- and y-intercepts of the graph of an equation given in any form.

Class Ex. #1

The equation of Graph 1 on the previous page is $3y = 5x + 15$.
Algebraically determine the values of the x-intercept and the y-intercept of Graph 1.

x-intercept	y-intercept

Class Ex. #2

The equation of Graph 3 on the previous page is $x^2 + y^2 = 16$.
Calculate the x-intercept and the y-intercept of the graph of $x^2 + y^2 = 16$. Give the answers as ordered pairs.

x-intercept	y-intercept

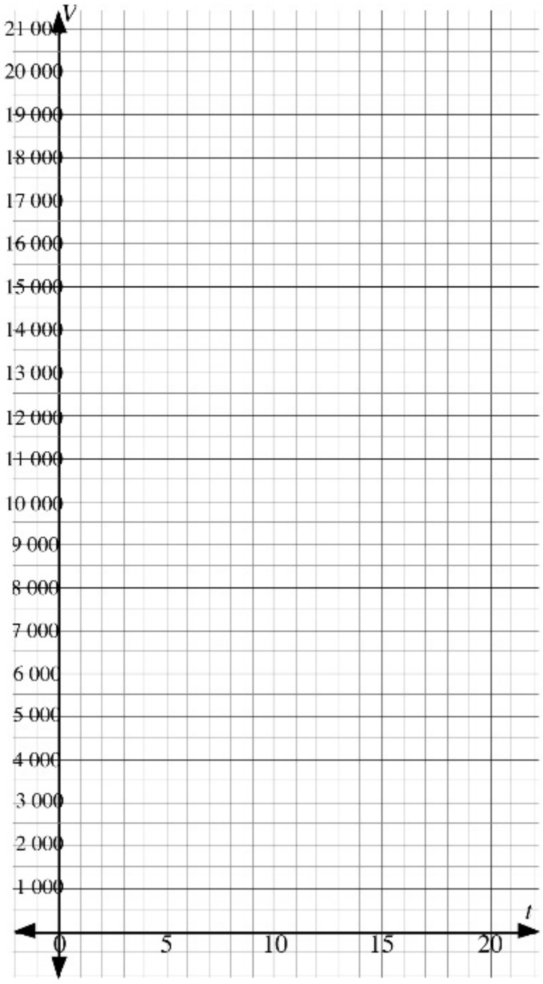
Complete Assignment Questions #1 - #3

Lisa purchases a new car for $20 000. The value of the car can be represented by the formula $V = 20\,000 - 1250t$, where V is the value of the car in dollars, and t is the age of the car in years.

a) Complete the table of values and plot the ordered pairs on the grid.

Input (t)	Output (V)	Ordered pair (t, V)
0		
2		
4		
6		

Connect the points with a straight line, and extend the line.



b) What does the ordered pair $(0, 20\,000)$ represent?

c) Use the graph to estimate the t-intercept. What does the t-intercept represent?

d) Use the graph to estimate the value of the car after
i) 3 years **ii)** 10 years **iii)** 14 years.

e) Use the formula to verify **d) ii)**.

f) Use the graph to estimate when the car will be worth
i) $5 000 **ii)** half of the purchase price.

g) Use the formula to verify **f) ii)**.

h) Complete the following statement to describe the relation:

The original value of the car is __________ . It depreciates in value by ________ per year and has no value after ___________ years.

Note

In this lesson, using algebra determines the **exact** values for intercepts, etc. whereas using graphs gives an **estimate** for intercepts, etc. In lesson 5 we use the features of a graphing calculator to determine more accurate results from a graph.

In part **d)i)** we were asked to use the graph to find values lying between given points. This process is called **interpolation**. Extending the graph to predict values outside the plotted points is called **extrapolation**. Examples of extrapolation are **d)ii)** and **d)iii)**.

Complete Assignment Questions #4 - #9

Assignment

1. Determine the value of the y-intercept of the graph of each equation.

a) $y = x - 5$ **b)** $y = 3x - 15$ **c)** $2y + 3x - 12 = 0$

d) $0.5x - 2.4y + 0.8 = 0$ **e)** $2y = x^2 - 60$ **f)** $y = 0.001x^2 - 0.001x + 12.44$

2. Determine the value of the x-intercept(s) of the graph of each equation.

a) $y = x - 2$ **b)** $y = 2x - 8$ **c)** $3y + 2x - 12 = 0$

d) $0.6x - 2y + 0.5 = 0$ **e)** $y = x^2 - 9$ **f)** $y = 12 - 3x$

3. Determine the x- and y-intercepts of each equation. Answer as ordered pairs.

a) $y = 4x + 7$ **b)** $y = 15 - 6x$ **c)** $4x - 2y + 16 = 0$

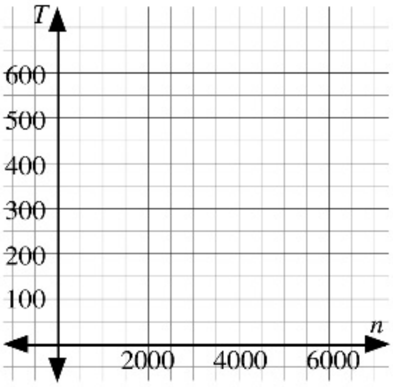
d) $y = \frac{x^2}{2} - 18$ **e)** $x^2 + y^2 = 25$ **f)** $y = 3x$

g) $y = x^2 + 4$ **h)** $9x^2 + y^2 = 81$ **i)** $9x^2 - y^2 = 81$

4. Triple A Car Rental charges \$100 per rental plus 10¢ per km. The total cost, T, in dollars of renting the car can be represented by the formula, $T = 100 + 0.10n$, where n is the number of km travelled.

a) Complete the table of values, and plot the ordered pairs on the grid provided.

Number of km (n)	Total Rental Cost (T) dollars
0	
1000	
3500	
5000	



Connect the points with a straight line, and extend the line in both directions.

b) What does the ordered pair (0, 100) represent?

c) Determine the n-intercept of the graph. Explain why it is not applicable to this problem.

d) Interpolate from the graph to estimate the cost for a journey of:
i) 2000 km **ii)** 4500 km

e) Use the formula to verify the answers in **d)**.

f) If the total cost of rental is \$650, use the graph to estimate the number of km travelled.

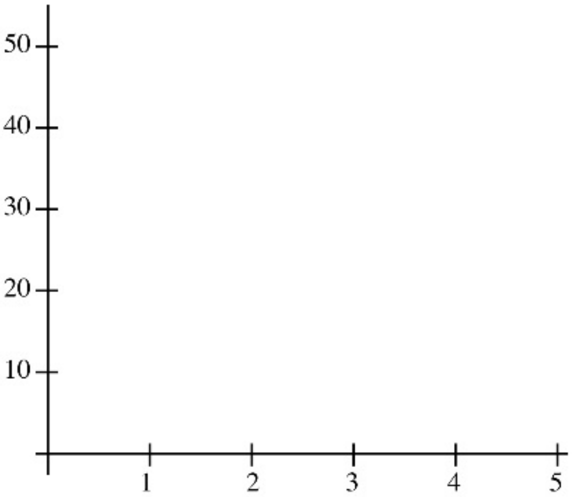
g) Verify the answer in **f)** using the formula.

5. An arrow is shot vertically into the air using a bow. The height, h metres, above the ground after t seconds, where $t \geq 0$ is approximated by the equation $h = -5t^2 + 20t + 25$.

a) The maximum height of the arrow is reached after 2 seconds.
Calculate the maximum height.

b) Complete the table of values, and plot the points on the grid.
Join the points with a smooth curve, and label the graph.

time (seconds)	height (metres)
0	
1	
2	
3	
4	
5	



c) Is this a linear or nonlinear relation?

d) For how many seconds is the arrow in the air?

e) What does the h-intercept represent in the context of the question?

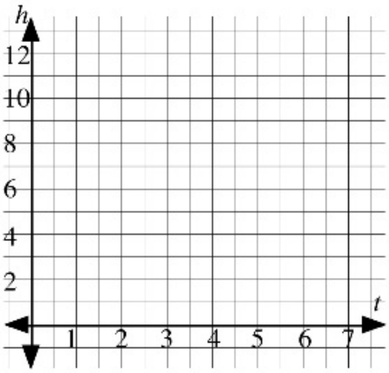
f) What does the t-intercept represent in the context of the question?

g) i) Use the graph to estimate the height of the arrow after 1.5 seconds.

ii) Use the equation to calculate the exact height of the arrow after 1.5 seconds.

h) Does it make sense to extend the graph of the relation $h = -5t^2 + 20t + 25$ further in a downward direction to the left or right? Explain.

6. A candle manufacturer determined that its "Long-Last" candles melted according to the formula $h = -2t + 12$, where h is the height of the candle, in cm, after t hours.



a) Make a table of values and use this to construct the graph of $h = -2t + 12$.

t				
h				

*Use your graph to answer **b** – **e**.*

b) How tall is the candle before it begins to melt?

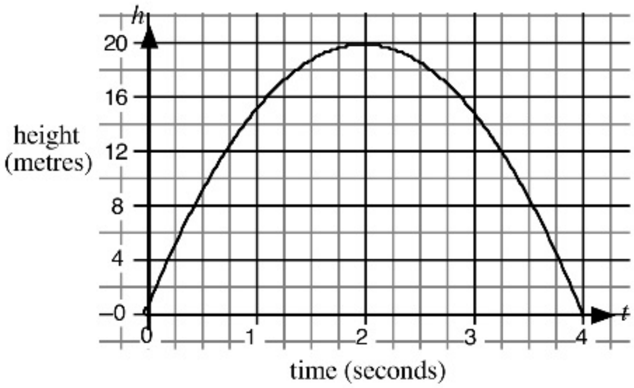
c) How many hours will the candle last before it will completely burn out?

d) How tall will the candle be after burning for 5 hours?

e) How long will it take for the candle to burn down to a height of 7 cm?

f) Verify the answers from **b)** - **e)** using the formula.

7. A football is kicked by a student. The graph of the relation between the height of the football above the ground and time is shown. The formula that represents the relation is given by $h = -4.9t^2 + 19.4t + 0.6$, where h is the height in metres above the ground and t is the time in seconds the football is in the air.



Use the graph to answer ***a – c****:*

a) Estimate, to the nearest metre, the maximum height of the football above the ground.

b) Estimate how long it takes for the football to reach the ground.

c) Estimate the height, to the nearest metre, of the football when it is in the air for 3 seconds.

d) Use the formula to calculate the exact answer to **c**).

e) Calculate the h-intercept, and describe what it represents in the context of the question.

Multiple Choice **8.** In which of the following relations does the graph of the relation have x- and y-intercepts with equal values?

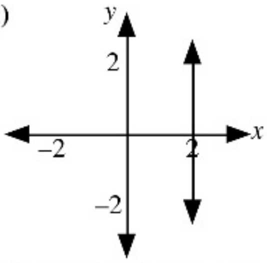
A. $y = x + 8$
B. $2x + 2y = 7$
C. $2x - 3y + 4 = 0$
D. none of the above

Numerical Response **9.** The graph of the relation $4x^2 + 9y^2 - 36 = 0$ has x-intercepts a and b, and y-intercepts c and d. The value of the product $abcd$ is _____ .

(Record your answer in the numerical response box from left to right)

10. Match each graph with the domain from A to F. Each domain may be used once, more than once, or not at all.

i)



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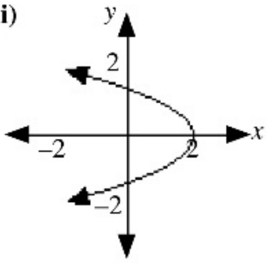

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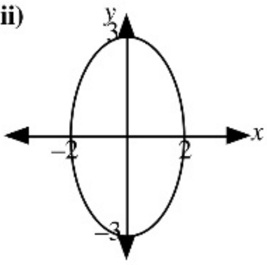

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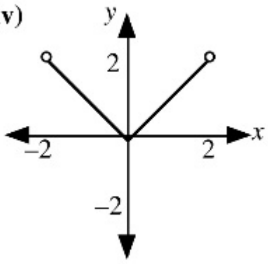

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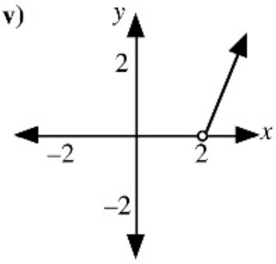

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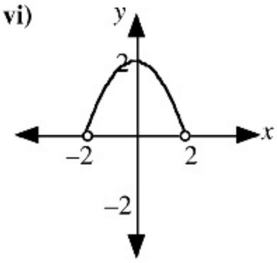
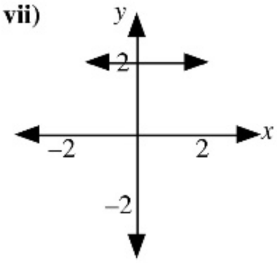


vii)



A. $x = 2$
B. $x < 2$
C. $x \leq 2$
D. $x > 2$
E. $x \geq 2$
F. $-2 < x < 2$
G. $-2 \leq x \leq 2$
H. $2 < x < -2$
I. $x \in R$

Relations Lesson #4:
Domain and Range

Number Systems Review

Number systems were studied in previous math courses. Recall the symbols and letters used by completing the following.

a) Each letter below is represents a number system. Write the name of the number system. $\overline{Q}$ has been completed.

N	W	I	Q	$\overline{Q}$	R
				Irrational Numbers	

b) The symbol $\in$ represents the phrase "

c) The brackets { } represents the phrase "

d) Recall the following by reading following notes regarding number systems.

Natural Numbers
- The natural numbers (counting numbers) are 1, 2, 3, 4, etc.
- There are an infinite number of natural numbers.
- The set of all natural numbers can be written as $N = \{1, 2, 3, 4, \ldots\}$.
- The number 11 belongs to the set of natural numbers and we can write $11 \in N$.

Whole Numbers
- The whole numbers are the natural numbers together with 0.
- The set of all whole numbers can be written as $W = \{0, 1, 2, 3, \ldots\}$.
- $6 \in N$ but $\frac{1}{2} \notin W$.

Integers
- The integers consist of positive and negative natural numbers, and zero.
- The set of all integers can be written as $I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.
- In some texts Z or J is used instead of I.

Rational Numbers
- Rational numbers are those numbers which can be expressed as a ratio between two integers, e.g. the fractions $\frac{1}{4}$ and $-\frac{2}{3}$ are both rational numbers. All the integers are included in the rational numbers since any integer can be written as $\frac{x}{1}$ e.g. $5 = \frac{5}{1}$.
- The set of rational numbers can be written as $Q = \left\{ \frac{a}{b},\ \text{where } a, b \in I,\ b \neq 0 \right\}$.

Irrational Numbers
- The set of irrational numbers can be written as $\overline{Q} = \{\text{non-terminating and non-repeating decimals}\}$ e.g. $\pi \in \overline{Q}$, $\sqrt{2} \in \overline{Q}$.

Real Numbers
- The real numbers consists of all rational numbers and irrational numbers.
- The set of all real numbers can be written as $R = \{Q \text{ and } \overline{Q}\}$.
- In some texts $\Re$ or Re are used instead of R.

Complete Assignment Question #1 and #2

Domain and Range

The **domain** of a relation is the set of all possible values which can be used for the **input** of the **independent variable** (x).

The **range** of a relation is the set of all possible values of the **output** of the **dependent variable** (y).

In lesson 2 on page 235 we described the relation in each of the following forms

- in words
- a mapping (or arrow) diagram
- a table of values
- an equation
- a set of ordered pairs
- a graph

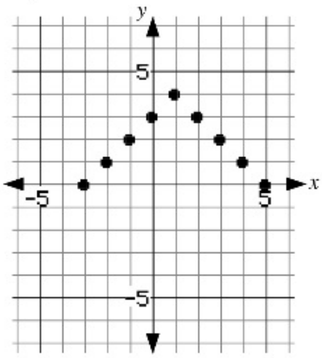
In this lesson we will study the domain and range given in any of these forms.

List the domain and range of the following set of ordered pairs.

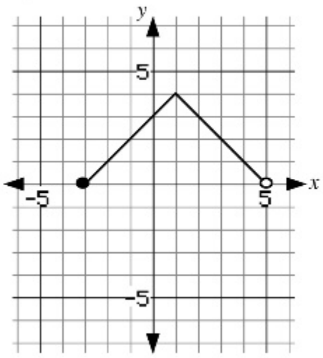
a) (1, 2), (0, 5), (3, 8), (5, 9), (–3, 2)　　**b)** (3, 3), (0, 3), (–3, 3), (2, 9), (–8, 3)

In each case, state the domain and range of the relation represented by the graph.

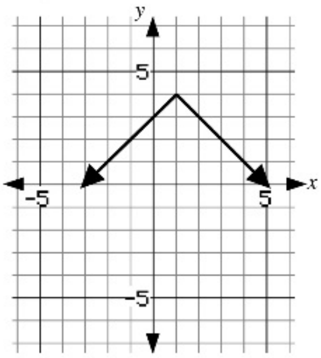
a)



b)



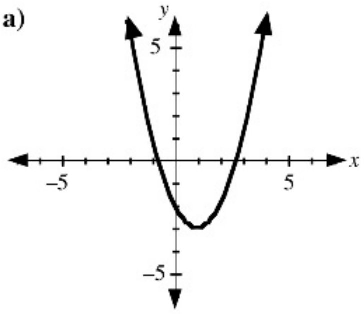
c)



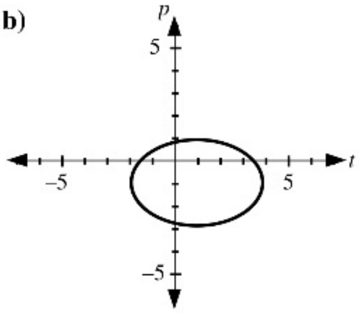
Class Ex. #3

State the domain and range of the following relations.

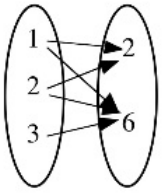
a)



b)



c)



d) A circle with centre $(-2, 3)$ and a radius of 3.

Class Ex. #4

a) Draw the graph of a relation which has domain $x \in R$, range $\{y \mid y \leq 2, y \in R\}$ and

i) only one x-intercept

ii) two x-intercepts

b) Explain why it is not possible to draw a graph which has domain $x \in R$, range $\{y \mid y \leq 2, y \in R\}$ and no x-intercepts.

Class Ex. #5

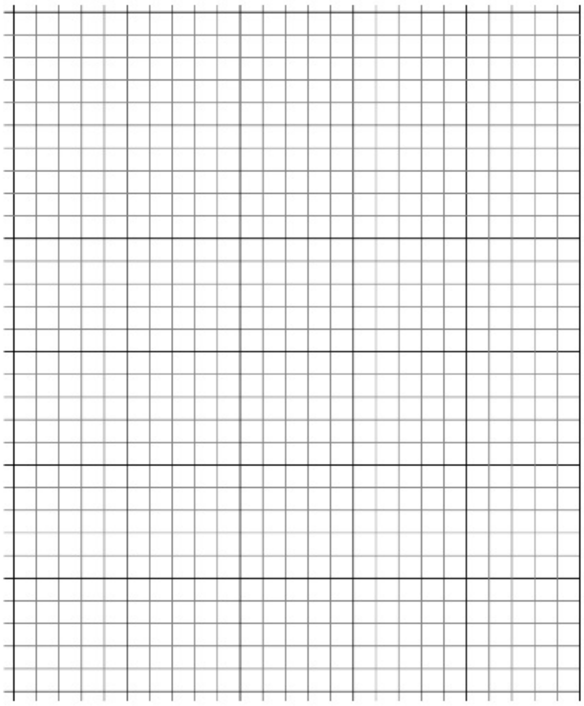
A high school football team is hosting a banquet to celebrate winning the championship. The caterer charges a set up fee of \$500 plus \$20 per person. The equation $C = 500 + 20n$ represents the cost of hosting the banquet for n people.

a) Make a table of values with 8 entries for a minimum of 100 and a maximum of 500 people.

n	C

b) Plot the eight ordered pairs from **a)** on the grid.

c) If all possible ordered pairs from **b)** were plotted on the grid, state the domain and range of the relation and explain why there are restrictions on both.



Complete Assignment Questions #3 - #12

Assignment

1. Use check marks to indicate all the sets to which each number belongs.

	N	W	I	Q	$\overline{Q}$	R
$1/3$						
123 983						
-2						
$7.53\overline{4}$						
9.5						
$\sqrt{75}$						
$-\pi$						
$-355/113$						
$-\sqrt{49}$						
0.000005						
2.232425...						

2. The addition of ANY two **natural** numbers is still a natural number.

Does this property hold true for the operations of subtraction, multiplication, or division (except for zero)?

Investigate for whole numbers, integers, rational numbers, and real numbers and write your conclusions.

3. State the domain and range of each relation.

a) $(2,3)$, $(0,2)$, $(4,8)$, $(-1,8)$, $(-3,1)$

b) $(-3,3)$, $(0,-5)$, $(-3,3)$, $(5,-2)$, $(-8,1)$

c)

Input (x)	Output (y)
0	3
2	4
4	5
6	3

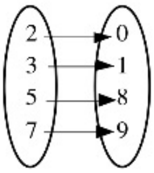
d)

Input (x)	Output (y)
2	3
0	4
–3	5
2	6

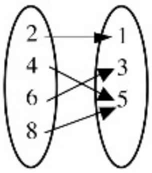
e)

Input (x)	Output (y)
1	5
–1	5
3	5
7	5

f)



g)



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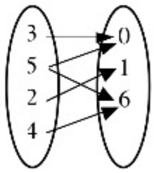
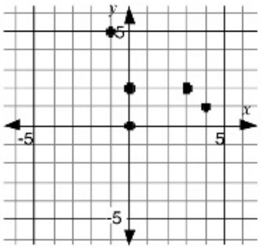
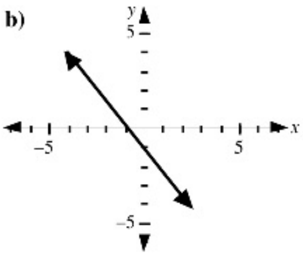


4. State the domain and range for each relation.

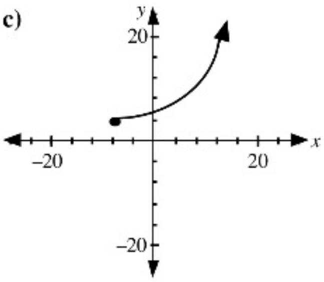
a)



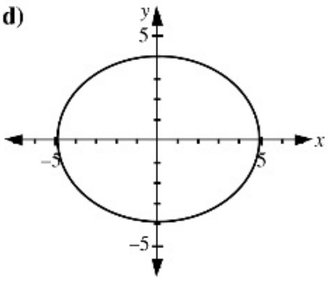
b)



c)



d)



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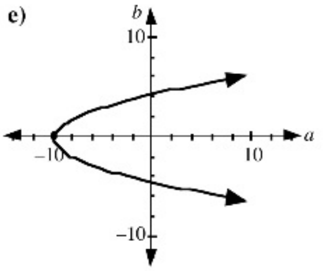

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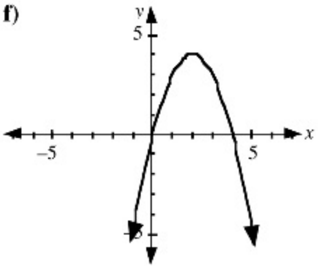

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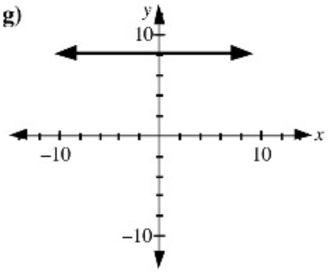
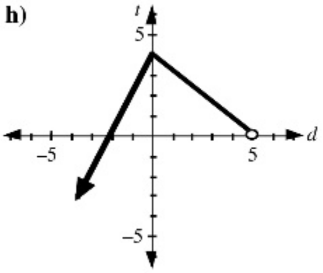


h)



5. In each case a relation is graphed on a grid. State the domain and range of the relation if the graph is

a) a circle whose centre is located at $(-1, 12)$ and with a radius of 5 units.

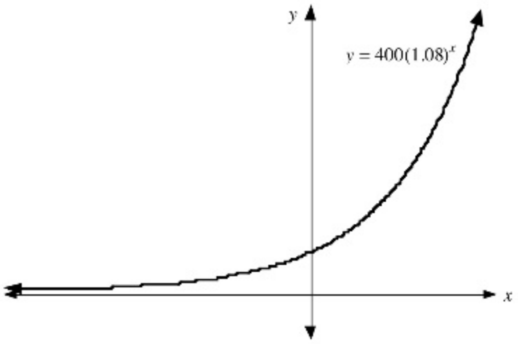
b) a circle with centre $(-3, -5)$ and diameter 40 units.

c) a rectangle with vertices $A(-8, 10)$, $B(-8, -2)$, $C(7, -2)$, and $D(7, 10)$.

d) a triangle with vertices $T(-50, -75)$, $U(-35, -25)$, and $V(-65, -25)$.

6. The graph of the relation $y = 400(1.08)^x$ is shown on the grid.

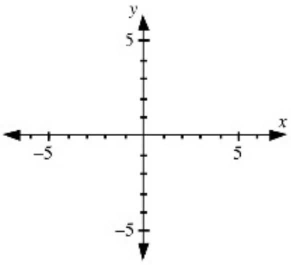
a) State the domain, range,
and y-intercept of the relation.



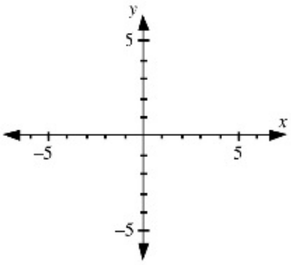
b) The relation $A = 400(1.08)^t$ represents the amount of money when an original investment of \$400 is compounded annually at 8% for a period of t years. State the domain and range of this relation, and explain why the answer is different from **a)**.

7. In each case draw a graph on the domain of real numbers which could represent a linear relation with

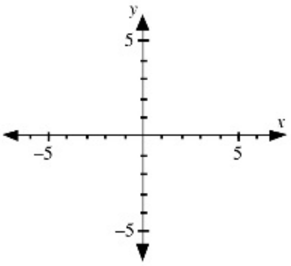
a) one x-intercept



b) no x-intercept

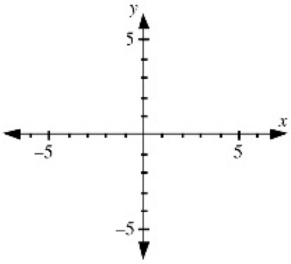


c) an infinite number of x-intercepts.

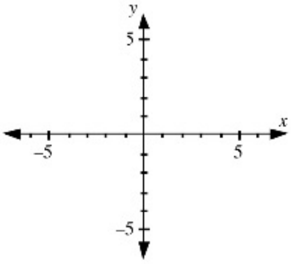


8. In each case draw a graph of a non-linear relation with

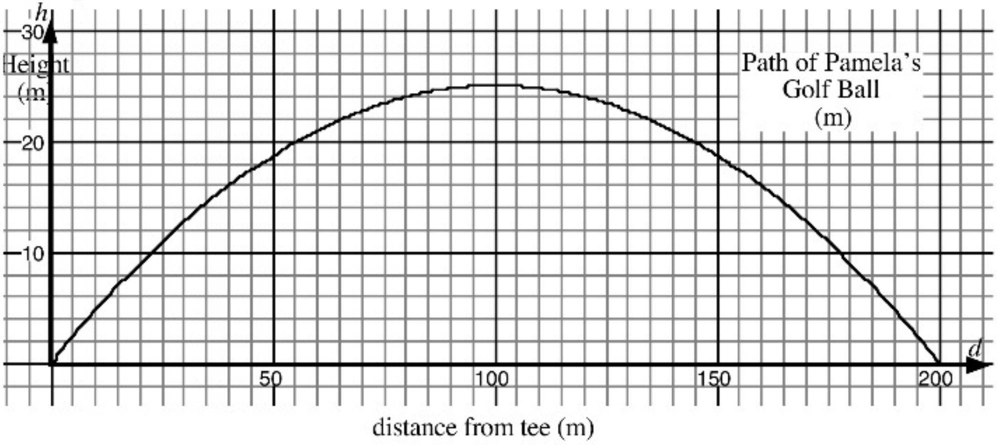
a) domain $x \in R$, range $y \geq -3, y \in R$
two x-intercepts and one y-intercept



b) domain $x \in R$, range $y \geq -3, y \in R$
one x-intercept and one y-intercept.



9. The graph shows the flight of Pamela's golf ball from the tee to a sand trap at the edge of the green.



a) State the h-intercept and the d-intercepts of the graph, and explain their significance in relation to the question.

b) State the maximum height of the golf ball, and explain its relevance to the domain or range of the relation.

c) State the domain and range of the relation.

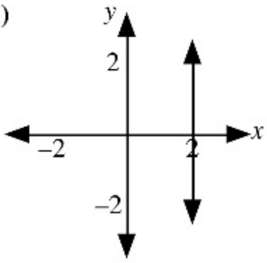
d) From the graph, estimate the horizontal distance the ball has traveled when it is 20 m in the air. Explain why there are two answers.

e) Using the graph, estimate the height of the golf ball when the horizontal distance from the tee is 80 m.

f) Give a brief description of the relationship between the height of the golf ball and the horizontal distance from the tee.

10. Match each graph with the domain from A to F. Each domain may be used once, more than once, or not at all.

i)



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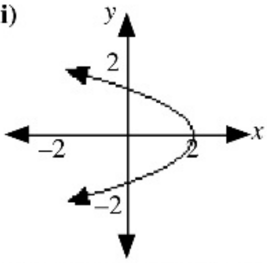

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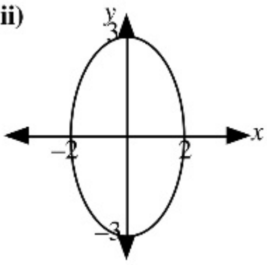

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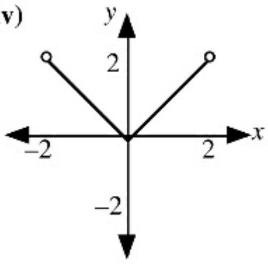

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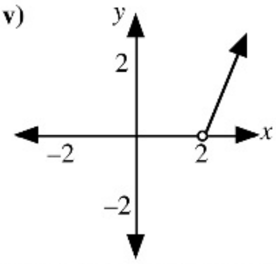

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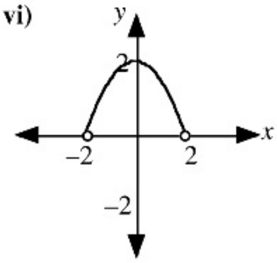
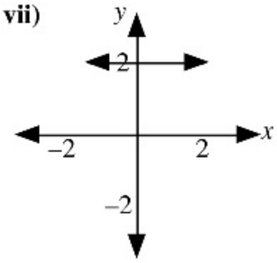
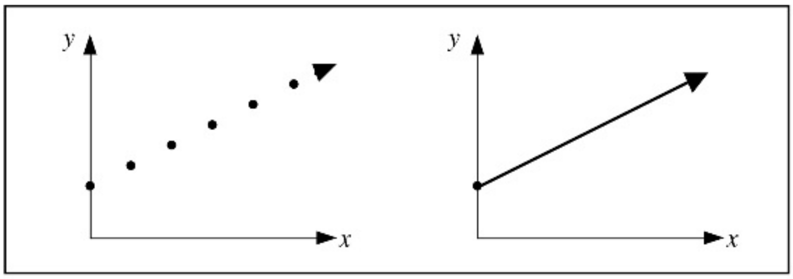


vii)



A. $x = 2$
B. $x < 2$
C. $x \leq 2$
D. $x > 2$
E. $x \geq 2$
F. $-2 < x < 2$
G. $-2 \leq x \leq 2$
H. $2 < x < -2$
I. $x \in R$

Multiple Choice **11.** The graphs of two relations are shown. Which of the following statements is true?



A. The domains are the same, but the ranges are different.
B. The ranges are the same, but the domains are different.
C. The domains are the same, and the ranges are the same.
D. The domains are different, and the ranges are different.

Numerical Response **12.** The relation between the distance traveled, d km, and the cost, C dollars, of renting a truck is given by the formula $C = 60 + 0.27d$. The domain of the relation can be expressed in the form $d \geq x$, and the range can be expressed in the form $C \geq y$. Write the value of y in the first two boxes and the value of x in the last two boxes.

(Record your answer in the numerical response box from left to right) ☐☐☐☐

Answer Key

1 . See chart below

	N	W	I	Q	$\overline{Q}$	R
$\frac{1}{3}$				√		√
123 983	√	√	√	√		√
-2			√	√		√
$7.53\overline{4}$				√		√
9.5				√		√
$\sqrt{75}$					√	√
$-\pi$					√	√
$-\frac{355}{113}$				√		√
$-\sqrt{49}$			√	√		√
0.000005				√		√
2.232425...					√	√

2 . See below.

- For natural numbers and whole numbers, the property is true for addition and multiplication only.
- For integers, the property is true for addition, subtraction, and multiplication.
- For rational numbers and real numbers, the property is true for addition, subtraction, multiplication, and division.

3. **a)** $D = \{2, 0, 4, -1, -3\}$ $R = \{3, 2, 8, 1\}$
b) $D = \{-3, 0, 5, -8\}$ $R = \{3, -5, -2, 1\}$
c) $D = \{0, 2, 4, 6\}$ $R = \{3, 4, 5\}$
d) $D = \{2, 0, -3\}$ $R = \{3, 4, 5, 6\}$
e) $D = \{1, -1, 3, 7\}$ $R = \{5\}$
f) $D = \{2, 3, 5, 7\}$ $R = \{0, 1, 8, 9\}$
g) $D = \{2, 4, 6, 8\}$ $R = \{1, 3, 5\}$
h) $D = \{3, 5, 2, 4\}$ $R = \{0, 1, 6\}$

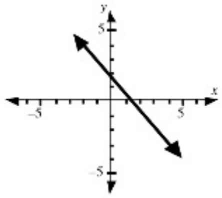
4. **a)** $D = \{-1, 0, 3, 4\}$ $R = \{5, 0, 2, 1\}$
b) $D = \{x \in R\}$ $R = \{y \in R\}$
c) $D = \{x \geq -8, x \in R\}$ $R = \{y \geq 4, y \in R\}$
d) $D = \{-5 \leq x \leq 5, x \in R\}$ $R = \{-4 \leq y \leq 4, y \in R\}$
e) $D = \{a \geq -10, a \in R\}$ $R = \{b \in R\}$
f) $D = \{x \in R\}$ $R = \{y \leq 4, y \in R\}$
g) $D = \{x \in R\}$ $R = \{8\}$
h) $D = \{d < 5, d \in R\}$ $R = \{t \leq 4, t \in R\}$

5. **a)** $D = \{-6 \leq x \leq 4, x \in R\}$ $R = \{7 \leq y \leq 17, y \in R\}$
b) $D = \{-23 \leq x \leq 17, x \in R\}$ $R = \{-25 \leq y \leq 15, y \in R\}$
c) $D = \{-8 \leq x \leq 7, x \in R\}$ $R = \{-2 \leq y \leq 10, y \in R\}$
d) $D = \{-65 \leq x \leq -35, x \in R\}$ $R = \{-75 \leq y \leq -25, y \in R\}$

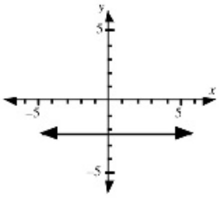
6. **a)** $D = \{x \in R\}$ $R = \{y > 0, y \in R\}$, y–int is 400
b) $D = \{t \geq 0, t \in R\}$ different from a) because time is never a negative value.
$R = \{A \geq 400, A \in R\}$ different from a) because the amount of money can never be less than \$400.

7. Answers may vary.

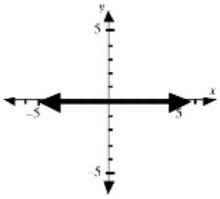
a)



b)

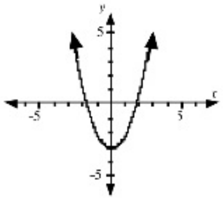


c)

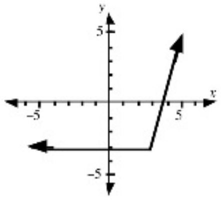


8. Answers may vary.

a)



b)



9. **a)** h–int = 0, d–int = 0 and 200. On the tee the ball is on the ground.
It returns to ground level 200 m from the tee.
b) max height = 25 m. The maximum height is the upper limit of the range.
c) $D = \{0 \leq d \leq 200, d \in R\}$ $R = \{0 \leq h \leq 25, h \in R\}$
d) 55 m from the tee when the ball is rising and 145m from the tee when the ball is descending.
e) 24 m
f) Starting from a height of 0 m at the tee, the golf ball increases in height to a maximum height of 25m, 100 m from the tee. Then the golf ball starts decreasing in height until it hits the ground 200 m from the tee.

10. **i)** A **ii)** C **iii)** G **iv)** F **v)** D **vi)** F **vii)** I

11. D

12.

6	0	0	0

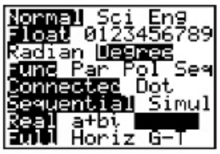
Relations Lesson #5:
Relations and the Graphing Calculator

- Although features of different types of graphing calculators may be similar to each other, the instructions in this unit refer to the TI-83/TI-84 series of graphing calculators.
- This lesson takes more than one class to complete.

Preparing the Calculator to Graph

Confirm that the calculator is in "Function" mode as shown.

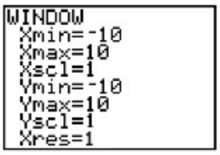
Press `MODE` and use the cursor keys to navigate to the proper settings, and then press `ENTER`.



```
Normal Sci Eng
Float 0123456789
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi
Full Horiz G-T
```

Set the display window of the calculator as shown.

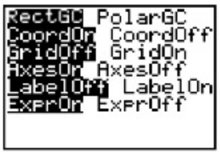
Press `WINDOW` and use the cursor keys to navigate to the proper settings, and then press `ENTER`, or press `ZOOM` `6`.



```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```

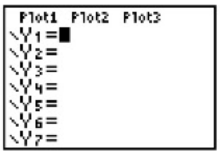
Set the format of the calculator as shown.

Press `2nd` `ZOOM` and use the cursor keys to navigate to the proper settings, and then press `ENTER`.



```
RectGC PolarGC
CoordOn CoordOff
GridOff GridOn
AxesOn AxesOff
LabelOff LabelOn
ExprOn ExprOff
```

Press `Y=` to confirm that the "Y= editor" window is as shown. If `Plot1`, `Plot2`, or `Plot3` are highlighted in black, then you must turn them off by scrolling to `Plot1`, `Plot2`, or `Plot3` and pressing `ENTER`.



```
Plot1 Plot2 Plot3
\Y1=
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

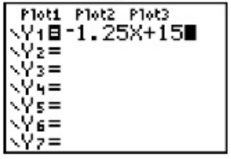
The relation with equation $y = -1.25x + 15$ is used throughout most of this lesson to illustrate some features of the graphing calculator.

Entering and Graphing a Relation using a Graphing Calculator

The following procedure may be used to display the graph of $y = -1.25x + 15$.

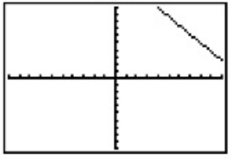
1. Access the "Y= editor" by pressing the `Y=` key.
2. Enter the equation in Y_1. (Write the equation in terms of the dependent variable if necessary).
3. Press the `GRAPH` key to display the graph.

Step 3



```
Plot1 Plot2 Plot3
\Y1=-1.25X+15
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

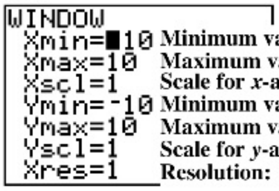
Step 4



Notice that the window we have chosen in this example does not allow us to see much of the graph. We need to alter the window setting to see the main features of the graph.

Using the Window and Zoom Features

Sometimes the standard display setting shown below does not allow us to see the behaviour of specific portions of a graph. For instance, if we wanted to see where the graph of $y = -1.25x + 15$ crosses the x-axis and the y-axis, we need to adjust the window settings of the calculator.

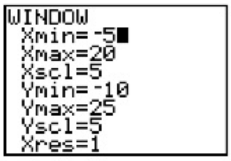


WINDOW
Xmin=-10 **Minimum value** seen on the screen for x
Xmax=10 **Maximum value** seen on the screen for x
Xscl=1 **Scale for x-axis** - The increments of the tic marks for x
Ymin=-10 **Minimum value** seen on the screen for y
Ymax=10 **Maximum value** seen on the screen for y
Yscl=1 **Scale for y-axis** - The increments of the tic marks for y
Xres=1 **Resolution:** 1 is best screen resolution. Leave at 1

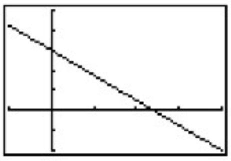
The following steps may be used to adjust the window settings:

1. Press the `WINDOW` key.

2. Use the cursor to enter in appropriate values to adjust the Cartesian plane and press the `GRAPH` key.



WINDOW
Xmin=-5
Xmax=20
Xscl=5
Ymin=-10
Ymax=25
Yscl=5
Xres=1



- An alternative method to "fit" the graph of an equation on the display screen is to use the `Zoom Out` feature of a graphing calculator. Although it may not give the exact window settings we would like, it can give a good first approximation.

  - Press the `ZOOM` `3` (or scroll down to `Zoom Out` and press `GRAPH`). Keep doing this until you see the graph.

  - Press `WINDOW` to see the new settings. Adjust the window.

- Another method to "fit" the graph of an equation on the display screen is to use the `ZoomFit` feature of a graphing calculator. Although it may not give the exact window settings we would like, it can also give a good first approximation.
- Press `ZOOM` `0` (or scroll down to `ZoomFit` and press `ENTER`).
- The standard display settings (as shown in the window at the top of this page) of a graphing calculator can be quickly entered by pressing `ZOOM` `6` (or scroll down to `ZStandard` and press `ENTER`).

- When displaying a graph on the calculator, it is generally a good idea to have the x- and y- intercepts of the graph visible.

- The following notation is used to write a graphing calculator window.

$$x{:}[x_{\min}, x_{\max}, x_{\text{scl}}] \qquad y{:}[y_{\min}, y_{\max}, y_{\text{scl}}]$$

For example, the window settings used above in step 1 can be written:

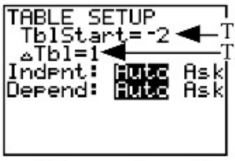
$$x{:}[-5, 20, 5] \qquad y{:}[-10, 25, 5]$$

Displaying the Table of Values for an Equation

The Table of Values feature may be used to display the ordered pairs of a relation defined by an equation. Before using this feature, we will need to set up the table using the Table Setup feature.

Accessing the Table Set Up Feature

- Press `2nd` `WINDOW` keys to access `TBLSET`. The illustration describes the features.



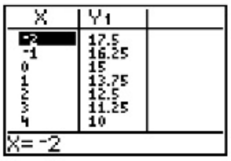
```
TABLE SETUP
 TblStart=-2
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

The value where the table starts.
The increments of the input.

- This will lead us to a table of values starting with $x = -2$ and increasing in units of 1.

Accessing the Table of Values

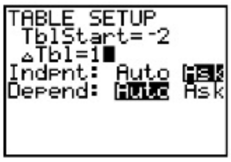
- Press `2nd` `GRAPH` keys to access the table feature of the calculator.
  We can use the cursor keys to scroll up and down the table to determine the value of y for any given value of x.



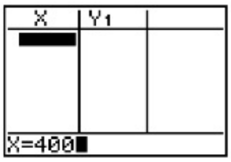
```
  X    Y1
 -2    17.5
 -1    16.25
 0     15
 1     13.75
 2     12.5
 3     11.25
 4     10
X=-2
```

If a value of x which would require continous cursoring is used, for example $x = 400$, a quicker method of calculating the value of y is given by the following procedure.

- Access `TBLSET` feature and change `Indpnt` to `Ask`,
- Return to the table of values, type in 400, and press `ENTER`.

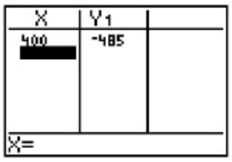


```
TABLE SETUP
 TblStart=-2
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```



```
  X    Y1

X=400
```



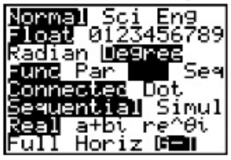
```
  X    Y1
 400   -485

X=
```

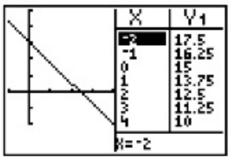
Accessing the Table of Values and the Graph Simultaneously

Return to Table Set Up as on the top of this page.

1. Press the `MODE` key, scroll down and across to "`G-T`", and then press `ENTER` and `GRAPH`.



```
Normal Sci Eng
Float 0123456789
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi re^θi
Full Horiz G-T
```



```
  X    Y1
 -2    17.5
 -1    16.25
 0     15
 1     13.75
 2     12.5
 3     11.25
 4     10
X=-2
```

2. Switching between the graph and table of values can be done by pressing the `GRAPH` key and the `2nd` `GRAPH` keys.

3. Using the `TRACE` key in the `G-T` mode allows us to scroll along the graph and up and down the table of values at the same time (use the left and right arrow keys).

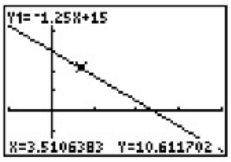
Complete Assignment Question #1

Finding the y- coordinate of a Point on the Graph of a Relation

To determine the output for a given input of a relation, the trace feature of a graphing calculator may be used.

Using the Trace Feature to Move Along the Graph

Return to Full screen.
The trace feature may be used to find the coordinates of points on the graph. The following procedures may be used.

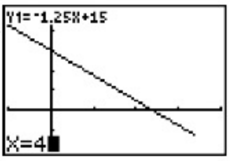


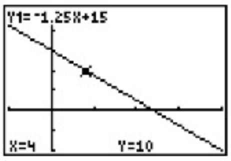
1. Set the window to x:[–5, 20, 5] y:[–10, 25, 5]

2. Press the TRACE key and use the scroll keys to move along the graph.

Using the Trace Feature to Determine the Value of *y* for a Given Value of *x*

- Press the TRACE key. Enter a value for the input, and press ENTER. For example, pressing TRACE 4 ENTER will calculate the value y when $x = 4$.
  The answer `Y = 10` will be displayed on the bottom right.





Finding the x- coordinate of a Point on the Graph of a Relation

To determine the input for a given output of a relation, the intersect feature of a graphing calculator may be used.

Using the Intersect Feature

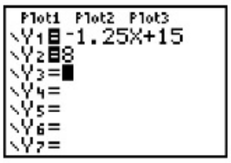
- The intersect feature can be used to determine the intersection point of the graphs of two relations. An application of this is to find the y-coordinate of a point on a graph given the x-coordinate.

- As a working example we will find the x-coordinate of $y = -1.25x + 15$ when $y = 8$.

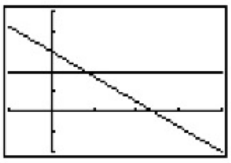
1. Enter the equation into Y_1.

2. Enter the given value of y into Y_2.

   Press GRAPH, and adjust the window if necessary.

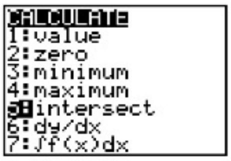


```
Plot1 Plot2 Plot3
\Y1=-1.25X+15
\Y2=8
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



3. Access the `CALC` menu by entering 2nd then TRACE.

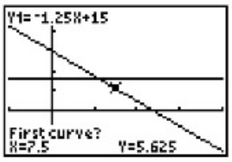
4. Select the "`intersect`" feature.



5. On the bottom left-hand side of the screen, the calculator will ask for the `First curve?` .

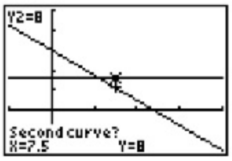
Scroll close to the intersection point, and press ENTER.

The values of x and y will vary depending on how close we scroll to the intersection point.


Y1=⁻1.25X+15
First curve?
X=7.5 Y=5.625

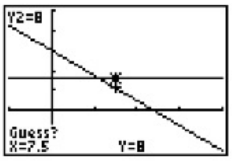
6. On the bottom left-hand side of the screen, the calculator will ask for a `Second curve?` .

Scroll close to the intersection point, and press ENTER.


Y2=8
Second curve?
X=7.5 Y=8

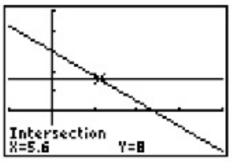
7. On the bottom left-hand side of the screen, the calculator will ask to `Guess?` .

Press ENTER.

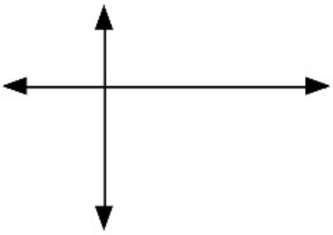

Y2=8
Guess?
X=7.5 Y=8

8. The x and y values of the intersection point will be displayed. In this example the intersection point is $(5.6, 8)$.

The answer to the question is $x = 5.6$.


Intersection
X=5.6 Y=8

Consider the equation $y = 2x - 25$.



a) Use a graphing calculator to sketch the graph on the grid provided. Write a suitable window.

x:[, ,] y:[, ,]

b) Use a graphing calculator to determine the value of y when $x = 4.5$.

c) Algebraically verify the answer in **b)**.

d) Use a graphing calculator to determine the value of x when $y = -6.4$.

e) Algebraically verify the answer in **d)**.

Complete Assignment Questions #2 - #5

Using a Graphing Calculator to Determine x-intercepts

Finding the *x*-intercept using the Intersect Feature

The intersect feature may be used to find the *x*-intercept by using the equation $y = 0$ for Y_2 and determining the intersection point of the graph and *x*-axis.

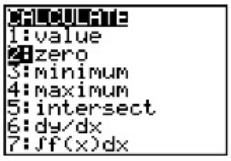
Finding the *x*-intercept using the Zero Feature

The *x*-intercept(s) on the graph of a relation can be determined using the **zero** feature on the calculator.

The following procedure may be used to determine the *x*-intercept on the graph.

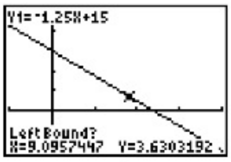
1. Enter the equation in Y_1 ($y = -1.25x + 15$, x:[−5, 20, 5], y:[−10, 25, 5]).

2. Press `2nd` `TRACE` keys to access the `CALCULATE` (`CALC`) menu.

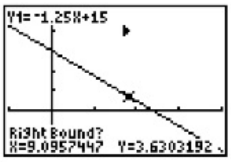


3. Choose `2:zero`, and press `ENTER`.

4. On the bottom left-hand side of the screen, the calculator will ask for a `LeftBound?`. The calculator is asking us to put the cursor on the **left side of the *x*-intercept**.



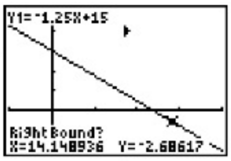
Left Bound?
X=9.0957447 Y=3.6303192



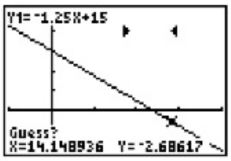
Right Bound?
X=9.0957447 Y=3.6303192

Place the flashing cursor on the left side of the *x*-intercept using the left and right cursor keys. Press `ENTER`. Note the arrow to the left and above the graph on the screen.

5. On the bottom left-hand side of the screen, the calculator will ask for a `RightBound?`. The calculator is asking us to put the cursor on the **right side of the *x*-intercept**.



Right Bound?
X=14.148936 Y=-2.68617



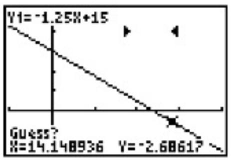
Guess?
X=14.148936 Y=-2.68617

Place the flashing cursor on the right side of the *x*-intercept using the left and right cursor keys. Press `ENTER`. Note the second arrow to the right and above the graph on the screen.

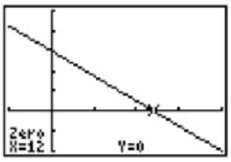
6. On the bottom left-hand side of the screen, the calculator will ask to `Guess?`.

Press `ENTER`.

The x value will be the x-intercept.



Guess?
X=14.148936 Y=-2.68617

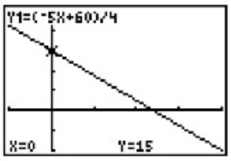

Zero
X=12 Y=0

Using a Graphing Calculator to Determine y-intercepts

Finding the *y*-intercept using the Trace Feature

1. Enter the equation in `Y1`, and adjust the window to the appropriate settings to display the *x*- and *y*-intercepts.

2. Press the `TRACE` key. Press `0` for the input, and then press `ENTER` for the output.



```
Y1=(-5X+60)/4
X=0          Y=15
```

Finding the *y*-intercept using the Value Feature

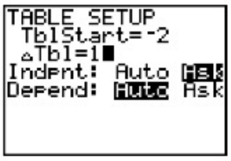
1. Enter the equation in `Y1`, and adjust the window to the appropriate settings to display the *x*- and *y*-intercepts.

2. Press `2nd` `TRACE` keys to access the `CALCULATE` (`CALC`) menu.

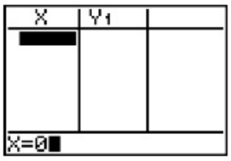
3. Choose `1:value`, and press `ENTER` `0` `ENTER`.

Finding the *y*-intercept using the Table Feature

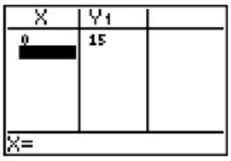
Use the procedure described earlier to find y when $x = 0$.



```
TABLE SETUP
 TblStart=-2
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```



```
   X   | Y1 |
X=0
```



```
   X   | Y1 |
0      | 15 |
X=
```

Class Ex. #2

In each case use the features of a graphing calculator to calculate the *x*- and *y*-intercepts of the graphs of the relation.

a) $y = \dfrac{5x + 15}{3}$

b) $y = -x^2 + x + 6$

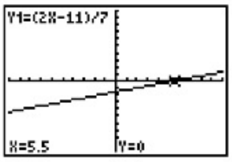
Converting a Decimal to a Fraction from a Graph Display

If the x- or y-intercept is not an integer, the graphing display will give the answer in decimal form. The following procedure may be used to write the decimal as an improper fraction (if applicable).

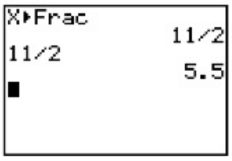
To demonstrate the procedure we will determine the x- and y-intercepts of the graph of the equation $y = \frac{(2x - 11)}{7}$.

Converting the x-value to a Fraction

1. Enter the equation in Y_1. Press `GRAPH`, and adjust the window to the appropriate settings.
2. Find the x-intercept using the zero feature.



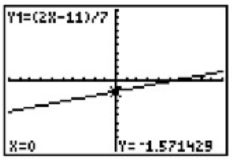
3. Press `2nd` `QUIT` to exit the graph display.
4. Press `X,T,θ,n` `MATH` `ENTER` `ENTER`.
   The decimal value may be verified by dividing the fraction.



```
X▸Frac
                11/2
11/2
                 5.5
```

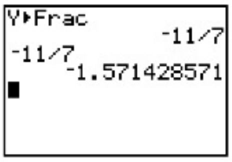
Converting the y-value to a Fraction

1. Enter the equation in Y_1. Press `GRAPH`, and adjust the window to the appropriate settings.
2. Find the y-intercept using the trace or value feature.



3. Press `2nd` `QUIT` to exit the graph display.
4. Press `ALPHA` `1` to access the letter Y, and then press `MATH` `ENTER` `ENTER`.

   The decimal value may be verified by dividing the fraction.

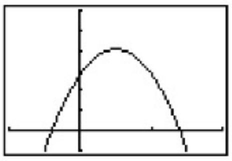


```
Y▸Frac
               -11/7
-11/7
        -1.571428571
```

Complete Assignment Questions #6 - #7

Extension: Using a Graphing Calculator to Determine a Maximum Value

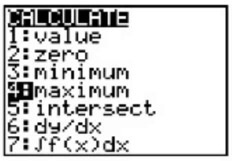
- The graphing calculator can be used to calculate the coordinates of the maximum or minimum points on a graph.
- The equation $y = -x^2 + 5x + 14$ will be used to illustrate the maximum features.



1. Enter the equation into Y_1; adjust the window to x: [–5, 10, 5] y: [–5, 30, 5], and press GRAPH.

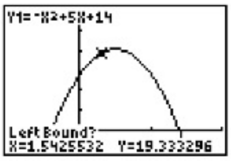
2. Access the CALC menu by entering 2nd then TRACE.

3. Select "maximum".

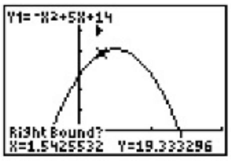


CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx

4. On the bottom left-hand side of the screen, the calculator will ask us for a LeftBound? . Select a value by scrolling to the left side of the maximum point and press ENTER.

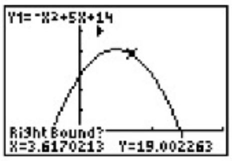


$Y1=-X^2+5X+14$
Left Bound?
X=1.5425532 Y=19.333296

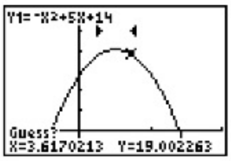


$Y1=-X^2+5X+14$
Right Bound?
X=1.5425532 Y=19.333296

5. On the bottom left-hand side of the screen, the calculator will ask us for a RightBound ?. Select a value by scrolling to the right side of the maximum point and press ENTER.

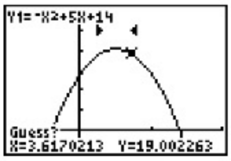


$Y1=-X^2+5X+14$
Right Bound?
X=3.6170213 Y=19.002263

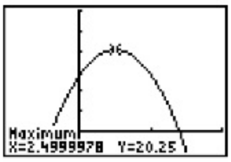


$Y1=-X^2+5X+14$
Guess?
X=3.6170213 Y=19.002263

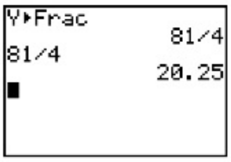
6. On the bottom left-hand side of the screen, the calculator will ask to Guess? . Press ENTER.



$Y1=-X^2+5X+14$
Guess?
X=3.6170213 Y=19.002263

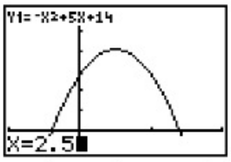


Maximum
X=2.4999978 Y=20.25

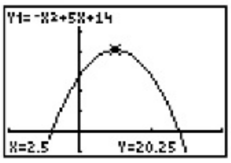


Y▶Frac 81/4
81/4 20.25

7. Due to the limitations of the calculator, the x value on the display may vary slightly from calculator to calculator. The answer obtained may depend on the window setting used and the bounds chosen. All the answers, however, should round to the exact answer; in this case 2.5. Test the point using the trace feature and entering $x = 2.5$.



$Y1=-X^2+5X+14$
X=2.5



$Y1=-X^2+5X+14$
X=2.5 Y=20.25

In this example the graph of $y = -x^2 + 5x + 14$ has a **maximum value** of 20.25 or $\frac{81}{4}$, and the **maximum point** has coordinates (2.5, 20.25).

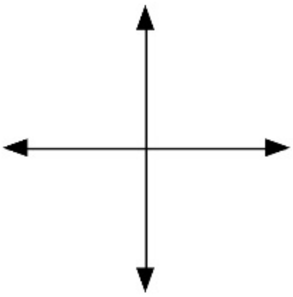
Complete Assignment Questions #8 - #9

Assignment

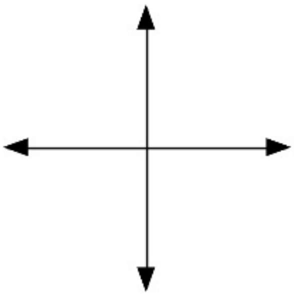
1. For each relation represented by an equation:

- Use a graphing calculator to graph the relation.
- Adjust the window to an appropriate setting so that the x- and y- intercepts (where applicable) are visible.
- Sketch the graph on the grid provided.
- Write an appropriate window setting.
- Reset the window to `ZStandard` before beginning a new graph.

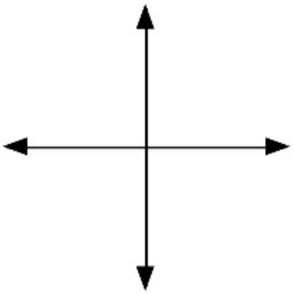
a) $y = 0.5x - 8$



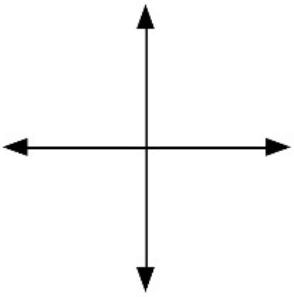
b) $y = 2x - 15$



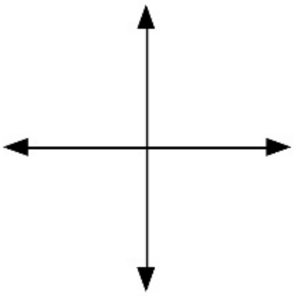
c) $y = -x + 25$



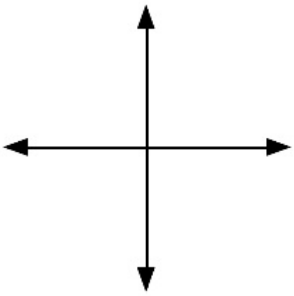
d) $y = -3x - 40$



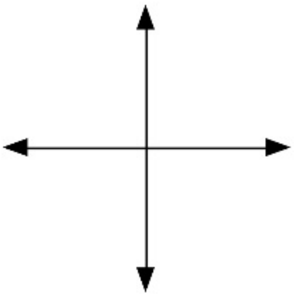
e) $y = -2x + 128$



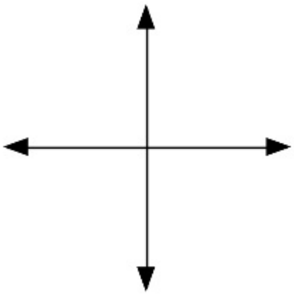
f) $y = x^2 - 15$



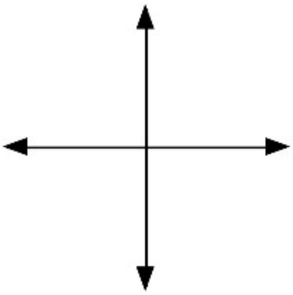
g) $y = x^2 + 20x$



h) $y = 30$

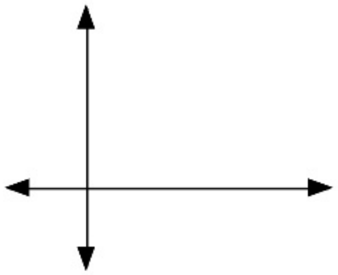


i) $y = 2^x$



2. Consider the equation $y = -3x + 36$.

a) Use a graphing calculator to sketch the graph on the grid provided. Write an appropriate window so that the x- and y- intercepts are visible.



b) Use a graphing calculator to determine the value of y when $x = 5$.

c) Algebraically verify the answer in **b)**.

d) Use a graphing calculator to determine the value of x when $y = 12$.

e) Algebraically verify the answer in **b)**.

3. Given the x-coordinate of each relation, determine the corresponding y-coordinate using the graphing features of a calculator.

a) $y = 8x - 5,\ x = 7,\ y =$

b) $y = 0.75x + 8,\quad x = 15,\ y =$

c) $y = -2x + 12, x = -17,\ y =$

d) $y = \frac{5}{3}x - 1,\ x = 0,\ y =$

e) $y = x^2 + 3x - 12,\ x = -5,\ y =$

f) $y = 2x^2 - 5x - 2,\ x = 3.5,\ y =$

g) $y = 2x^3 + 15,\ x = 2,\ y =$

h) $y = |x| - 7,\ x = -15,\ y =$

4. Given the y-coordinate of each relation, determine the corresponding x-coordinate using the graphing features of a calculator.
Write your answer to the nearest hundredth where necessary.

a) $y = 7x - 3,\ x =$, $y = 8$

b) $y = 5x + 8,\ x =$, $y = -3$

c) $y = -2x + 12,\ x =$, $y = 15$

d) $y = \frac{5}{3}x - 1,\ x =$, $y = -12$

e) $y = x^2 + 2x - 15,\ x =$ and , $y = 3$

f) $y = 2x^3 + 15,\ x =$, $y = 10$

g) $y = -x^2 - 10x - 21,\ x =$ and , $y = -15$

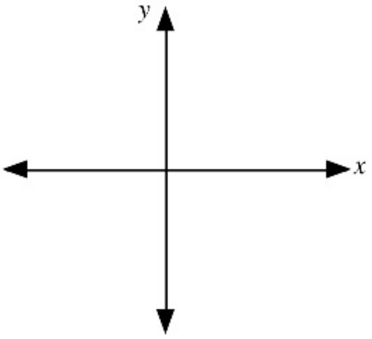
h) $y = |x| - 7,\ x =$, $y = 20$

5. Consider the relation given by the equation $y = -0.5x^2 + 8$.

a) Use the table set and table features of the calculator to complete the table of values below.

Input (x)	Output (y)
–6	
–4	
–2	
0	
2	
4	

b) Sketch the graph of the relation using an appropriate window.



c) State the window setting x:[, ,] y:[, ,].

d) Use the appropriate features of a calculator to determine:

i) the value of y when $x = 10$

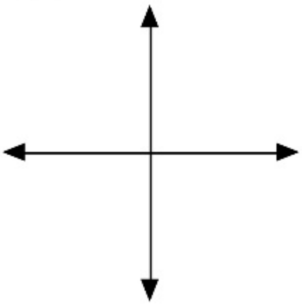
ii) the values of x when $y = 3.5$

e) Compare the answers to **a)**, **b)**, and **d ii)** with those from Lesson #2, assignment question #6.

6. For each equation use a graphing calculator to:

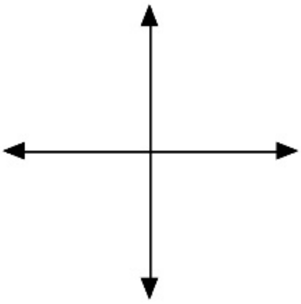
- sketch the graph of the relation on the grid provided
- **list** in the chart provided any x- and y-intercepts as **exact values**
- **write** the coordinates of the points representing the x- and y-intercepts on the graph of the relation.

a) $y = -3 - x$



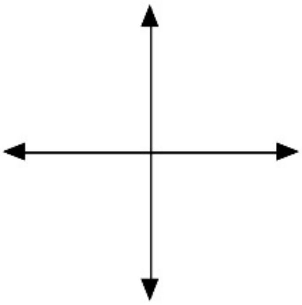
x-intercept	
y-intercept	

b) $y = -3x - 1$



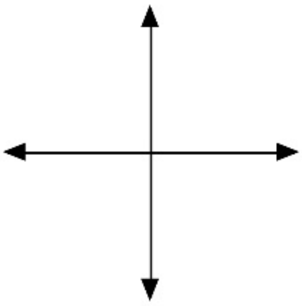
x-intercept	
y-intercept	

c) $y = 2x - 5$



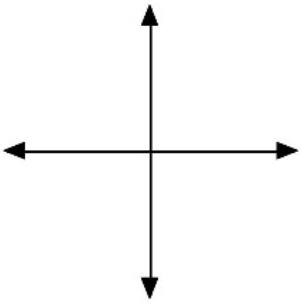
x-intercept	
y-intercept	

d) $y = -6x - 10$



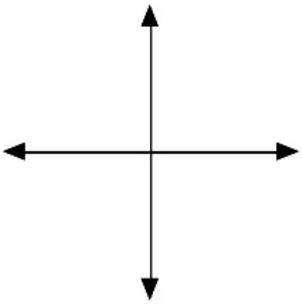
x-intercept	
y-intercept	

e) $y = \dfrac{-4x + 6}{3}$



x-intercept	
y-intercept	

f) $y = 5$

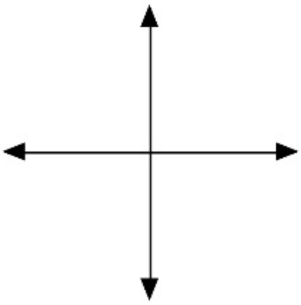


x-intercept	
y-intercept	

7. For each equation use a graphing calculator to:

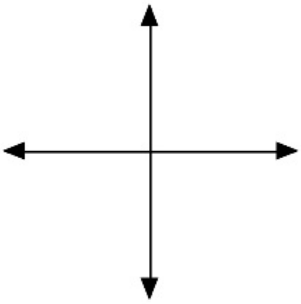
- sketch the graph of the relation on the grid provided
- **list** in the chart provided any x- and y-intercepts as **exact values**
- **write** the coordinates of the points representing the x- and y-intercepts on the graph of the relation.

a) $y = x^2 - 5x - 6$



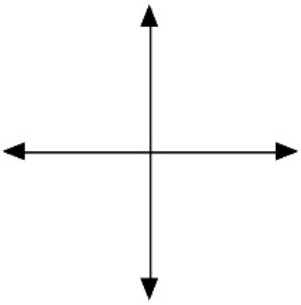
x-intercept(s)	
y-intercept	

b) $y = x^2 - 16$



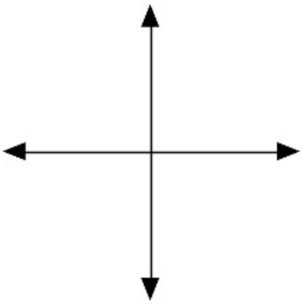
x-intercept(s)	
y-intercept	

c) $y = 2x^2 - 7x + 8$



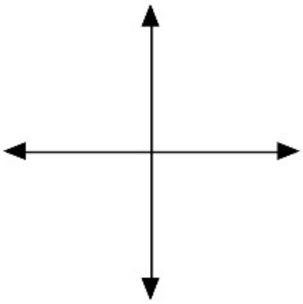
x-intercept(s)	
y-intercept	

d) $y = -x^2 + 3x + 18$



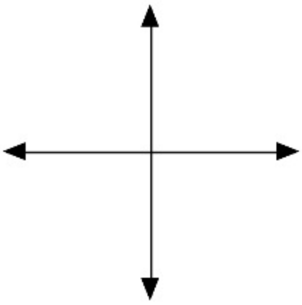
x-intercept(s)	
y-intercept	

e) $y = -2x^2 - 7x - 12$



x-intercept(s)	
y-intercept	

f) $y = 2x^3 + x^2 - 16x - 15$



x-intercept(s)	
y-intercept	

8. Determine the coordinates of the maximum point of each graph. Answer using exact values.

a) $y = -0.3x^2 + 5x - 10$ **b)** $y = -0.1x^2 + 5x + 7$ **c)** $y = -9x^2 - 60x + 100$

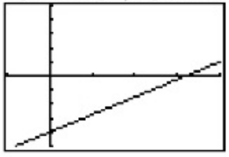
9. Determine the coordinates of the minimum point of each graph. Answer using exact values.

a) $y = x^2 + 2x + 35$ **b)** $y = 0.15x^2 - 2x - 8$ **c)** $y = 0.5x^2 + 2x - 16$

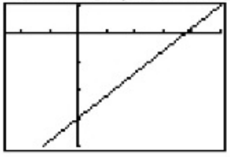
Answer Key

1. *Window setting answers may vary.*

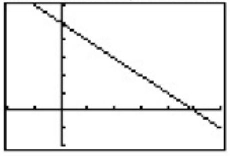
a) x: [-5,20,5], y: [-10,10,2]



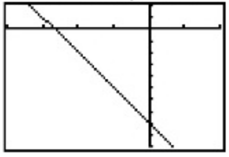
b) x: [-5,10,2], y: [-20,5,5]



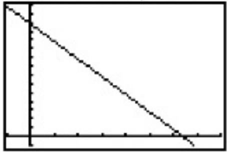
c) x: [-10,30,5], y: [-10,30,5]



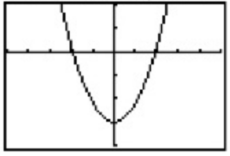
d) x: [-20,10,5], y: [-50,10,10]



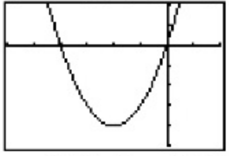
e) x: [-10,80,10], y: [-10,150,10]



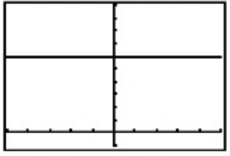
f) x: [-10,10,2], y: [–20,10,5]



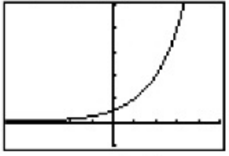
g) x: [-30,10,5], y: [-125,50,25]



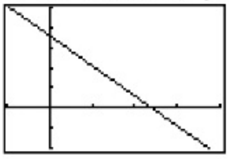
h) x: [-10,10,2], y: [-5,50,5]



i) x: [-2,5,1], y: [-5,30,5]



2. a) x: [-5,20,5], y: [-20,50,10] **b)** 21 **c)** 21 **d)** 8 **e)** 8



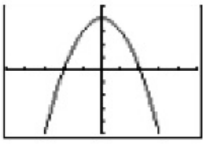
3. a) 51 **b)** 19.25 **c)** 46 **d)** –1
e) –2 **f)** 5 **g)** 31 **h)** 8

4. a) 1.57 **b)** –2.2 **c)** –1.5 **d)** –6.6
e) –5.36 and 3.36 **f)** –1.36 **g)** –9.36 and –0.64 **h)** ±27

5 a)

Input (x)	Output (y)
–6	-10
–4	0
–2	6
0	8
2	6
4	0

b)

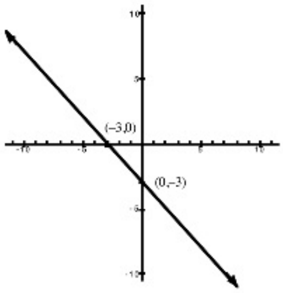


c) x:[–10, 10, 2] y:[–10, 10, 2]

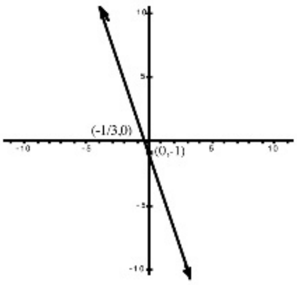
d) i) –42 **ii)** –3 and 3

e) exactly the same

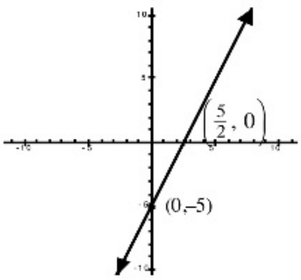
6. **a)** x-int $= -3$, y-int $= -3$



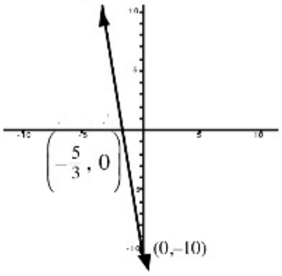
b) x-int $= -\frac{1}{3}$, y-int $= -1$



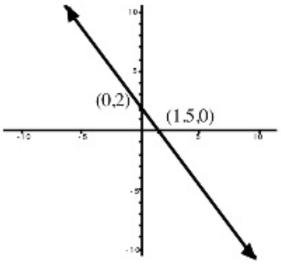
c) x-int $= \frac{5}{2}$, y-int $= -5$



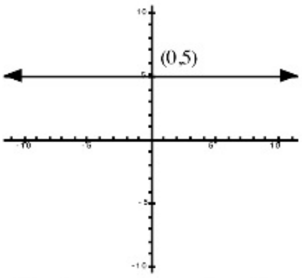
d) x-int $= -\frac{5}{3}$, y-int $= -10$



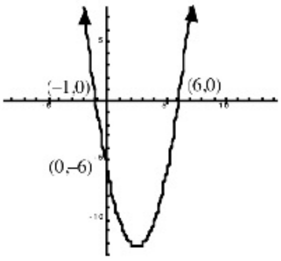
e) x-int $= \frac{3}{2}$, y-int $= 2$



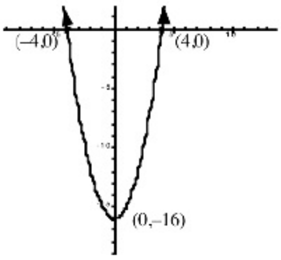
f) no x-int, y-int $= 5$



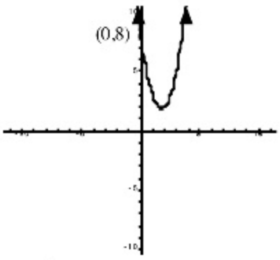
7. **a)** x-int $= -1$ and 6, y-int $= -6$



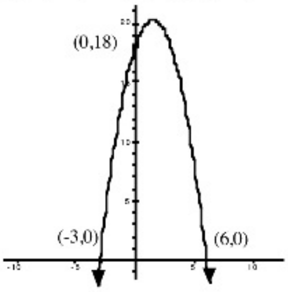
b) x-int $= -4$ and 4, y-int $= -16$



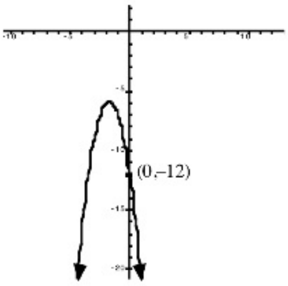
c) no x-int, y-int $= 8$



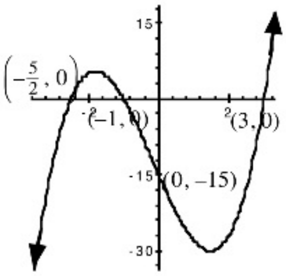
d) x-int $= -3$ and 6, y-int $= 18$



e) no x-int, y-int $= -12$



f) x-int $= -\frac{5}{2}$, -1 and 3 y-int $= -15$



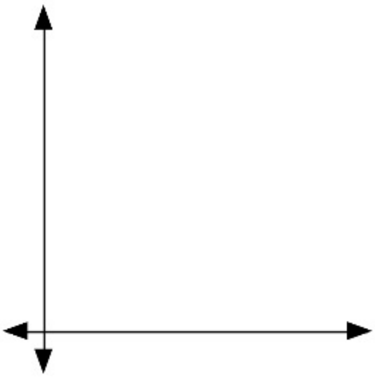
8. **a)** $\left(\frac{25}{3}, \frac{65}{6}\right)$ **b)** $\left(25, \frac{139}{2}\right)$ **c)** $\left(-\frac{10}{3}, 200\right)$

9. **a)** $(-1, 34)$ **b)** $\left(\frac{20}{3}, -\frac{44}{3}\right)$ **c)** $(-2, -18)$

Relations Lesson #6:
Interpreting Relations Using a Graphing Calculator

Consider the scenario from Class Ex. #3 from Lesson #3 of this unit.

"Lisa purchases a new car for \$20 000.
The value of the car can be represented by the formula $V = 20\,000 - 1250t$, where V is the value of the car in dollars and t is the age of the car in years."



a) • Graph $V = 20\,000 - 1250t$ using a graphing calculator.

- Use the ZoomFit feature as a guide to adjust the graphing window. Then use the **Window** key to align the graph appropriately for this scenario. Write the window setting below.

- Sketch the graph on the grid provided.

b) Calculate the t-intercept of the graph using a graphing calculator, and label it on the sketch. Describe what this value represents in the context of the question.

c) Calculate the V-intercept of the graph using a graphing calculator, and label it on the sketch. Describe what this value represents in the context of the question.

d) Use the trace feature of a graphing calculator to determine what the car will be worth in 5 years.

e) Use the intersect feature of a graphing calculator to determine when the car will be worth half of the purchase price. Illustrate this on your sketch.

f) Write an appropriate domain and range for the function which describes the value of the car over time.

The height of a human cannon ball, "Cano", can be described by the formula $h = 12 + 6t - t^2$, where h is the height in metres above ground level and t is the time in seconds. Cano is projected out of a cannon from the top of a building and lands on a soft mat. The mat is placed in a hole in the ground so that the top of the mat is level with the ground.

a) Display the graph of $h = 12 + 6t - t^2$ on a graphing calculator.

b) Write down a window setting which would be appropriate for this situation.

c) Sketch $h = 12 + 6t - t^2$ on the grid provided.

d) What is the height of the cannon above the ground?

e) What is the maximum height Cano reaches?

f) How many seconds does it take Cano to reach the highest point on the path he is travelling?

g) To the nearest hundredth of a second, how long does it take Cano to land on the mat?

h) How high is Cano one second after he is launched?

i) When will Cano be at the height in **h)** again?

j) In words, describe the relation connecting height and time.

k) Write an appropriate domain and range for the relation described in **j)**.

Complete Assignment Questions #1 - #5

Assignment

In this assignment, round answers to 2 decimal places unless otherwise stated.

1. Students at a senior high school produce an art literary magazine. The cost for this magazine can be modelled by the formula $C = 2n + 30$, where C is the total cost of the magazine in dollars and n is the number of magazines produced.

a) Sketch the graph on the grid provided. Explain why the graph is not a continuous straight line.

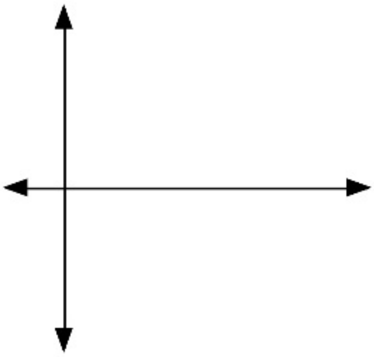
b) Is the n-intercept relevant to the graph of this relation?

c) What would be the cost for 30 magazines?

d) How many magazines are produced if the total cost is $126?

e) Describe the significance of the C-intercept.

2. For a science experiment, a pot of water is heated to a certain temperature and then put in a freezer and allowed to freeze. The rate of freezing can be estimated by the formula $T = -0.4t + 50$, where the temperature, T, in degrees Celsius is recorded for each time, t in minutes.



a) Sketch the graph on the grid using a graphing calculator.

b) What was the temperature of the water at 12 minutes?

c) How long did it take for the pot of water to cool down to 40°C?

d) Describe the significance of the t-intercept in this question.

e) Describe the relation in words.

3. During a high school football game, the height of a punt can be modelled by the relation

$$h = -5t^2 + 14t + 1$$

where

t is the number of seconds which have elapsed since the football was punted, and

h is the number of metres above the ground after t seconds

a) Sketch the graph on the grid.

b) What was the height of the football above the ground as the punter makes contact with the football?

c) What was the height of the football above the ground 1 second after contact?

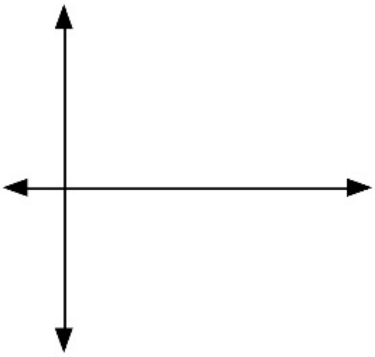
d) What is the maximum height of the football above the ground?

e) How many seconds had elapsed when the football reached its maximum height?

f) The punt is not caught by the opposing team, and the football hits the ground. How many seconds, to the nearest hundredth, did it take for the football to hit the ground?

4. Student Council is planning a school dance and sells tickets at $5 per student. The expenses for the dance are $800. The treasurer of the council generates a formula to determine the potential profit for the dance to be $P = 5t - 800$, where P is the profit made on the dance and t is the number of tickets sold. The maximum number of tickets which can be sold is 450.

a) Sketch the graph on the grid using a graphing calculator.



b) Determine how many tickets need to be sold to break even.

c) How many tickets need to be sold to make a $1250 profit?

d) How much profit is made if 200 tickets are sold?

e) Write the domain and range for this relation.

5. In a soccer game the ball is passed back to the goal keeper, and she kicks the ball from ground level up the field. The height, h metres, of the ball above the ground can be modelled by the equation $h = -0.033d^2 + 1.6d - 10$, where d is the distance in metres from the goal line.

a) Sketch the graph on the grid.

b) What was the distance between the goal line and the point where the ball was kicked?

c) What is the maximum height the soccer ball reaches?

d) How far from the goal line is the ball when it reaches its maximum height?

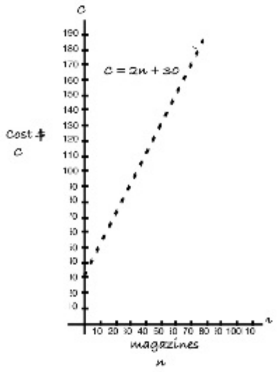
e) What horizontal distance has the ball travelled when it hits the ground for the first time?

f) Write a domain and range for this relation.

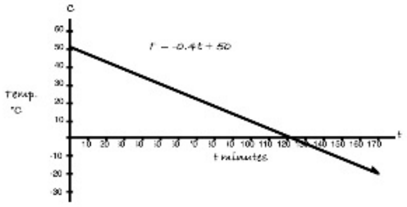
g) At what distance from the goal line is the ball 5 metres in the air?

Assignment Key

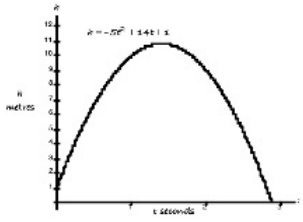
1 . a) See grid at side. The data is discrete so there are gaps in the line.
b) No, since the number of magazines cannot be negative.
c) $90
d) 48
e) The C-intercept of 30 represents a fixed charge of $30 irrespective of the number of magazines produced.



2 . a) See grid at side
b) 45.2° C
c) 25 min.
d) The t intercept of 125 represents the number of minutes it takes the water to freeze.
e) A pot of water is heated to 50°C and placed in a freezer and allowed to freeze. It cools at a constant rate and after 125 min the water is frozen.

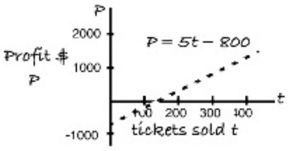


3 . a) See grid below



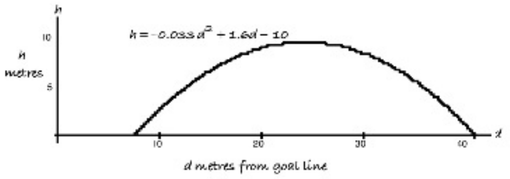
b) 1 m
c) 10 m
d) 10.8 m
e) 1.4 sec.
f) 2.87 sec.

4 . a) See grid below
b) 160
c) 410
d) $200



e) domain: 0, 1, 2, 3 450
range: –800, –795, –790, –785 1450
(increasing in steps of 5)

5 . a) See grid at side
b) 7.37 m **c)** 9.39 m
d) 24.24 m **e)** 33.74 m
f) domain: $\{7.37 \le d \le 41.11, d \in R\}$
range: $\{0 \le h \le 9.39, h \in R\}$
g) 12.70 m and 35.78 m



Relations Lesson #7:
Practice Test

1. Which of the following variables is discrete?

A. The time taken to run 50 metres.
B. The distance travelled by a train.
C. The number of animals in a zoo.
D. The weight of a dog.

2. Which of the following statements is false?

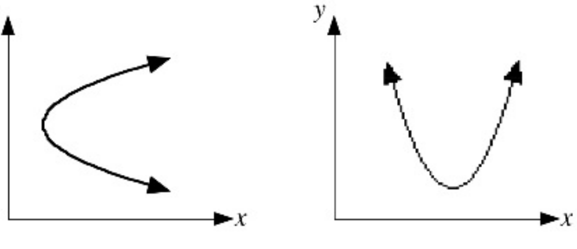
A. The domain of a relation is associated with values of the independent variable.
B. The dependent variable is represented by the first coordinate of an ordered pair.
C. The inputs of a relation are shown on the horizontal axis of a Cartesian Plane.
D. The range of a relation is represented on the vertical axis of the graph of the relation.

Numerical Response **1.** To the nearest hundredth, the positive *x*-intercept on the graph of the relation $2x^2 + 6y^2 = 7$ is _____ .
(Record your answer in the numerical response box from left to right)

☐☐☐☐

Use the following information to answer the next question.

The graphs of two relations are shown.



3. Which of the following statements is true?
A. The domains are the same, but the ranges are different.
B. The ranges are the same, but the domains are different.
C. The domains are the same, and the ranges are the same.
D. The domains are different, and the ranges are different.

Numerical Response **2.** The graph of the relation $4x^2 - 3y = 12$ has x-intercepts a and b, and y-intercept c. The value of the product abc is _____ .

(Record your answer in the numerical response box from left to right) ☐☐☐☐

Use the following information to answer the next two questions.

The cost ($ C) of publishing a school newspaper is partially fixed and partially depends on the number (n) of newspapers printed. The cost is given by the formula $C = 98 + 0.12n$.

4. The cost of publishing 780 newspapers is

A. $191.60
B. $773.76
C. $94.58
D. $9 458.00

Numerical Response **3.** The domain of the relation can be expressed in the form $n \geq x$, and the range can be expressed in the form $C \geq y$. Write the value of y in the first two boxes, and the value of x in the last two boxes.

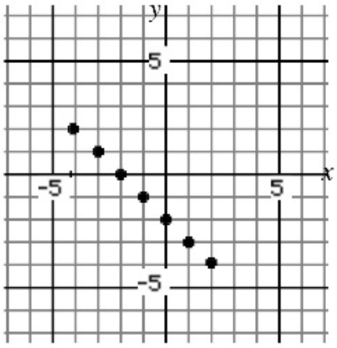
(Record your answer in the numerical response box from left to right) ☐☐☐☐

5. Dianna is driving back to Squamish Nation from Tsawwassen First Nation. Consider the relation in which the quantity of fuel in the gas tank of her truck is related to the distance she has travelled. The domain of this relation can be measured in

A. km
B. litres
C. km/litre
D. litres/km

Use the following information to answer the next two questions.

The diagram shows the graph of a relation.



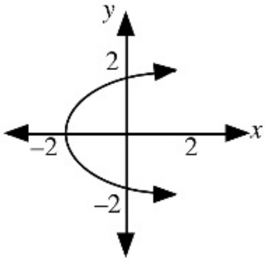
6. The domain of the relation can be written in the form

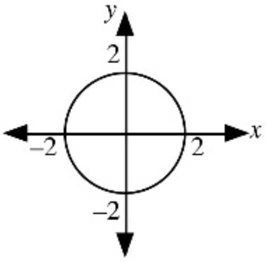
A. $\{x \mid -4 \leq x \leq 2, x \in R\}$
B. $\{x \mid -4 \leq x \leq 2, x \in W\}$
C. $\{x \mid -4 \leq x \leq 2, x \in I\}$
D. $\{x \mid -4 \leq x \leq 2, x \in N\}$

7. The range can be written in the form

A. $\{y \mid y \geq -4, y \in I\}$
B. $\{x \mid y \leq 2, y \in I\}$
C. $\{y \mid -4 < y < 2, y \in I\}$
D. $\{y \mid -5 < y < 3, y \in I\}$

8. The graphs of two relations are shown. Which of the following statements is true?





A. The range of both relations is $\{y \mid -2 \leq y \leq 2, y \in R\}$.
B. The range of both relations is $y \in R$.
C. The domain of both relations is $\{x \mid -2 \leq x \leq 2, x \in R\}$.
D. None of the above.

Use the following information to answer the next four questions.

A student has graphed the relation $y = 4x^3 - 7x + 3$ on her graphing calculator.

9. Which of the following is not an x-intercept?

A. $-\frac{3}{2}$
B. $\frac{1}{3}$
C. $\frac{1}{2}$
D. 1

10. The y-intercept is

A. 0
B. 4
C. –7
D. 3

11. When $y = 2$, the number of possible values of x is

A. one
B. two
C. three
D. infinite

12. When $y = -2$, the value of x, to the nearest tenth, is

A. –1.6
B. –1.5
C. 1.5
D. 1.6

Use the following information to answer the next five questions.

The height of a soccer ball above the ground after a free kick for the Hawks is given by the equation $h = 0.02(-d^2 + 40d)$, where h is the height in metres and d is the horizontal distance in metres the ball travels.

13. The height, in metres, that the soccer ball is above the ground when the ball is kicked is

A. 0
B. 0.02
C. 0.05
D 0.2

14. The maximum height reached by the soccer ball is

A. 4 metres
B. 8 metres
C. 10 metres
D. 20 metres

15. When the ball reaches its maximum height, the horizontal distance it has travelled is

A. 8 metres
B. 16 metres
C. 20 metres
D. 40 metres

Numerical Response **4.** Before the ball strikes the ground, a defender heads the ball after it has travelled a horizontal distance of 37 m. The height of the ball above the ground, to the nearest tenth of a metre, when the defender heads the ball, is _____ .

(Record your answer in the numerical response box from left to right)

Numerical Response **5.** After the ball is kicked, it is exactly 5 m above the ground at the top of its trajectory. To the nearest tenth of a metre, the horizontal distance that the ball has travelled between the two instances when it is 5 m above the ground is _____ .

(Record your answer in the numerical response box from left to right)

Written Response - 5 marks

1. During an airshow, the path of a stunt dive of a jet can be modelled by the equation $h = t^2 - 9t + 81$, where h is the height in metres after t seconds. The end of the stunt occurs when the plane achieves its starting height.

- How high is the jet at the start of the dive?
- How high is the jet above ground level at its lowest point?
- When does the jet reach its lowest point in the dive?
- How high is the jet two seconds into its dive?
- After how many seconds does this height occur again within the jet's stunt dive?

Answer Key

1. C **2.** B **3.** D **4.** A **5.** B **6.** C **7.** D **8.** D
9. B **10.** D **11.** C **12.** A **13.** A **14.** B **15.** C

Numerical Response

1.

1	.	8	7

2.

1	2		

3.

9	8	0	0

4.

2	.	2	

5.

2	4	.	5

Written Response

1.
- 81 m
- 60.75 m
- 4.5 sec
- 67 m
- 7 sec

Relations Lesson #2:
Relationships Between Two Quantities

Relations

Much of mathematics involves the search for patterns and relationships between sets of data. Many real life applications of mathematics investigate the relationship between two quantities.

For example:

- the value of a computer is related to its age
- the price of a watermelon is related to its weight
- the time taken for a person to walk to school is related to the walking distance.

In mathematics, a comparison between two sets of elements is called a **relation**.

List one more example of a relation.

Representing the Relationship Between Two Quantities

In the next two units we will consider seven ways in which the relationship between two quantities can be represented.

- in words
- a table of values
- a set of ordered pairs
- a mapping (or arrow) diagram
- an equation
- a graph
- function notation (some relations can be represented in this way as in the next unit)

We will use the relation below as an example.

Investigating a Relation

Consider the following relation:

> **" The cost, C (cents per km), of driving a car is related to the speed, s (km/h), at which it is driven."**

We will use this relation to introduce some ideas which will be developed throughout the course of this unit. Our task is to represent this relation in some form.

The example illustrates a relationship between two **variables**, C and s.
In the statement of the relation, the cost depends on the speed.

C is called the **dependent variable** and s is called the **independent variable**.

When representing a relation, we often regard the values of the independent variable as the **input** and the values of the dependent variable as the **output**.

Before considering how to represent this relation we need some data: we need input values and output values.

The input values make up the **domain** of the relation, and the output values make up the **range** of the relation. These concepts will be discussed in more detail later.

Obviously we would not attempt to collect data for every possible input value (i.e. for every possible speed at which the car could be driven). Suppose that we choose as input values speeds of 20, 30, 40, ...120 km/h. and that the output values are as given in the diagrams below.

Table of Values

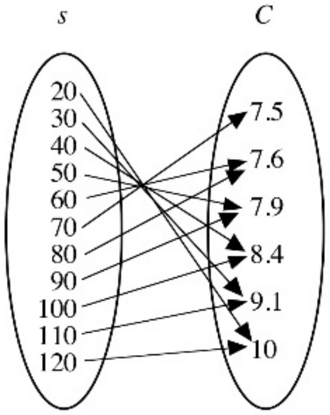
Input (s)	Output (C)
20	10
30	9.1
40	8.4
50	7.9
60	7.6
70	7.5
80	7.6
90	7.9
100	8.4
110	9.1
120	10

Ordered Pairs

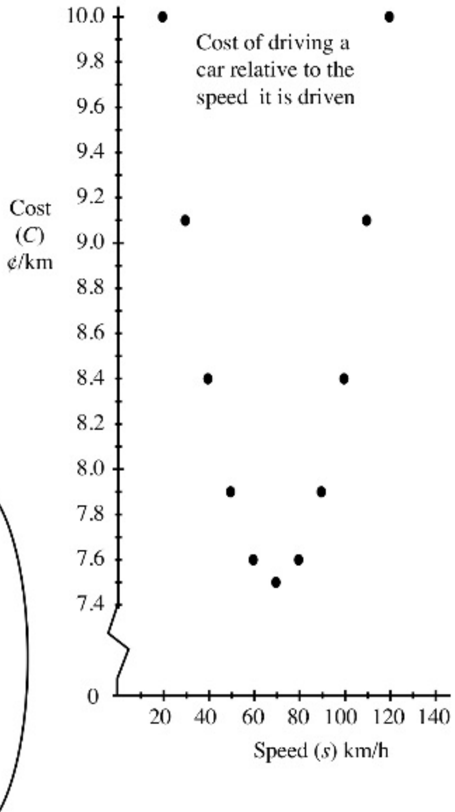
(s, C)

$(20, 10)$
$(30, 9.1)$
$(40, 8.4)$
$(50, 7.9)$
$(60, 7.6)$
$(70, 7.5)$
$(80, 7.6)$
$(90, 7.9)$
$(100, 8.4)$
$(110, 9.1)$
$(120, 10)$

Mapping Diagram



Graph



The diagrams above also illustrate how the information collected can be represented

- as ordered pairs in a table of values
- as a mapping diagram
- graphically

We have only chosen some of the possible input values, however it is obvious that an output value could have been determined for any input value greater than zero and up to the maximum speed of the car. It makes sense then to connect the points on the graph in some way.

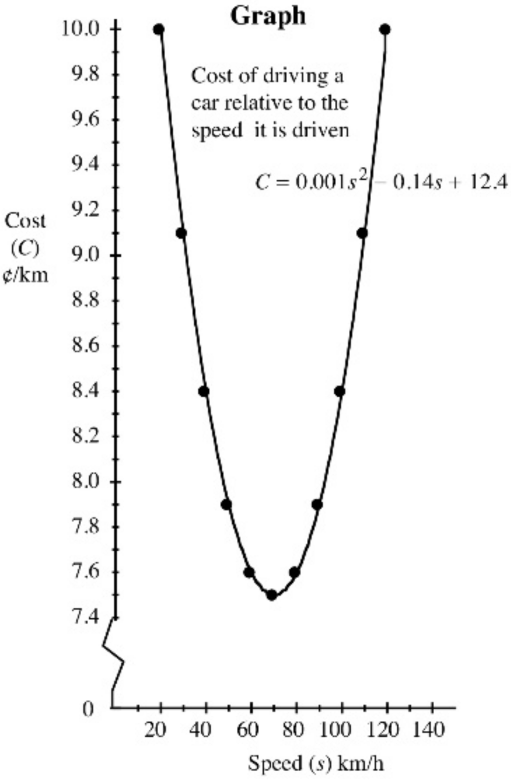
Later we will learn how this can be done and how an equation can be determined that best represents the data.

The graph and equation for the relation are given below. Note that the equation is only valid for certain input values which make up the domain of the relation. For example, the equation would not be valid for $s = 5000$!

Equation

$$C = 0.001s^2 - 0.14s + 12.4$$

Graph



Independent and Dependent Variables in a Relation

The values of the independent variable represent the inputs, and the corresponding values of the dependent variable are the outputs.

- In an ordered pair, the values of the first coordinate are those of the independent variable and the values of the second coordinate are values of the dependent variable.
- In a table of values, the independent variable is usually given first - either to the left or above the values of the dependent variable.
- In a mapping diagram, the arrows point from the independent variable to the dependent variable.
- On a graph, the independent variable is on the horizontal axis, often the x-axis, and the dependent variable is on the vertical axis, often the y-axis.
- In an equation, we usually try to isolate the dependent variable to the left side.

The illustration below uses the equation $y = 3x - 5$ as an example to illustrate the independent and dependent variables of an equation.

$$y = 3x - 5$$

- The **dependent variable**.
- Values of the dependent variable represent the outputs of the relation.
- Values of the dependent variable are represented by the **second** coordinate of an ordered pair and are on the **vertical axis**.

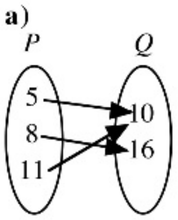
- The **independent variable.**
- Values of the independent variable represent the inputs of the relation.
- Values of the independent variable are represented by the **first** coordinate of an ordered pair and are on the **horizontal axis**.

The diagrams show relations expressed in different ways. In each case

i) state the independent variable **ii)** state the dependent variable

iii) list the inputs **iv)** list the outputs

a)



b)

V	A
4	15
10	12
25	15

c)

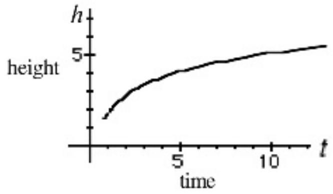
(B, c): $(3, 7)$, $(4, 11)$, $(5, 15)$, $(6, 19)$

Relations are expressed in different ways below. In each case

i) state the independent variable **ii)** state the dependent variable

a) $C = 2\pi r$

b)



c) The amount of sap, s, obtained from a maple tree is dependent on the time, t, a container is left attached to the maple tree.

Complete Assignment Questions #1 - #4

Investigating Relationships by Plotting Ordered Pairs

In this section we will consider relations defined by an equation, and sketch a graph by plotting ordered pairs using the following steps:

- Make a table of inputs by choosing replacements for the independent variable.
- For each of the input values, calculate the corresponding value (the output) of the dependent variable.
- Plot the ordered pairs on a Cartesian plane.

Class Ex. #4

Consider the relation described by the equation $y = 2x - 5$.

a) Complete the first five rows of the following table of values which shows some of the possible input values.

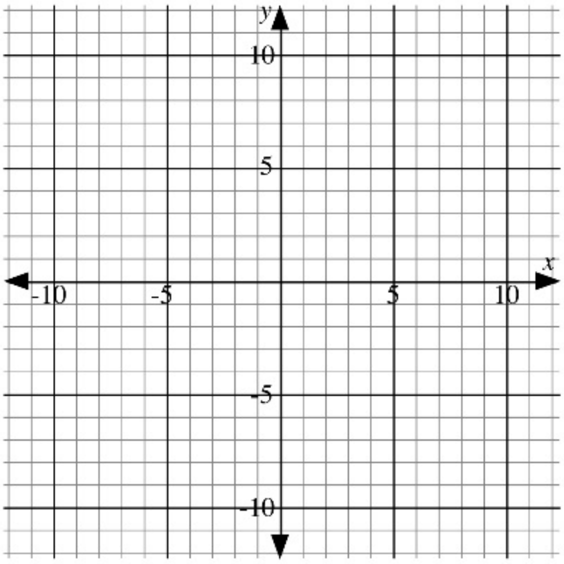
Input (x)	Output (y)	Ordered pair (x, y)
-2		
-1		
0		
1		
2		

b) Plot the ordered pairs in **a)** on the grid provided.

c) Connect the points on the grid, and extend the line in both directions with arrows at both ends.

d) Use the graph to determine the value of y when $x = 6$.

e) Use the equation to determine the value of y when $x = 6$, and verify the answer in **d)**.



f) Write the value of y when $x = 6$ in the table of values using the first blank space in **a)**.

g) Use the graph to determine the value of x when $y = 3$. Put this information in the last row in **a)**.

h) Complete the following statement:

This relation is called a _______________ relation because the graph of the relation is a straight line.

Class Ex. #5

Consider the relation described by the equation $y = x^2 - 6$.

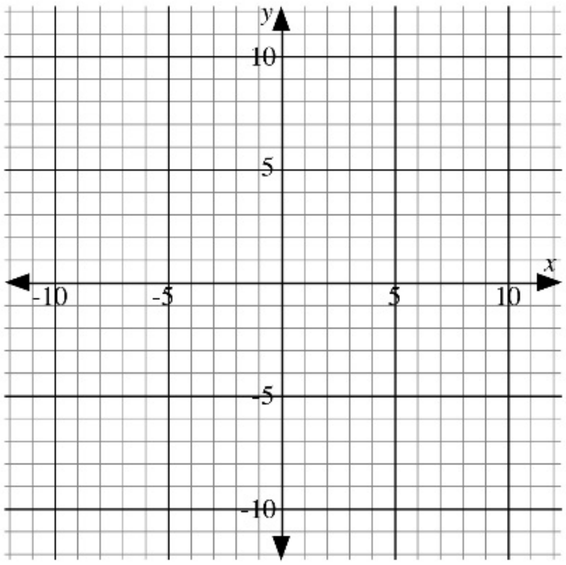
a) Complete the table of values to the right which shows some of the possible input values.

Input (x)	Output (y)	Ordered pair (x, y)
4		
3		
2		
1		
0		
–1		
–2		
–3		

b) Plot the ordered pairs in **a)** on the grid provided.

c) Use the symmetry of the graph or table to predict the value of y when $x = -4$.

d) Use the equation to determine the value of y when $x = -4$, and verify the answer in **c)**.



e) Write the value of y when $x = -4$ in the table of values using the first blank space in **a)**.

f) Connect the points on the grid with a smooth curve.

g) Why do you think this type of relation is called a **nonlinear relation**?

Complete Assignment Questions #5 - #9

Assignment

1. Complete the following.

a) The mathematical relationship between two quantities is called a ____________ .

b) The variable used for inputs in a relation is known as the _____________ variable.

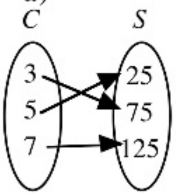
c) The variable used for outputs in a relation is known as the _____________ variable.

d) In the equation $A = \pi r^2$, the independent variable is _____, and the dependent variable is ____.

2. The diagrams show relations expressed in different ways. In each case:

i) state the independent and dependent variables **ii)** list the inputs and outputs.

a)



b)

C	n
8	22
20	19
50	35

c)

(f, e): $(2, 3)$, $(-2, 19)$, $(8, 17)$, $(0, 2)$

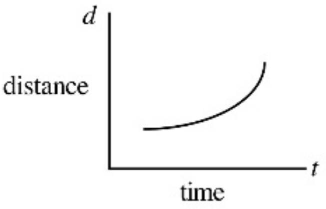
3. For each of the following relations, state

i) the independent variable **ii)** the dependent variable

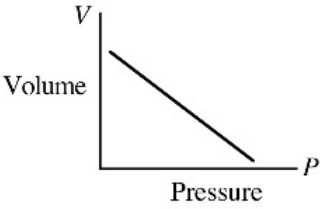
a) $V = \frac{4}{3}\pi r^3$

b) $C = \frac{5}{9}(F - 32)$

c)



d)



e) A truck's value, v, depends on its age, a.

f) The cost, C, of producing business cards is dependent on the number of cards, n, produced.

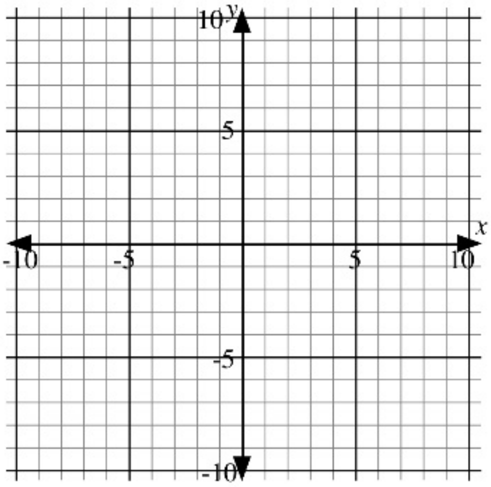
4. List the different ways a relation may be represented.

5. Consider the relation described by the equation $y = -x - 2$.

a) Identify the independent and dependent variables.

b) Complete the following table of values.

Input (x)	Output (y)	Ordered pair (x, y)
–3		
–1		
0		
1		



c) Plot the ordered pairs in **b)** on the grid provided.

d) Connect the points on the grid, and then extend the line in both directions with arrows at both ends.

e) Use the graph to determine the value of y when $x = 5$.

f) Use the equation to determine the value of y when $x = 5$, and verify the answer in **e)**.

g) Write the value of y when $x = 5$ in the table of values in **b)**.

h) Use the graph to determine the value of x when $y = 0$. Include this ordered pair in the table of values.

i) Use the graph to determine the value of x when $y = 4$. Include this ordered pair in the table of values.

j) Verify the answer in i) using the equation.

k) Is this a linear or a nonlinear relation?

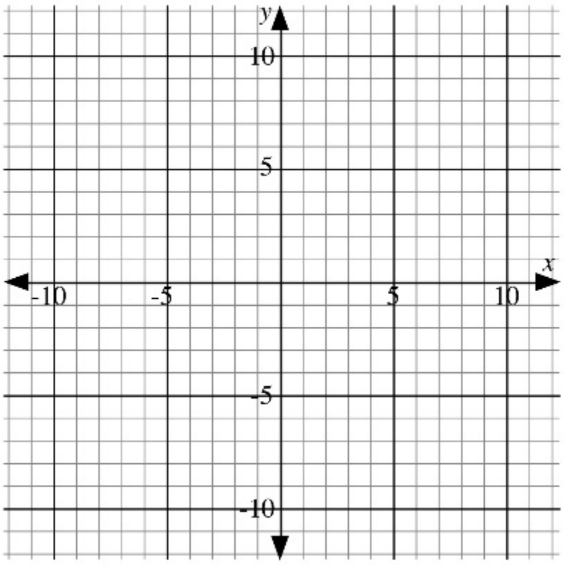
6. Consider the relation described by the equation $y = -0.5x^2 + 8$.

a) Identify the independent and dependent variables.

b) Complete the following table of values.

Input (x)	Output (y)	Ordered pair (x, y)
-6		
-4		
-2		
0		
2		
4		

c) Plot the ordered pairs in **b)** on the grid provided.



d) Use the plotted points and table to predict the value of y when $x = 6$. Plot this point on the grid.

e) Use the equation to determine the value of y when $x = 6$, and verify the answer in **d)**.

f) Connect the points on the grid with a smooth curve.

g) Is this a linear or a nonlinear relation?

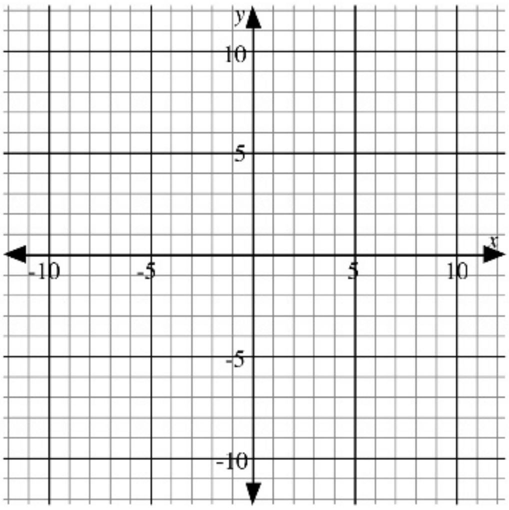
h) Use the graph to determine the values of x when $y = 3.5$. Verify your answer by using the equation.

7. For the following relations:

i) Complete the table of values choosing your own input values where necessary.
ii) Plot the ordered pairs on the grid, and sketch the graph of the relation.
iii) State whether the relation is linear or nonlinear.

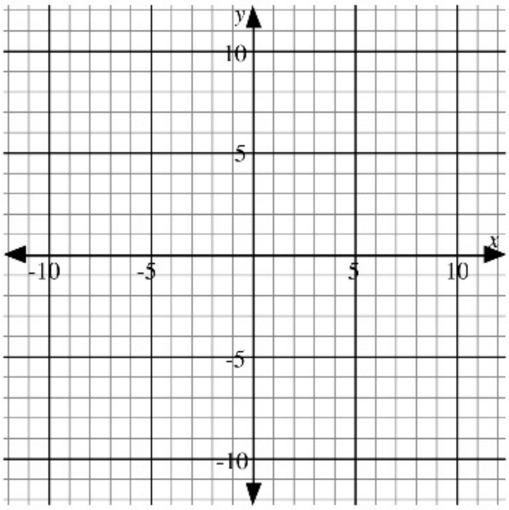
a) $y = -2x + 3$

Input (x)	Output (y)	Ordered pair (x, y)
0		
3		



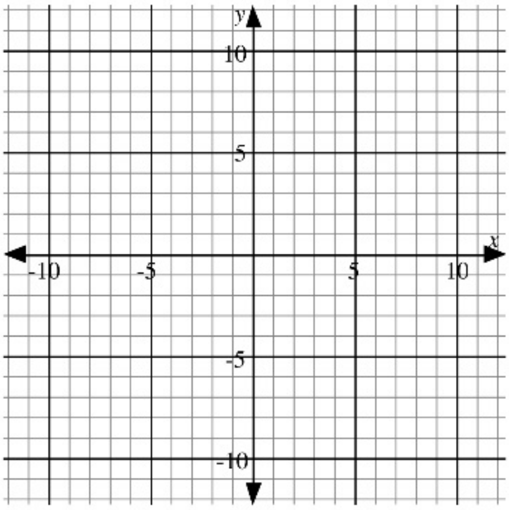
b) $y = 0.5x - 8$

Input (x)	Output (y)	Ordered pair (x, y)
–8		
0		
6		



c) $y = -x^2 + 5$

Input (x)	Output (y)	Ordered pair (x, y)
4		
3		
2		
1		
0		
–1		
–2		
–3		
–4		



Multiple Choice **8.** Which of the following statements is false?

A. The dependent variable is represented on the vertical axis of a Cartesian Plane.
B. The independent variable is represented by the first coordinate of an ordered pair.
C. The outputs of a relation are shown on the horizontal axis of a Cartesian Plane.
D. The independent variable is usually shown on the right side of an equation.

Numerical Response **9.** Consider the relation described by the equation $y = 1.5^{x-2}$. If the input is 4, then the output is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** relation **b)** independent **c)** dependent **d)** r, A

2. **a)** **i)** independent - C
dependent - S
ii) input - 3, 5, 7
output - 25, 75, 125

b) **i)** independent - C
dependent - n
ii) input - 8, 20, 50
output - 22, 19, 35

c) **i)** independent - f
dependent - e
ii) input - 2, –2, 8, 0
output - 3, 19, 17, 2

3. **a)** **i)** independent - r
ii) dependent - V

b) **i)** independent - F
ii) dependent - C

c) **i)** independent - time
ii) dependent - distance

d) **i)** independent - pressure
ii) dependent - volume

e) **i)** independent - a
ii) dependent - v

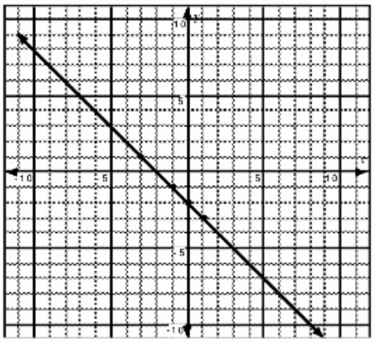
f) **i)** independent - n
ii) dependent - C

4. words, table of values, set of ordered pairs, mapping, equation, graph, function notation

5. **a)** Independent → x, Dependent → y

b) See table below **c), d)** See graph below **e)** –7 **f)** –7

	Input (x)	Output (y)	Ordered pair (x, y)
	–3	**1**	**(–3, 1)**
	–1	**–1**	**(–1, –1)**
	0	**–2**	**(0, –2)**
	1	**–3**	**(1, –3)**
g)	5	**–7**	**(5, –7)**
h)	**–2**	**0**	**(–2, 0)**
i)	**–6**	**4**	**(–6, 4)**

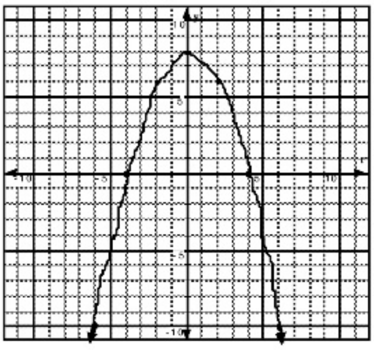


k) linear

6. **a)** Independent → x Dependent → y

b) See table below. **c), f)** See graph below. **d)** –10 **e)** $y = -0.5(6)^2 + 8\,y = -10$ **g)** non-linear

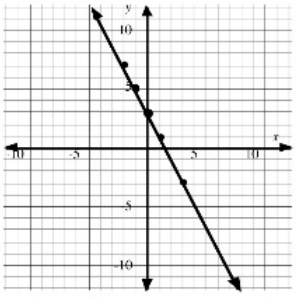
Input (x)	Output (y)	Ordered pair (x, y)
–6	**–10**	**(–6, –10)**
–4	**0**	**(–4, 0)**
–2	**6**	**(–2, 6)**
0	**8**	**(0, 8)**
2	**6**	**(2, 6)**
4	**0**	**(4, 0)**



h) $x = \pm 3$

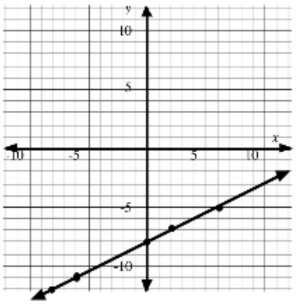
7. a) i) See table below. Inputs may vary. **ii)** See grid below. **iii)** linear

Input (x)	Output (y)	Ordered pair (x, y)
–2	**7**	**(–2, 7)**
–1	**5**	**(–1, 5)**
0	**3**	**(0, 3)**
1	**1**	**(1, 1)**
3	**–3**	**(3, –3)**



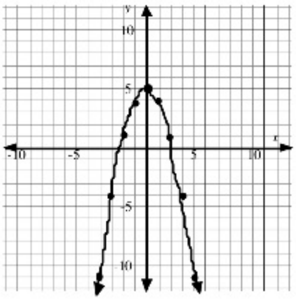
b) i) See table below. Inputs may vary. **ii)** See grid below. **iii)** linear

Input (x)	Output (y)	Ordered pair (x, y)
–8	**–12**	**(–8, –12)**
–6	**–11**	**(–6, –11)**
0	**–8**	**(0, –8)**
2	–7	**(2, –7)**
6	**–5**	**(6, –5)**



c) i) See table below. **ii)** See grid below. **iii)** non-linear

Input (x)	Output (y)	Ordered pair (x, y)
4	**–11**	**(4, –11)**
3	**–4**	**(3, –4)**
2	**1**	**(2, 1)**
1	**4**	**(1, 4)**
0	**5**	**(0, 5)**
–1	**4**	**(–1, 4)**
–2	**1**	**(–2, 1)**
–3	**–4**	**(–3, –4)**
–4	**–11**	**(–4, –11)**



8. C

9.

2	.	2	5

Relations Lesson #3:
x- and y-intercepts and Interpreting Relations

Review

a) A **relation** is a connection between two quantities. A relation can be represented graphically by a set of ______________ ___________ .

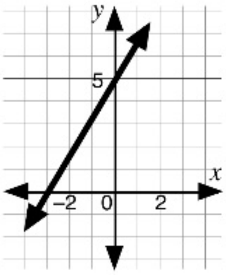
b) The first component of a set of ordered pairs is the ___ coordinate, also known as the input. Values of the input are values of the________________ variable.

c) The second component of a set of ordered pairs is the ___ coordinate, also known as the output. Values of the output are values of the________________ variable.

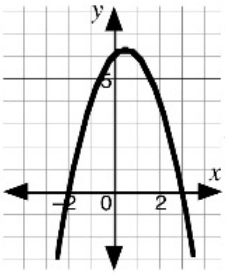
Exploring *x*- and *y*-intercepts

Consider the following graphs.

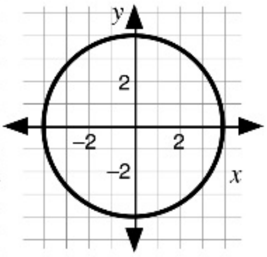
Graph 1



Graph 2



Graph 3



a) List the coordinates of the point(s) where each graph crosses the x-axis.

- Graph 1 crosses the x-axis at (,).
- Graph 2 crosses the x-axis at (,) and (,).
- Graph 3 crosses the x-axis at (,) and (,).

b) What do all the points in a) have in common?

c) List the coordinates of the point(s) where each graph crosses the y-axis.

- Graph 1 crosses the y-axis at (,).
- Graph 2 crosses the y-axis at (,).
- Graph 3 crosses the y-axis at (,) and (,).

d) What do all the points in c) have in common?

x- and y- intercepts of a Graph

The **x-intercept** of a graph is the x-coordinate of the ordered pair where the graph intersects the x-axis. An x-intercept occurs at a point on the graph where the y-coordinate is zero. The x-intercept can be given as a value or as an ordered pair.

The **y-intercept** of a graph is the y-coordinate of the ordered pair where the graph intersects the y-axis. A y-intercept occurs at a point on the graph where the x-coordinate is zero. The y-intercept can be given as a value or as an ordered pair.

1. Given the equation of the graph of a relation:
- to determine the x-intercept, set $y = 0$ and solve for x.
- to determine the y-intercept, set $x = 0$ and solve for y.

2. The equation of a graph can be written in different forms, all of which are equivalent. The equation of Graph 1 on the previous page is $y = \frac{5}{3}x + 5$, which can be written as $3y = 5x + 15$ or $5x - 3y + 15 = 0$. Equivalent forms of an equation will be studied in detail in a later unit. For the time being, use the instruction in note 1 to find the x- and y-intercepts of the graph of an equation given in any form.

The equation of Graph 1 on the previous page is $3y = 5x + 15$. Algebraically determine the values of the x-intercept and the y-intercept of Graph 1.

x-intercept	y-intercept

The equation of Graph 3 on the previous page is $x^2 + y^2 = 16$.
Calculate the x-intercept and the y-intercept of the graph of $x^2 + y^2 = 16$. Give the answers as ordered pairs.

x-intercept	y-intercept

Complete Assignment Questions #1 - #3

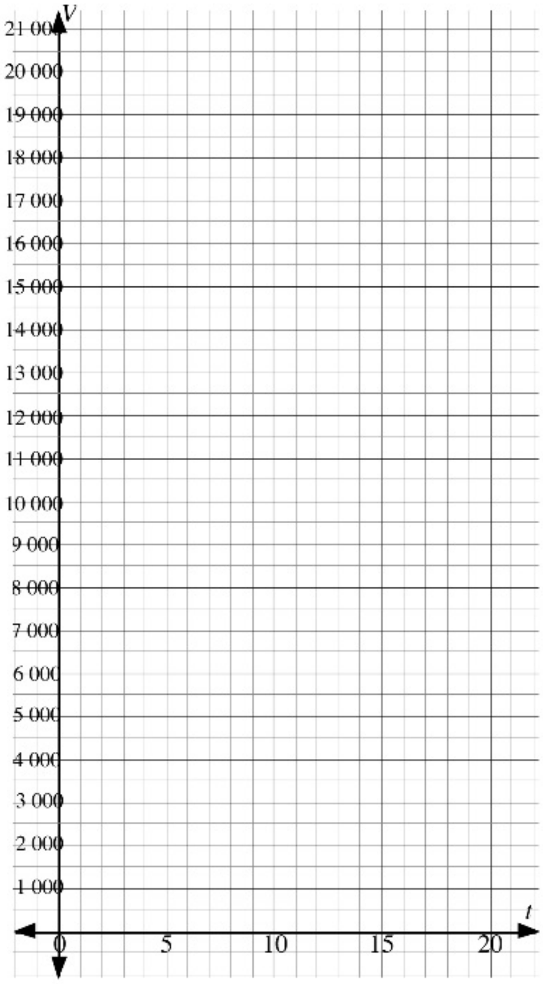
Class Ex. #3

Lisa purchases a new car for \$20 000. The value of the car can be represented by the formula $V = 20\,000 - 1250t$, where V is the value of the car in dollars, and t is the age of the car in years.

a) Complete the table of values and plot the ordered pairs on the grid.

Input (t)	Output (V)	Ordered pair (t, V)
0		
2		
4		
6		

Connect the points with a straight line, and extend the line.



b) What does the ordered pair $(0, 20\,000)$ represent?

c) Use the graph to estimate the t-intercept. What does the t-intercept represent?

d) Use the graph to estimate the value of the car after
i) 3 years **ii)** 10 years **iii)** 14 years.

e) Use the formula to verify **d) ii)**.

f) Use the graph to estimate when the car will be worth
i) \$5 000 **ii)** half of the purchase price.

g) Use the formula to verify **f) ii)**.

h) Complete the following statement to describe the relation:

The original value of the car is __________ . It depreciates in value by ________ per year and has no value after ___________ years.

In this lesson, using algebra determines the **exact** values for intercepts, etc. whereas using graphs gives an **estimate** for intercepts, etc. In lesson 5 we use the features of a graphing calculator to determine more accurate results from a graph.

In part **d)i)** we were asked to use the graph to find values lying between given points. This process is called **interpolation**. Extending the graph to predict values outside the plotted points is called **extrapolation**. Examples of extrapolation are **d)ii)** and **d)iii)**.

Complete Assignment Questions #4 - #9

Assignment

1. Determine the value of the y-intercept of the graph of each equation.

a) $y = x - 5$ **b)** $y = 3x - 15$ **c)** $2y + 3x - 12 = 0$

d) $0.5x - 2.4y + 0.8 = 0$ **e)** $2y = x^2 - 60$ **f)** $y = 0.001x^2 - 0.001x + 12.44$

2. Determine the value of the x-intercept(s) of the graph of each equation.

a) $y = x - 2$ **b)** $y = 2x - 8$ **c)** $3y + 2x - 12 = 0$

d) $0.6x - 2y + 0.5 = 0$ **e)** $y = x^2 - 9$ **f)** $y = 12 - 3x$

3. Determine the x- and y-intercepts of each equation. Answer as ordered pairs.

a) $y = 4x + 7$ **b)** $y = 15 - 6x$ **c)** $4x - 2y + 16 = 0$

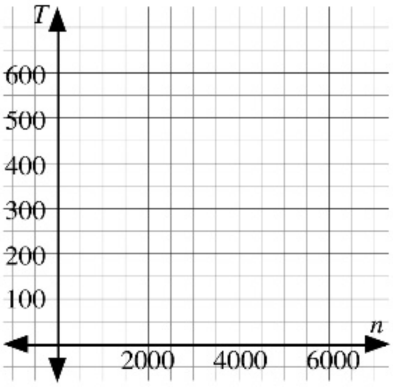
d) $y = \frac{x^2}{2} - 18$ **e)** $x^2 + y^2 = 25$ **f)** $y = 3x$

g) $y = x^2 + 4$ **h)** $9x^2 + y^2 = 81$ **i)** $9x^2 - y^2 = 81$

4. Triple A Car Rental charges $\$100$ per rental plus $10¢$ per km. The total cost, T, in dollars of renting the car can be represented by the formula, $T = 100 + 0.10n$, where n is the number of km travelled.

a) Complete the table of values, and plot the ordered pairs on the grid provided.

Number of km (n)	Total Rental Cost (T) dollars
0	
1000	
3500	
5000	



Connect the points with a straight line, and extend the line in both directions.

b) What does the ordered pair (0, 100) represent?

c) Determine the n-intercept of the graph. Explain why it is not applicable to this problem.

d) Interpolate from the graph to estimate the cost for a journey of:
i) 2000 km **ii)** 4500 km

e) Use the formula to verify the answers in **d)**.

f) If the total cost of rental is $\$650$, use the graph to estimate the number of km travelled.

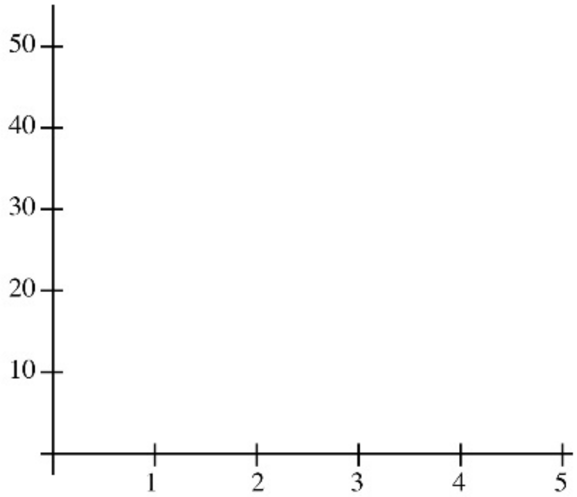
g) Verify the answer in **f)** using the formula.

5. An arrow is shot vertically into the air using a bow. The height, h metres, above the ground after t seconds, where $t \geq 0$ is approximated by the equation $h = -5t^2 + 20t + 25$.

a) The maximum height of the arrow is reached after 2 seconds.
Calculate the maximum height.

b) Complete the table of values, and plot the points on the grid.
Join the points with a smooth curve, and label the graph.

time (seconds)	height (metres)
0	
1	
2	
3	
4	
5	



c) Is this a linear or nonlinear relation?

d) For how many seconds is the arrow in the air?

e) What does the h-intercept represent in the context of the question?

f) What does the t-intercept represent in the context of the question?

g) **i)** Use the graph to estimate the height of the arrow after 1.5 seconds.

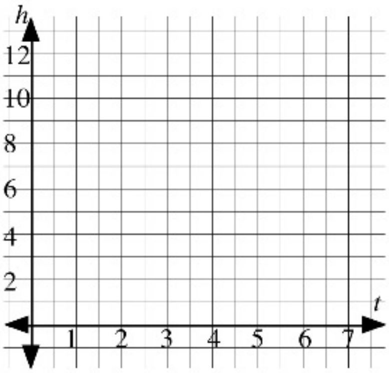
ii) Use the equation to calculate the exact height of the arrow after 1.5 seconds.

h) Does it make sense to extend the graph of the relation $h = -5t^2 + 20t + 25$ further in a downward direction to the left or right? Explain.

6. A candle manufacturer determined that its "Long-Last" candles melted according to the formula $h = -2t + 12$, where h is the height of the candle, in cm, after t hours.

a) Make a table of values and use this to construct the graph of $h = -2t + 12$.

t				
h				



Use your graph to answer ***b – e***.

b) How tall is the candle before it begins to melt?

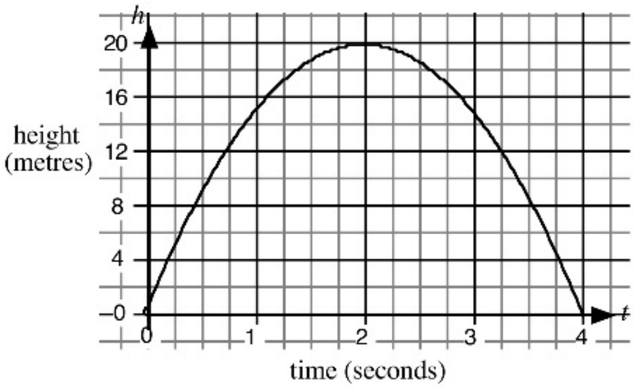
c) How many hours will the candle last before it will completely burn out?

d) How tall will the candle be after burning for 5 hours?

e) How long will it take for the candle to burn down to a height of 7 cm?

f) Verify the answers from **b)** - **e)** using the formula.

7. A football is kicked by a student. The graph of the relation between the height of the football above the ground and time is shown. The formula that represents the relation is given by $h = -4.9t^2 + 19.4t + 0.6$, where h is the height in metres above the ground and t is the time in seconds the football is in the air.



*Use the graph to answer **a – c**:*

a) Estimate, to the nearest metre, the maximum height of the football above the ground.

b) Estimate how long it takes for the football to reach the ground.

c) Estimate the height, to the nearest metre, of the football when it is in the air for 3 seconds.

d) Use the formula to calculate the exact answer to **c**).

e) Calculate the h-intercept, and describe what it represents in the context of the question.

Multiple Choice **8.** In which of the following relations does the graph of the relation have x- and y-intercepts with equal values?

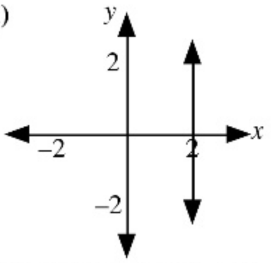
A. $y = x + 8$
B. $2x + 2y = 7$
C. $2x - 3y + 4 = 0$
D. none of the above

Numerical Response **9.** The graph of the relation $4x^2 + 9y^2 - 36 = 0$ has x-intercepts a and b, and y-intercepts c and d. The value of the product $abcd$ is _____ .

(Record your answer in the numerical response box from left to right) ☐☐☐☐

10. Match each graph with the domain from A to F. Each domain may be used once, more than once, or not at all.

i)



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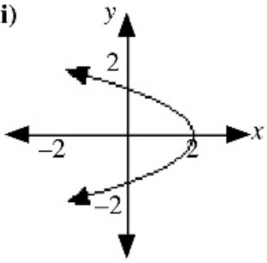

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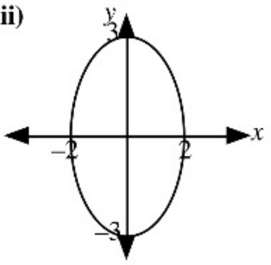

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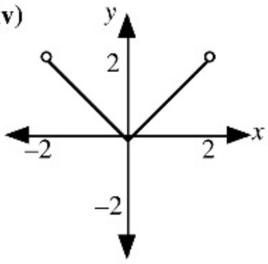

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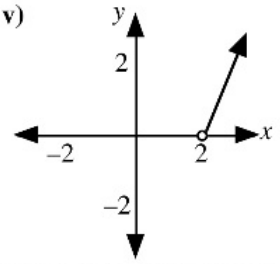

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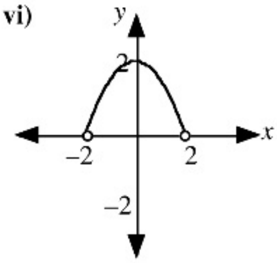

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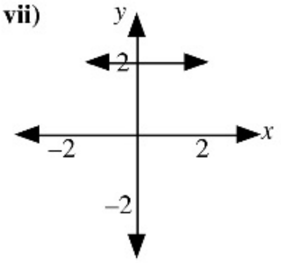


A. $x = 2$

B. $x < 2$

C. $x \leq 2$

D. $x > 2$

E. $x \geq 2$

F. $-2 < x < 2$

G. $-2 \leq x \leq 2$

H. $2 < x < -2$

I. $x \in R$

Relations Lesson #4:
Domain and Range

Number Systems Review

Number systems were studied in previous math courses. Recall the symbols and letters used by completing the following.

a) Each letter below is represents a number system. Write the name of the number system. $\overline{Q}$ has been completed.

N	W	I	Q	$\overline{Q}$	R
				Irrational Numbers	

b) The symbol $\in$ represents the phrase "

c) The brackets { } represents the phrase "

d) Recall the following by reading following notes regarding number systems.

Natural Numbers

- The natural numbers (counting numbers) are 1, 2, 3, 4, etc.
- There are an infinite number of natural numbers.
- The set of all natural numbers can be written as $N = \{1, 2, 3, 4, ...\}$.
- The number 11 belongs to the set of natural numbers and we can write $11 \in N$.

Whole Numbers

- The whole numbers are the natural numbers together with 0.
- The set of all whole numbers can be written as $W = \{0, 1, 2, 3, ...\}$.
- $6 \in N$ but $\frac{1}{2} \notin W$.

Integers

- The integers consist of positive and negative natural numbers, and zero.
- The set of all integers can be written as $I = \{ ..., -3, -2, -1, 0, 1, 2, 3, ... \}$.
- In some texts Z or J is used instead of I.

Rational Numbers

- Rational numbers are those numbers which can be expressed as a ratio between two integers, e.g. the fractions $\frac{1}{4}$ and $-\frac{2}{3}$ are both rational numbers. All the integers are included in the rational numbers since any integer can be written as $\frac{x}{1}$ e.g. $5 = \frac{5}{1}$.
- The set of rational numbers can be written as $Q = \left\{ \frac{a}{b},\ \text{where } a, b \in I,\ b \neq 0 \right\}$.

Irrational Numbers

- The set of irrational numbers can be written as $\overline{Q}$ = {non-terminating and non-repeating decimals} e.g. $\pi \in \overline{Q}$, $\sqrt{2} \in \overline{Q}$.

Real Numbers

- The real numbers consists of all rational numbers and irrational numbers.
- The set of all real numbers can be written as $R = \{Q \text{ and } \overline{Q}\}$.
- In some texts $\Re$ or Re are used instead of R.

Complete Assignment Question #1 and #2

Domain and Range

The **domain** of a relation is the set of all possible values which can be used for the **input** of the **independent variable** (x).

The **range** of a relation is the set of all possible values of the **output** of the **dependent variable** (y).

In lesson 2 on page 235 we described the relation in each of the following forms

- in words
- a mapping (or arrow) diagram
- a table of values
- an equation
- a set of ordered pairs
- a graph

In this lesson we will study the domain and range given in any of these forms.

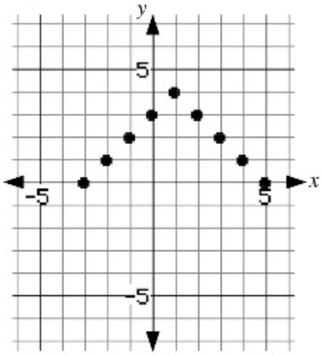
List the domain and range of the following set of ordered pairs.

a) $(1, 2)$, $(0, 5)$, $(3, 8)$, $(5, 9)$, $(-3, 2)$

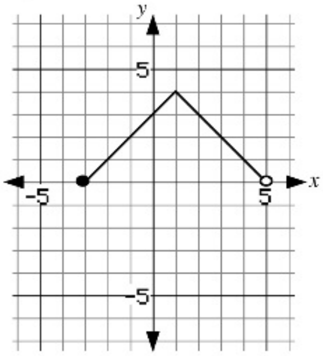
b) $(3, 3)$, $(0, 3)$, $(-3, 3)$, $(2, 9)$, $(-8, 3)$

In each case, state the domain and range of the relation represented by the graph.

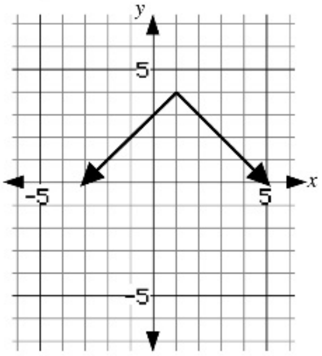
a)



b)

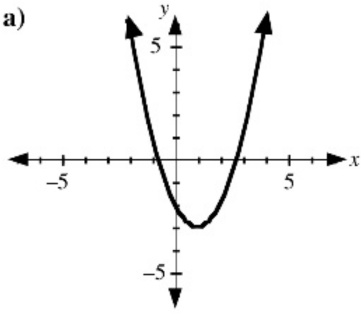


c)

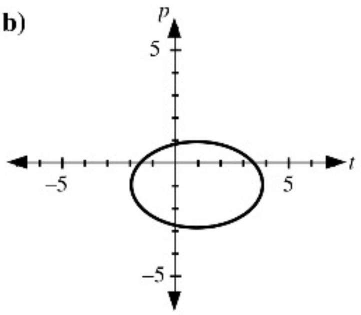


State the domain and range of the following relations.

a)



b)



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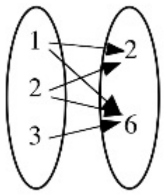


d) A circle with centre $(-2, 3)$ and a radius of 3.

a) Draw the graph of a relation which has domain $x \in R$, range $\{y \mid y \le 2, y \in R\}$ and

i) only one x-intercept **ii)** two x-intercepts

b) Explain why it is not possible to draw a graph which has domain $x \in R$, range $\{y \mid y \le 2, y \in R\}$ and no x-intercepts.

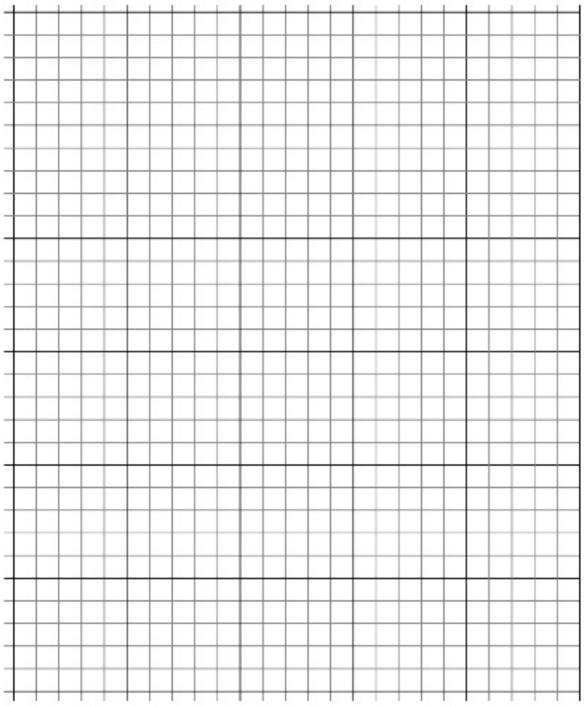
A high school football team is hosting a banquet to celebrate winning the championship. The caterer charges a set up fee of \$500 plus \$20 per person. The equation $C = 500 + 20n$ represents the cost of hosting the banquet for n people.

a) Make a table of values with 8 entries for a minimum of 100 and a maximum of 500 people.

n	C

b) Plot the eight ordered pairs from **a)** on the grid.

c) If all possible ordered pairs from **b)** were plotted on the grid, state the domain and range of the relation and explain why there are restrictions on both.



Complete Assignment Questions #3 - #12

Assignment

1. Use check marks to indicate all the sets to which each number belongs.

	N	W	I	Q	$\overline{Q}$	R
$\frac{1}{3}$						
123 983						
-2						
$7.53\overline{4}$						
9.5						
$\sqrt{75}$						
$-\pi$						
$-\frac{355}{113}$						
$-\sqrt{49}$						
0.000005						
2.232425...						

2. The addition of ANY two **natural** numbers is still a natural number.

Does this property hold true for the operations of subtraction, multiplication, or division (except for zero)?

Investigate for whole numbers, integers, rational numbers, and real numbers and write your conclusions.

3. State the domain and range of each relation.

a) $(2,3)$, $(0,2)$, $(4,8)$, $(-1,8)$, $(-3,1)$ **b)** $(-3,3)$, $(0,-5)$, $(-3,3)$, $(5,-2)$, $(-8,1)$

c)

Input (x)	Output (y)
0	3
2	4
4	5
6	3

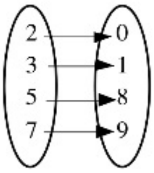
d)

Input (x)	Output (y)
2	3
0	4
–3	5
2	6

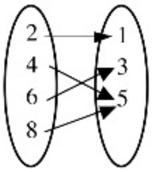
e)

Input (x)	Output (y)
1	5
–1	5
3	5
7	5

f)



g)



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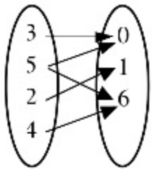
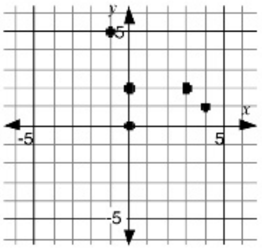
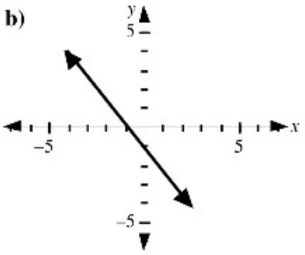


4. State the domain and range for each relation.

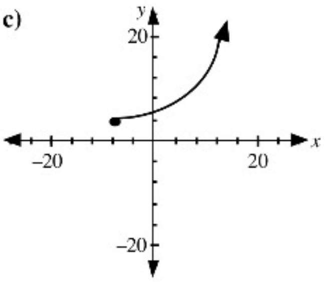
a)



b)



c)



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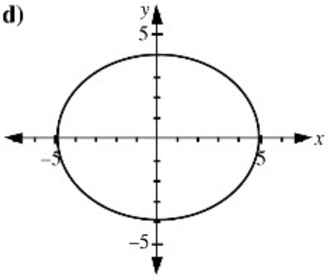

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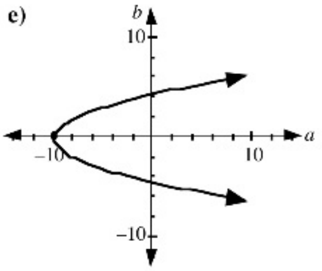

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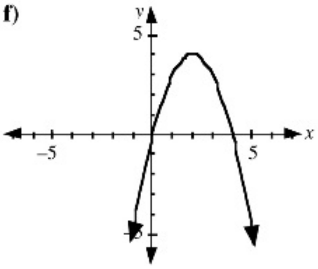

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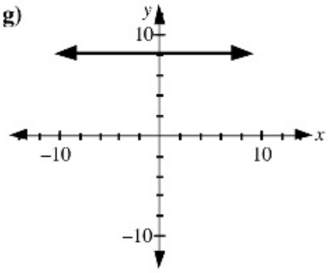

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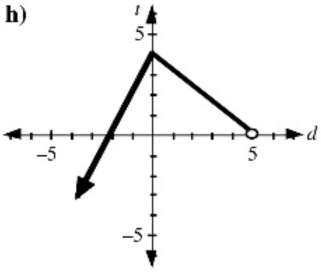


5. In each case a relation is graphed on a grid. State the domain and range of the relation if the graph is

a) a circle whose centre is located at $(-1, 12)$ and with a radius of 5 units.

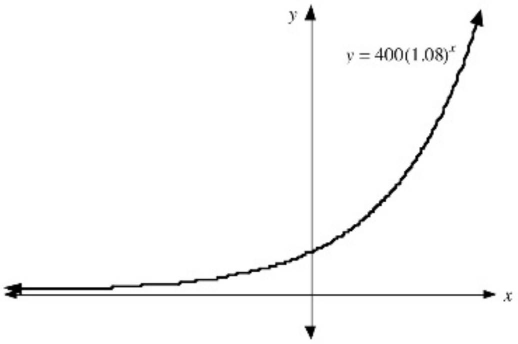
b) a circle with centre $(-3, -5)$ and diameter 40 units.

c) a rectangle with vertices $A(-8, 10)$, $B(-8, -2)$, $C(7, -2)$, and $D(7, 10)$.

d) a triangle with vertices $T(-50, -75)$, $U(-35, -25)$, and $V(-65, -25)$.

6. The graph of the relation $y = 400(1.08)^x$ is shown on the grid.

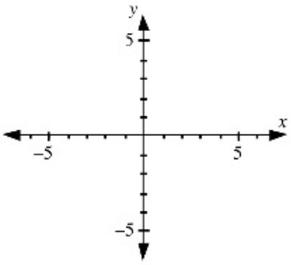
a) State the domain, range, and y-intercept of the relation.

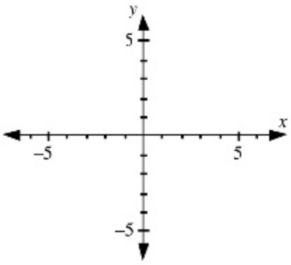


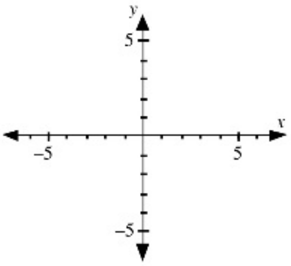
b) The relation $A = 400(1.08)^t$ represents the amount of money when an original investment of \$400 is compounded annually at 8% for a period of t years. State the domain and range of this relation, and explain why the answer is different from **a)**.

7. In each case draw a graph on the domain of real numbers which could represent a linear relation with

a) one x-intercept **b)** no x-intercept **c)** an infinite number of x-intercepts.



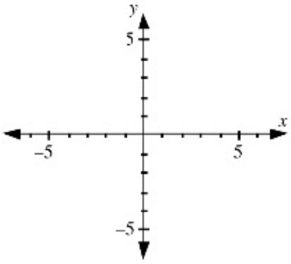


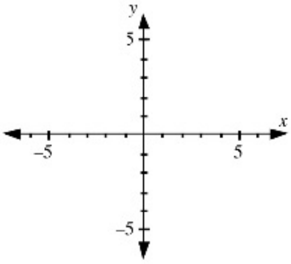


8. In each case draw a graph of a non-linear relation with

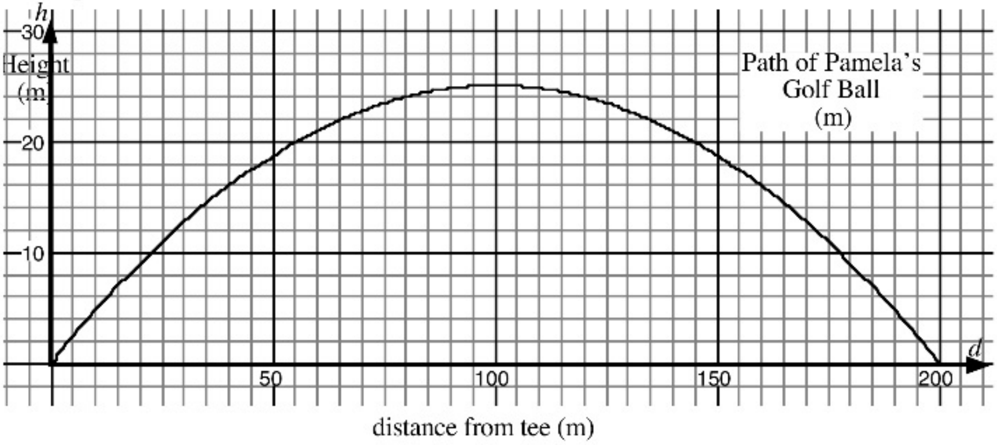
a) domain $x \in R$, range $y \geq -3, y \in R$
two x-intercepts and one y-intercept

b) domain $x \in R$, range $y \geq -3, y \in R$
one x-intercept and one y-intercept.





9. The graph shows the flight of Pamela's golf ball from the tee to a sand trap at the edge of the green.



a) State the h-intercept and the d-intercepts of the graph, and explain their significance in relation to the question.

b) State the maximum height of the golf ball, and explain its relevance to the domain or range of the relation.

c) State the domain and range of the relation.

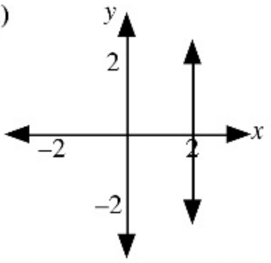
d) From the graph, estimate the horizontal distance the ball has traveled when it is 20 m in the air. Explain why there are two answers.

e) Using the graph, estimate the height of the golf ball when the horizontal distance from the tee is 80 m.

f) Give a brief description of the relationship between the height of the golf ball and the horizontal distance from the tee.

10. Match each graph with the domain from A to F. Each domain may be used once, more than once, or not at all.

i)



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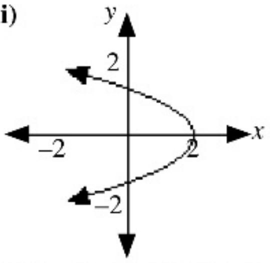

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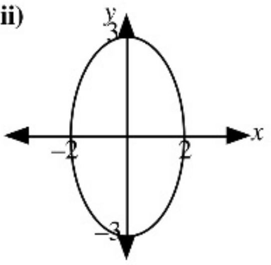

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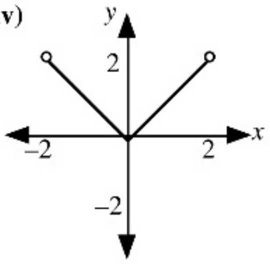

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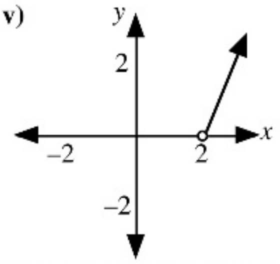
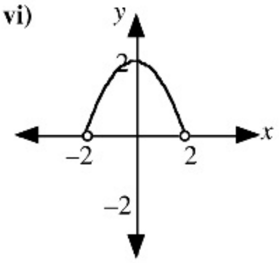
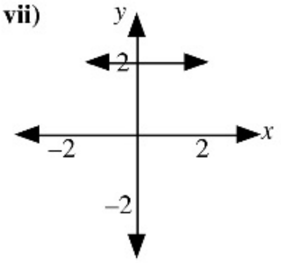


vi)

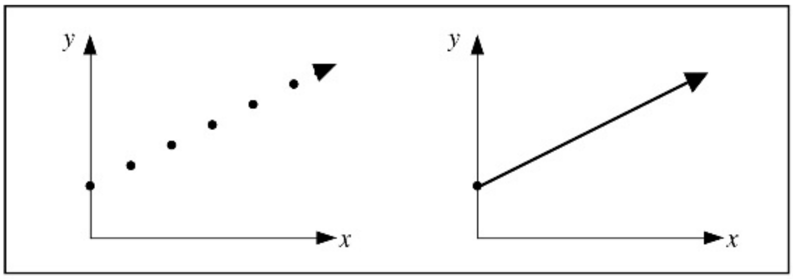


vii)



A. $x = 2$
B. $x < 2$
C. $x \leq 2$
D. $x > 2$
E. $x \geq 2$
F. $-2 < x < 2$
G. $-2 \leq x \leq 2$
H. $2 < x < -2$
I. $x \in R$

Multiple Choice **11.** The graphs of two relations are shown. Which of the following statements is true?



A. The domains are the same, but the ranges are different.
B. The ranges are the same, but the domains are different.
C. The domains are the same, and the ranges are the same.
D. The domains are different, and the ranges are different.

Numerical Response **12.** The relation between the distance traveled, d km, and the cost, C dollars, of renting a truck is given by the formula $C = 60 + 0.27d$. The domain of the relation can be expressed in the form $d \geq x$, and the range can be expressed in the form $C \geq y$. Write the value of y in the first two boxes and the value of x in the last two boxes.

(Record your answer in the numerical response box from left to right) ☐☐☐☐

Answer Key

1. See chart below

	N	W	I	Q	$\overline{Q}$	R
$\frac{1}{3}$				√		√
123 983	√	√	√	√		√
-2			√	√		√
$7.5\overline{31}$				√		√
9.5				√		√
$\sqrt{75}$					√	√
$-\pi$					√	√
$-\frac{355}{113}$				√		√
$-\sqrt{49}$			√	√		√
0.000005				√		√
2.232425...					√	√

2. See below.

- For natural numbers and whole numbers, the property is true for addition and multiplication only.
- For integers, the property is true for addition, subtraction, and multiplication.
- For rational numbers and real numbers, the property is true for addition, subtraction, multiplication, and division.

3. **a)** $D = \{2, 0, 4, -1, -3\}$ $R = \{3, 2, 8, 1\}$
b) $D = \{-3, 0, 5, -8\}$ $R = \{3, -5, -2, 1\}$
c) $D = \{0, 2, 4, 6\}$ $R = \{3, 4, 5\}$
d) $D = \{2, 0, -3\}$ $R = \{3, 4, 5, 6\}$
e) $D = \{1, -1, 3, 7\}$ $R = \{5\}$
f) $D = \{2, 3, 5, 7\}$ $R = \{0, 1, 8, 9\}$
g) $D = \{2, 4, 6, 8\}$ $R = \{1, 3, 5\}$
h) $D = \{3, 5, 2, 4\}$ $R = \{0, 1, 6\}$

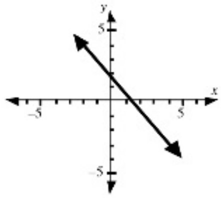
4. **a)** $D = \{-1, 0, 3, 4\}$ $R = \{5, 0, 2, 1\}$
b) $D = \{x \in R\}$ $R = \{y \in R\}$
c) $D = \{x \geq -8, x \in R\}$ $R = \{y \geq 4, y \in R\}$
d) $D = \{-5 \leq x \leq 5, x \in R\}$ $R = \{-4 \leq y \leq 4, y \in R\}$
e) $D = \{a \geq -10, a \in R\}$ $R = \{b \in R\}$
f) $D = \{x \in R\}$ $R = \{y \leq 4, y \in R\}$
g) $D = \{x \in R\}$ $R = \{8\}$
h) $D = \{d < 5, d \in R\}$ $R = \{t \leq 4, t \in R\}$

5. **a)** $D = \{-6 \leq x \leq 4, x \in R\}$ $R = \{7 \leq y \leq 17, y \subset R\}$
b) $D = \{-23 \leq x \leq 17, x \in R\}$ $R = \{-25 \leq y \leq 15, y \subset R\}$
c) $D = \{-8 \leq x \leq 7, x \in R\}$ $R = \{-2 \leq y \leq 10, y \in R\}$
d) $D = \{-65 \leq x \leq -35, x \in R\}$ $R = \{-75 \leq y \leq -25, y \in R\}$

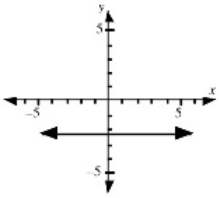
6. **a)** $D = \{x \in R\}$ $R = \{y > 0, y \in R\}$, y–int is 400
b) $D = \{t \geq 0, t \in R\}$ different from a) because time is never a negative value.
$R = \{A \geq 400, A \in R\}$ different from a) because the amount of money can never be less than \$400.

7. Answers may vary.

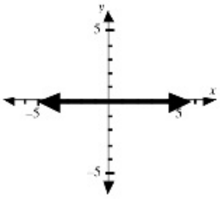
a)



b)

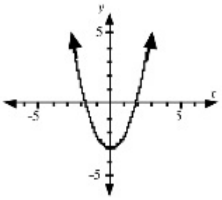


c)

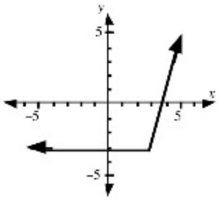


8. Answers may vary.

a)



b)



9. **a)** h–int = 0, d–int = 0 and 200. On the tee the ball is on the ground.
It returns to ground level 200 m from the tee.
b) max height = 25 m. The maximum height is the upper limit of the range.
c) $D = \{0 \leq d \leq 200, d \in R\}$ $R = \{0 \leq h \leq 25, h \in R\}$
d) 55 m from the tee when the ball is rising and 145m from the tee when the ball is descending.
e) 24 m
f) Starting from a height of 0 m at the tee, the golf ball increases in height to a maximum height of 25m, 100 m from the tee. Then the golf ball starts decreasing in height until it hits the ground 200 m from the tee.

10. **i)** A **ii)** C **iii)** G **iv)** F **v)** D **vi)** F **vii)** I

11. D

12.

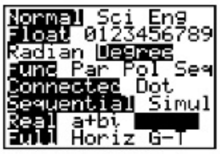
6	0	0	0

Relations Lesson #5:
Relations and the Graphing Calculator

- Although features of different types of graphing calculators may be similar to each other, the instructions in this unit refer to the TI-83/TI-84 series of graphing calculators.
- This lesson takes more than one class to complete.

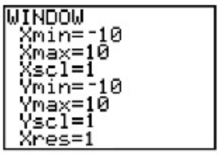
Preparing the Calculator to Graph

Confirm that the calculator is in "Function" mode as shown.
Press `MODE` and use the cursor keys to navigate to the proper settings, and then press `ENTER`.



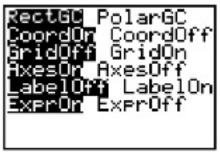
```
Normal Sci Eng
Float 0123456789
Radian Degree
Func Par Pol Seq
Connected Dot
Sequential Simul
Real a+bi
Full Horiz G-T
```

Set the display window of the calculator as shown.
Press `WINDOW` and use the cursor keys to navigate to the proper settings, and then press `ENTER`, or press `ZOOM` `6`.



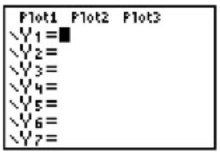
```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```

Set the format of the calculator as shown.
Press `2nd` `ZOOM` and use the cursor keys to navigate to the proper settings, and then press `ENTER`.



```
RectGC PolarGC
CoordOn CoordOff
GridOff GridOn
AxesOn AxesOff
LabelOff LabelOn
ExprOn ExprOff
```

Press `Y=` to confirm that the "Y= editor" window is as shown.
If `Plot1`, `Plot2`, or `Plot3` are highlighted in black, then you must turn them off by scrolling to `Plot1`, `Plot2`, or `Plot3` and pressing `ENTER`.



```
  Plot1  Plot2  Plot3
\Y1=█
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

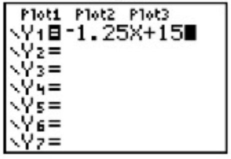
The relation with equation $y = -1.25x + 15$ is used throughout most of this lesson to illustrate some features of the graphing calculator.

Entering and Graphing a Relation using a Graphing Calculator

The following procedure may be used to display the graph of $y = -1.25x + 15$.

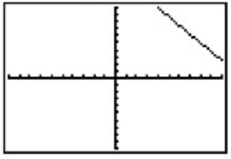
1. Access the "Y= editor" by pressing the `Y=` key.
2. Enter the equation in Y_1. (Write the equation in terms of the dependent variable if necessary).
3. Press the `GRAPH` key to display the graph.

Step 3



```
  Plot1  Plot2  Plot3
\Y1=-1.25X+15█
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

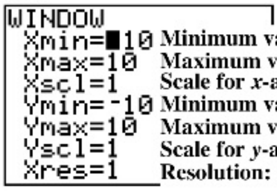
Step 4



Notice that the window we have chosen in this example does not allow us to see much of the graph. We need to alter the window setting to see the main features of the graph.

Using the Window and Zoom Features

Sometimes the standard display setting shown below does not allow us to see the behaviour of specific portions of a graph. For instance, if we wanted to see where the graph of $y = -1.25x + 15$ crosses the x-axis and the y-axis, we need to adjust the window settings of the calculator.

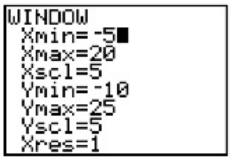


WINDOW	
Xmin=-10	**Minimum value** seen on the screen for x
Xmax=10	**Maximum value** seen on the screen for x
Xscl=1	**Scale for x-axis** - The increments of the tic marks for x
Ymin=-10	**Minimum value** seen on the screen for y
Ymax=10	**Maximum value** seen on the screen for y
Yscl=1	**Scale for y-axis** - The increments of the tic marks for y
Xres=1	**Resolution:** 1 is best screen resolution. Leave at 1

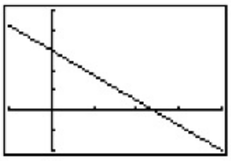
The following steps may be used to adjust the window settings:

1. Press the `WINDOW` key.

2. Use the cursor to enter in appropriate values to adjust the Cartesian plane and press the `GRAPH` key.



```
WINDOW
 Xmin=-5
 Xmax=20
 Xscl=5
 Ymin=-10
 Ymax=25
 Yscl=5
 Xres=1
```



- An alternative method to "fit" the graph of an equation on the display screen is to use the `Zoom Out` feature of a graphing calculator. Although it may not give the exact window settings we would like, it can give a good first approximation.

  - Press the `ZOOM` `3` (or scroll down to `Zoom Out` and press `GRAPH`). Keep doing this until you see the graph.

  - Press `WINDOW` to see the new settings. Adjust the window.

- Another method to "fit" the graph of an equation on the display screen is to use the `ZoomFit` feature of a graphing calculator. Although it may not give the exact window settings we would like, it can also give a good first approximation.
- Press `ZOOM` `0` (or scroll down to `ZoomFit` and press `ENTER`).
- The standard display settings (as shown in the window at the top of this page) of a graphing calculator can be quickly entered by pressing `ZOOM` `6` (or scroll down to `ZStandard` and press `ENTER`).

- When displaying a graph on the calculator, it is generally a good idea to have the x- and y- intercepts of the graph visible.

- The following notation is used to write a graphing calculator window.

  x:$[x_{\text{min}}, x_{\text{max}}, x_{\text{scl}}]$ y:$[y_{\text{min}}, y_{\text{max}}, y_{\text{scl}}]$

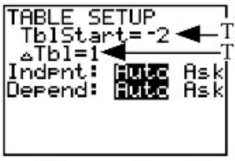
  For example, the window settings used above in step 1 can be written:

  x:$[-5, 20, 5]$ y:$[-10, 25, 5]$

Displaying the Table of Values for an Equation

The Table of Values feature may be used to display the ordered pairs of a relation defined by an equation. Before using this feature, we will need to set up the table using the Table Setup feature.

Accessing the Table Set Up Feature

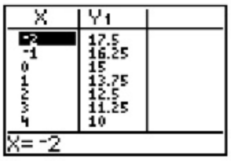


```
TABLE SETUP
 TblStart=-2
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

The value where the table starts.
The increments of the input.

- Press `2nd` `WINDOW` keys to access `TBLSET`. The illustration describes the features.
- This will lead us to a table of values starting with $x = -2$ and increasing in units of 1.

Accessing the Table of Values

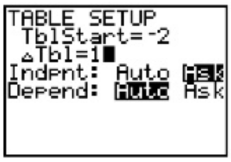


```
   X   | Y1  |
 -2    17.5
 -1    16.25
 0     15
 1     13.75
 2     12.5
 3     11.25
 4     10
X= -2
```

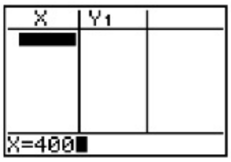
- Press `2nd` `GRAPH` keys to access the table feature of the calculator.
  We can use the cursor keys to scroll up and down the table to determine the value of y for any given value of x.

If a value of x which would require continous cursoring is used, for example $x = 400$, a quicker method of calculating the value of y is given by the following procedure.

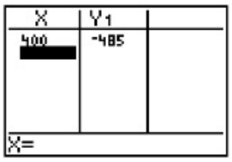
- Access `TBLSET` feature and change `Indpnt` to `Ask`,
- Return to the table of values, type in 400, and press `ENTER`.



```
TABLE SETUP
 TblStart=-2
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```



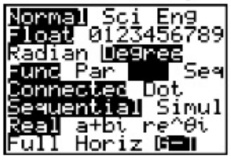
```
   X   | Y1  |

X=400
```



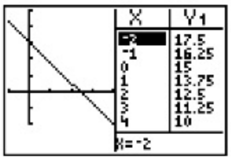
```
   X   | Y1  |
 400    -485

X=
```

Accessing the Table of Values and the Graph Simultaneously

Return to Table Set Up as on the top of this page.



```
Normal Sci Eng
Float 0123456789
Radian Degree
Func Par    Seq
Connected Dot
Sequential Simul
Real a+bi re^θi
Full Horiz G-T
```



```
   X   | Y1
 -2    17.5
 -1    16.25
 0     15
 1     13.75
 2     12.5
 3     11.25
 4     10
X=-2
```

1. Press the `MODE` key, scroll down and across to "`G-T`", and then press `ENTER` and `GRAPH`.
2. Switching between the graph and table of values can be done by pressing the `GRAPH` key and the `2nd` `GRAPH` keys.
3. Using the `TRACE` key in the `G-T` mode allows us to scroll along the graph and up and down the table of values at the same time (use the left and right arrow keys).

Complete Assignment Question #1

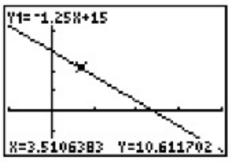
Finding the y- coordinate of a Point on the Graph of a Relation

To determine the output for a given input of a relation, the trace feature of a graphing calculator may be used.

Using the Trace Feature to Move Along the Graph

Return to Full screen.

The trace feature may be used to find the coordinates of points on the graph. The following procedures may be used.

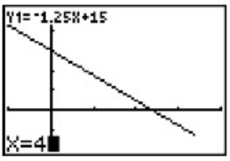


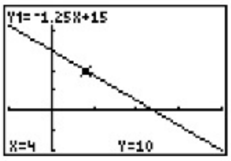
1. Set the window to x:[–5, 20, 5] y:[–10, 25, 5]

2. Press the TRACE key and use the scroll keys to move along the graph.

Using the Trace Feature to Determine the Value of y for a Given Value of x

- Press the TRACE key. Enter a value for the input, and press ENTER. For example, pressing TRACE 4 ENTER will calculate the value y when $x = 4$.
  The answer `Y = 10` will be displayed on the bottom right.





Finding the x- coordinate of a Point on the Graph of a Relation

To determine the input for a given output of a relation, the intersect feature of a graphing calculator may be used.

Using the Intersect Feature

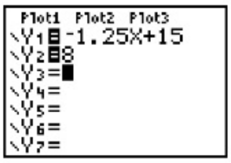
- The intersect feature can be used to determine the intersection point of the graphs of two relations. An application of this is to find the y-coordinate of a point on a graph given the x-coordinate.

- As a working example we will find the x-coordinate of $y = -1.25x + 15$ when $y = 8$.

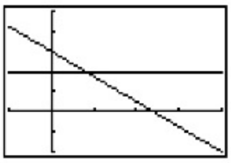
1. Enter the equation into Y_1.

2. Enter the given value of y into Y_2.

Press GRAPH, and adjust the window if necessary.

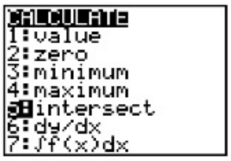


```
\Y1=-1.25X+15
\Y2=8
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



3. Access the `CALC` menu by entering 2nd then TRACE.

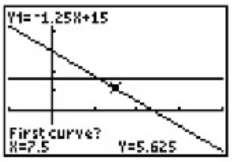
4. Select the "`intersect`" feature.



5. On the bottom left-hand side of the screen, the calculator will ask for the `First curve?` .

Scroll close to the intersection point, and press `ENTER` .

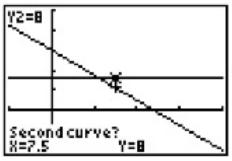
The values of x and y will vary depending on how close we scroll to the intersection point.



```
Y1=-1.25X+15
First curve?
X=7.5         Y=5.625
```

6. On the bottom left-hand side of the screen, the calculator will ask for a `Second curve?` .

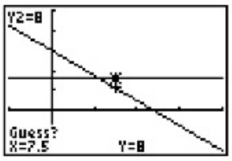
Scroll close to the intersection point, and press `ENTER` .



```
Y2=8
Second curve?
X=7.5         Y=8
```

7. On the bottom left-hand side of the screen, the calculator will ask to `Guess?` .

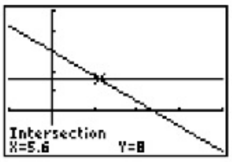
Press `ENTER` .



```
Y2=8
Guess?
X=7.5         Y=8
```

8. The x and y values of the intersection point will be displayed. In this example the intersection point is $(5.6, 8)$.

The answer to the question is $x = 5.6$.



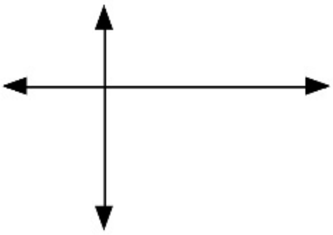
```
Intersection
X=5.6         Y=8
```

Class Ex. #1

Consider the equation $y = 2x - 25$.

a) Use a graphing calculator to sketch the graph on the grid provided. Write a suitable window.

x:[, ,] y:[, ,]



b) Use a graphing calculator to determine the value of y when $x = 4.5$.

c) Algebraically verify the answer in **b)**.

d) Use a graphing calculator to determine the value of x when $y = -6.4$.

e) Algebraically verify the answer in **d)**.

Complete Assignment Questions #2 - #5

Using a Graphing Calculator to Determine x-intercepts

Finding the x-intercept using the Intersect Feature

The intersect feature may be used to find the x-intercept by using the equation $y = 0$ for Y_2 and determining the intersection point of the graph and x-axis.

Finding the x-intercept using the Zero Feature

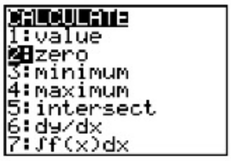
The x-intercept(s) on the graph of a relation can be determined using the **zero** feature on the calculator.

The following procedure may be used to determine the x-intercept on the graph.

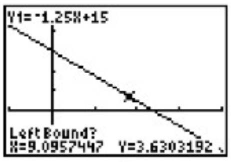
1. Enter the equation in Y_1 ($y = -1.25x + 15$, x:[−5, 20, 5], y:[−10, 25, 5]).

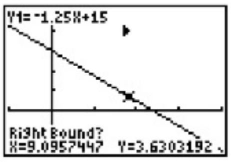
2. Press `2nd` `TRACE` keys to access the `CALCULATE` (`CALC`) menu.

3. Choose `2:zero`, and press `ENTER`.


CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx

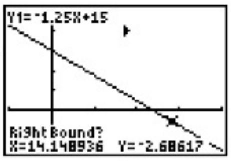
4. On the bottom left-hand side of the screen, the calculator will ask for a `LeftBound?`. The calculator is asking us to put the cursor on the **left side of the x-intercept**.

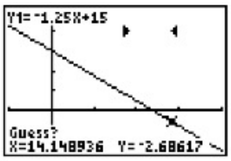

Y1=-1.25X+15
Left Bound?
X=9.0957447 Y=3.6303192


Y1=-1.25X+15
Right Bound?
X=9.0957447 Y=3.6303192

Place the flashing cursor on the <u>left</u> side of the x-intercept using the left and right cursor keys. Press `ENTER`. Note the arrow to the <u>left</u> and above the graph on the screen.

5. On the bottom left-hand side of the screen, the calculator will ask for a `RightBound?`. The calculator is asking us to put the cursor on the **right side of the x-intercept**.


Y1=-1.25X+15
Right Bound?
X=14.148936 Y=-2.68617

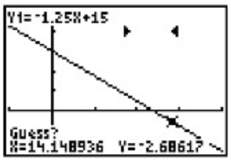

Y1=-1.25X+15
Guess?
X=14.148936 Y=-2.68617

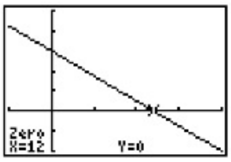
Place the flashing cursor on the <u>right</u> side of the x-intercept using the left and right cursor keys. Press `ENTER`. Note the second arrow to the <u>right</u> and above the graph on the screen.

6. On the bottom left-hand side of the screen, the calculator will ask to `Guess?`.

Press `ENTER`.

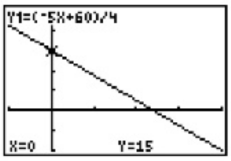
The x value will be the x-intercept.


Y1=-1.25X+15
Guess?
X=14.148936 Y=-2.68617



Using a Graphing Calculator to Determine y-intercepts

Finding the *y*-intercept using the Trace Feature



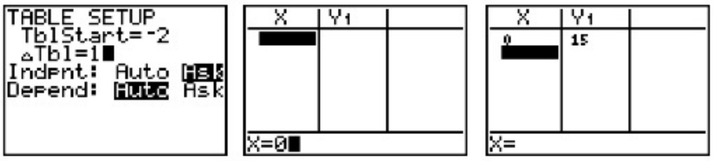
1. Enter the equation in Y_1, and adjust the window to the appropriate settings to display the *x*- and *y*-intercepts.
2. Press the TRACE key. Press 0 for the input, and then press ENTER for the output.

Finding the *y*-intercept using the Value Feature

1. Enter the equation in Y_1, and adjust the window to the appropriate settings to display the *x*- and *y*-intercepts.
2. Press 2nd TRACE keys to access the `CALCULATE` (`CALC`) menu.
3. Choose `1:value`, and press ENTER 0 ENTER.

Finding the *y*-intercept using the Table Feature

Use the procedure described earlier to find y when $x = 0$.



```
TABLE SETUP
 TblStart=-2
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```

```
  X   | Y1 |
X=0
```

```
  X   | Y1 |
  0   | 15 |
X=
```

In each case use the features of a graphing calculator to calculate the *x*- and *y*-intercepts of the graphs of the relation.

a) $y = \dfrac{5x + 15}{3}$

b) $y = -x^2 + x + 6$

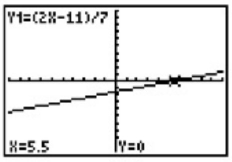
Converting a Decimal to a Fraction from a Graph Display

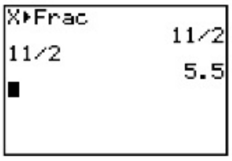
If the x- or y-intercept is not an integer, the graphing display will give the answer in decimal form. The following procedure may be used to write the decimal as an improper fraction (if applicable).

To demonstrate the procedure we will determine the x- and y-intercepts of the graph of the equation $y = \frac{(2x - 11)}{7}$.

Converting the x-value to a Fraction

1. Enter the equation in Y_1. Press GRAPH , and adjust the window to the appropriate settings.
2. Find the x-intercept using the zero feature.
3. Press 2nd QUIT to exit the graph display.
4. Press X,T,θ,n MATH ENTER ENTER .
   The decimal value may be verified by dividing the fraction.



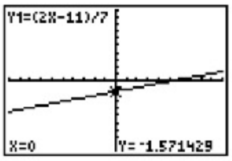


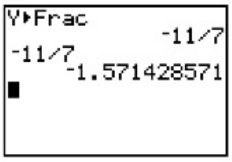
```
X▸Frac
                11/2
11/2
                 5.5
■
```

Converting the y-value to a Fraction

1. Enter the equation in Y_1. Press GRAPH , and adjust the window to the appropriate settings.
2. Find the y-intercept using the trace or value feature.
3. Press 2nd QUIT to exit the graph display.
4. Press ALPHA 1 to access the letter Y, and then press MATH ENTER ENTER .

   The decimal value may be verified by dividing the fraction.





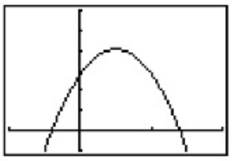
```
Y▸Frac
               -11/7
-11/7
        -1.571428571
■
```

Complete Assignment Questions #6 - #7

Extension: Using a Graphing Calculator to Determine a Maximum Value

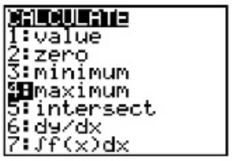
- The graphing calculator can be used to calculate the coordinates of the maximum or minimum points on a graph.
- The equation $y = -x^2 + 5x + 14$ will be used to illustrate the maximum features.

1. Enter the equation into `Y1`; adjust the window to x: [–5, 10, 5] y: [–5, 30, 5], and press `GRAPH`.

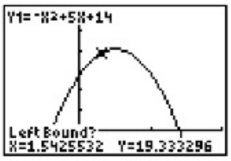


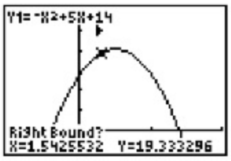
2. Access the CALC menu by entering `2nd` then `TRACE`.

3. Select "maximum".

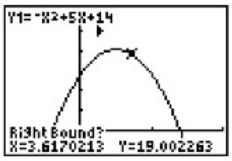


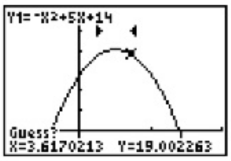
4. On the bottom left-hand side of the screen, the calculator will ask us for a LeftBound? . Select a value by scrolling to the left side of the maximum point and press `ENTER`.


Y1=-X²+5X+14
Left Bound?
X=1.5425532 Y=19.333296

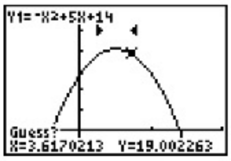

Y1=-X²+5X+14
Right Bound?
X=1.5425532 Y=19.333296

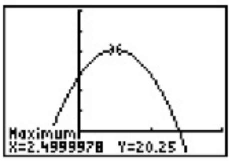
5. On the bottom left-hand side of the screen, the calculator will ask us for a RightBound ?. Select a value by scrolling to the right side of the maximum point and press `ENTER`.

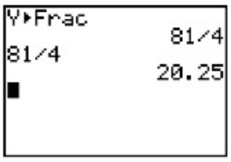

Y1=-X²+5X+14
Right Bound?
X=3.6170213 Y=19.002263


Y1=-X²+5X+14
Guess?
X=3.6170213 Y=19.002263

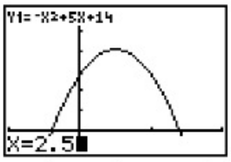
6. On the bottom left-hand side of the screen, the calculator will ask to Guess? . Press `ENTER`.

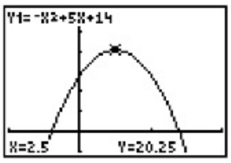

Y1=-X²+5X+14
Guess?
X=3.6170213 Y=19.002263


Maximum
X=2.4999978 Y=20.25


Y▸Frac
81/4
81/4
20.25

7. Due to the limitations of the calculator, the x value on the display may vary slightly from calculator to calculator. The answer obtained may depend on the window setting used and the bounds chosen. All the answers, however, should round to the exact answer; in this case 2.5. Test the point using the trace feature and entering $x = 2.5$.


X=2.5


Y1=-X²+5X+14
X=2.5 Y=20.25

In this example the graph of $y = -x^2 + 5x + 14$ has a **maximum value** of 20.25 or $\frac{81}{4}$, and the **maximum point** has coordinates (2.5, 20.25).

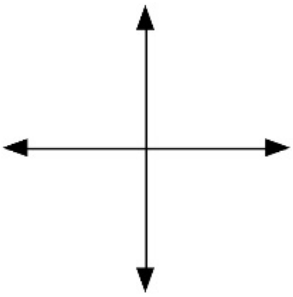
Complete Assignment Questions #8 - #9

Assignment

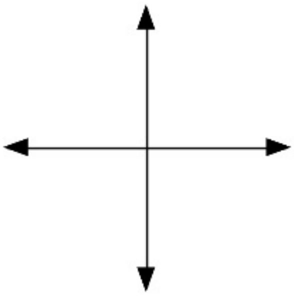
1. For each relation represented by an equation:

- Use a graphing calculator to graph the relation.
- Adjust the window to an appropriate setting so that the x- and y- intercepts (where applicable) are visible.
- Sketch the graph on the grid provided.
- Write an appropriate window setting.
- Reset the window to `ZStandard` before beginning a new graph.

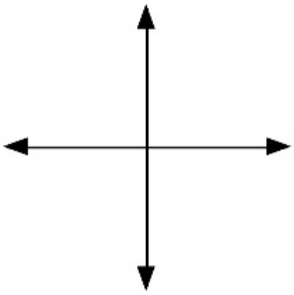
a) $y = 0.5x - 8$



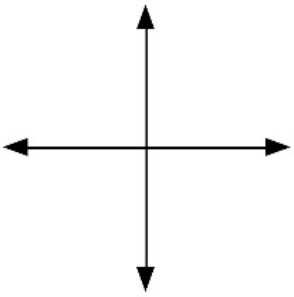
b) $y = 2x - 15$



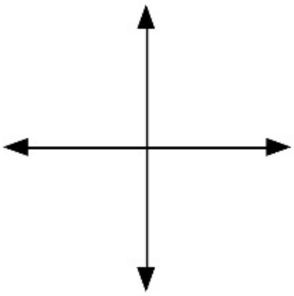
c) $y = -x + 25$



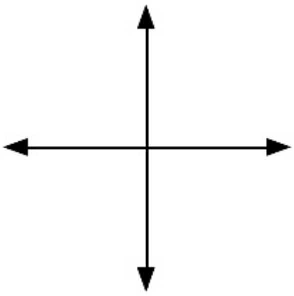
d) $y = -3x - 40$



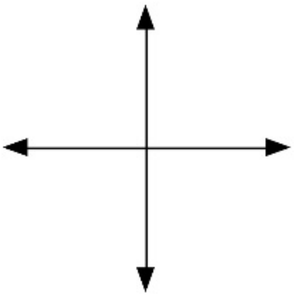
e) $y = -2x + 128$



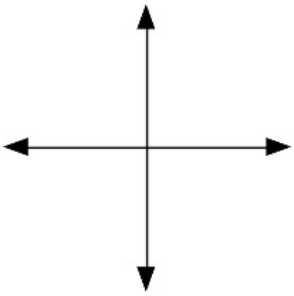
f) $y = x^2 - 15$



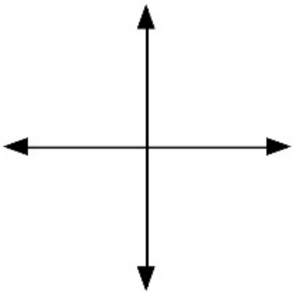
g) $y = x^2 + 20x$



h) $y = 30$



i) $y = 2^x$

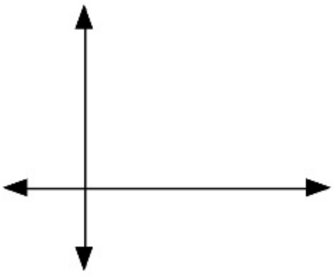


2. Consider the equation $y = -3x + 36$.

a) Use a graphing calculator to sketch the graph on the grid provided. Write an appropriate window so that the x- and y- intercepts are visible.

b) Use a graphing calculator to determine the value of y when $x = 5$.

c) Algebraically verify the answer in **b)**.



d) Use a graphing calculator to determine the value of x when $y = 12$.

e) Algebraically verify the answer in **b)**.

3. Given the x-coordinate of each relation, determine the corresponding y-coordinate using the graphing features of a calculator.

a) $y = 8x - 5,\ x = 7,\ y =$

b) $y = 0.75x + 8,\quad x = 15,\ y =$

c) $y = -2x + 12, x = -17,\ y =$

d) $y = \frac{5}{3}x - 1,\ x = 0,\ y =$

e) $y = x^2 + 3x - 12,\ x = -5,\ y =$

f) $y = 2x^2 - 5x - 2,\ x = 3.5,\ y =$

g) $y = 2x^3 + 15,\ x = 2,\ y =$

h) $y = |x| - 7,\ x = -15,\ y =$

4. Given the y-coordinate of each relation, determine the corresponding x-coordinate using the graphing features of a calculator.
Write your answer to the nearest hundredth where necessary.

a) $y = 7x - 3,\ x =$ $\qquad$ $, y = 8$

b) $y = 5x + 8,\ x =$ $\qquad$ $, y = -3$

c) $y = -2x + 12,\ x =$ $\qquad$ $, y = 15$

d) $y = \frac{5}{3}x - 1,\ x =$ $\qquad$ $, y = -12$

e) $y = x^2 + 2x - 15,\ x =$ $\qquad$ and $\qquad$ $, y = 3$

f) $y = 2x^3 + 15,\ x =$ $\qquad$ $, y = 10$

g) $y = -x^2 - 10x - 21,\ x =$ $\qquad$ and $\qquad$ $, y = -15$

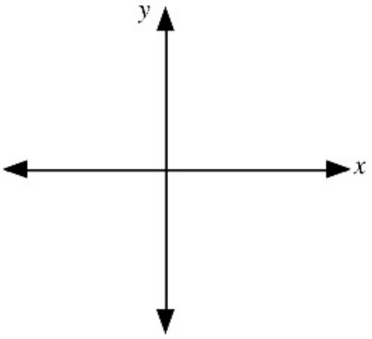
h) $y = |x| - 7,\ x =$ $\qquad$ $, y = 20$

5. Consider the relation given by the equation $y = -0.5x^2 + 8$.

a) Use the table set and table features of the calculator to complete the table of values below.

Input (x)	Output (y)
–6	
–4	
–2	
0	
2	
4	

b) Sketch the graph of the relation using an appropriate window.



c) State the window setting x:[, ,] y:[, ,].

d) Use the appropriate features of a calculator to determine:

i) the value of y when $x = 10$

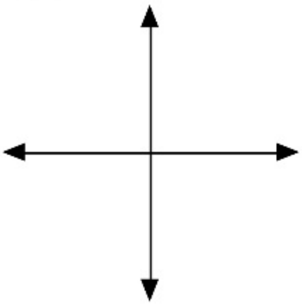
ii) the values of x when $y = 3.5$

e) Compare the answers to **a)**, **b)**, and **d ii)** with those from Lesson #2, assignment question #6.

6. For each equation use a graphing calculator to:

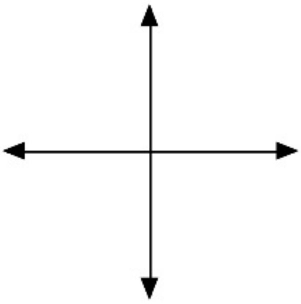
- sketch the graph of the relation on the grid provided
- **list** in the chart provided any x- and y-intercepts as **exact values**
- **write** the coordinates of the points representing the x- and y-intercepts on the graph of the relation.

a) $y = -3 - x$



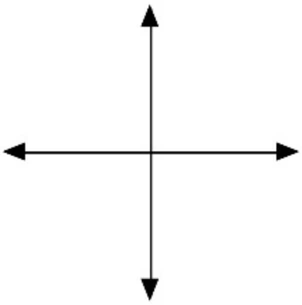
x-intercept	
y-intercept	

b) $y = -3x - 1$



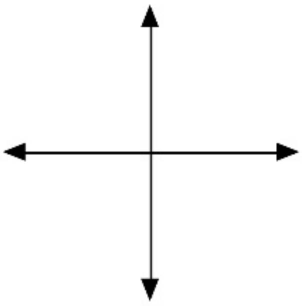
x-intercept	
y-intercept	

c) $y = 2x - 5$



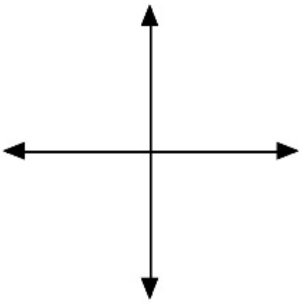
x-intercept	
y-intercept	

d) $y = -6x - 10$



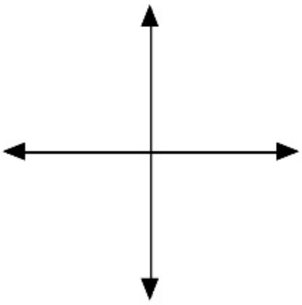
x-intercept	
y-intercept	

e) $y = \dfrac{-4x + 6}{3}$



x-intercept	
y-intercept	

f) $y = 5$

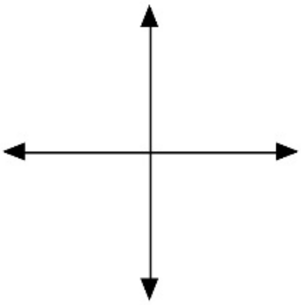


x-intercept	
y-intercept	

7. For each equation use a graphing calculator to:

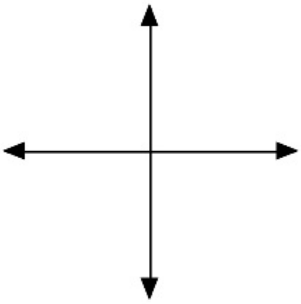
- sketch the graph of the relation on the grid provided
- **list** in the chart provided any x- and y-intercepts as **exact values**
- **write** the coordinates of the points representing the x- and y-intercepts on the graph of the relation.

a) $y = x^2 - 5x - 6$



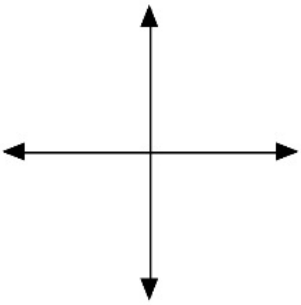
x-intercept(s)	
y-intercept	

b) $y = x^2 - 16$



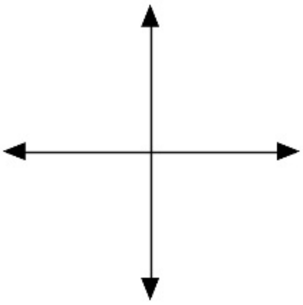
x-intercept(s)	
y-intercept	

c) $y = 2x^2 - 7x + 8$



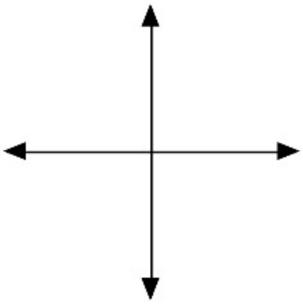
x-intercept(s)	
y-intercept	

d) $y = -x^2 + 3x + 18$



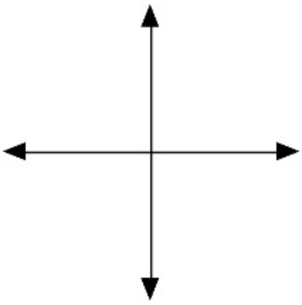
x-intercept(s)	
y-intercept	

e) $y = -2x^2 - 7x - 12$



x-intercept(s)	
y-intercept	

f) $y = 2x^3 + x^2 - 16x - 15$



x-intercept(s)	
y-intercept	

8. Determine the coordinates of the maximum point of each graph. Answer using exact values.

a) $y = -0.3x^2 + 5x - 10$ **b)** $y = -0.1x^2 + 5x + 7$ **c)** $y = -9x^2 - 60x + 100$

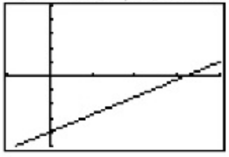
9. Determine the coordinates of the minimum point of each graph. Answer using exact values.

a) $y = x^2 + 2x + 35$ **b)** $y = 0.15x^2 - 2x - 8$ **c)** $y = 0.5x^2 + 2x - 16$

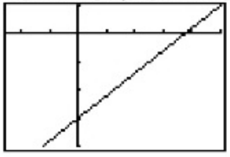
Answer Key

1. *Window setting answers may vary.*

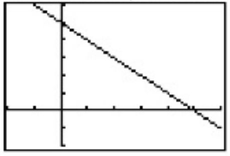
a) x: [-5,20,5], y: [-10,10,2]



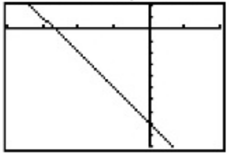
b) x: [-5,10,2], y: [-20,5,5]



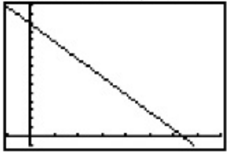
c) x: [-10,30,5], y: [-10,30,5]



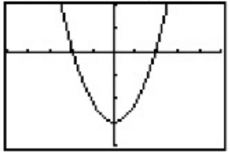
d) x: [-20,10,5], y: [-50,10,10]



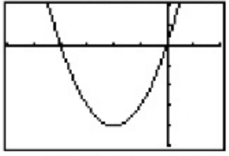
e) x: [-10,80,10], y: [-10,150,10]



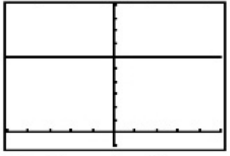
f) x: [-10,10,2], y: [–20,10,5]



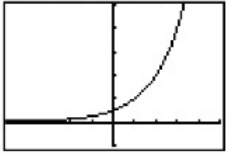
g) x: [-30,10,5], y: [-125,50,25]



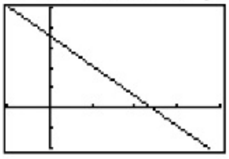
h) x: [-10,10,2], y: [-5,50,5]



i) x: [-2,5,1], y: [-5,30,5]



2. a) x: [-5,20,5], y: [-20,50,10] **b)** 21 **c)** 21 **d)** 8 **e)** 8



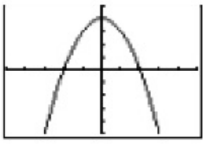
3. a) 51 **b)** 19.25 **c)** 46 **d)** –1
e) –2 **f)** 5 **g)** 31 **h)** 8

4. a) 1.57 **b)** –2.2 **c)** –1.5 **d)** –6.6
e) –5.36 and 3.36 **f)** –1.36 **g)** –9.36 and –0.64 **h)** ±27

5 a)

Input (x)	Output (y)
–6	-10
–4	0
–2	6
0	8
2	6
4	0

b)

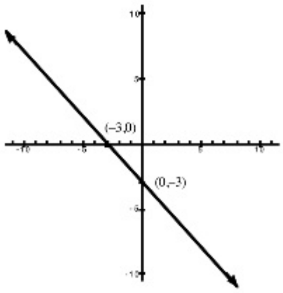


c) x:[–10, 10, 2] y:[–10, 10, 2]

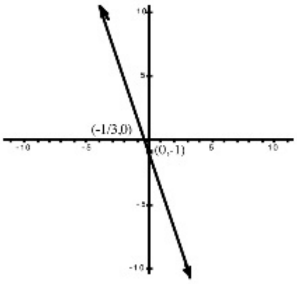
d) i) –42 **ii)** –3 and 3

e) exactly the same

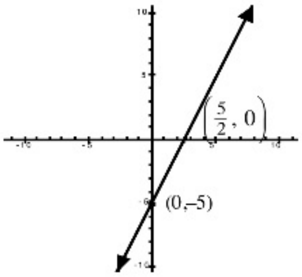
6. a) x-int $= -3$, y-int $= -3$



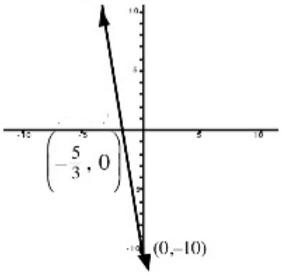
b) x-int $= -\frac{1}{3}$, y-int $= -1$



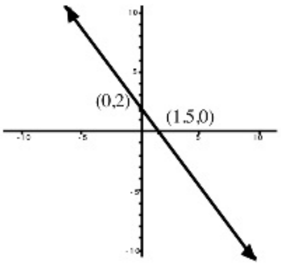
c) x-int $= \frac{5}{2}$, y-int $= -5$



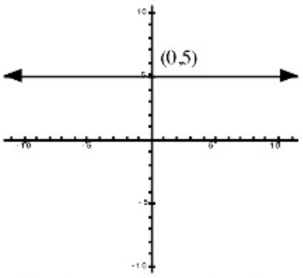
d) x-int $= -\frac{5}{3}$, y-int $= -10$



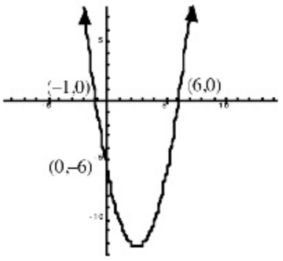
e) x-int $= \frac{3}{2}$, y-int $= 2$



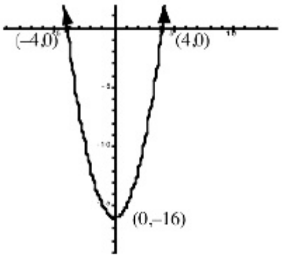
f) no x-int, y-int $= 5$



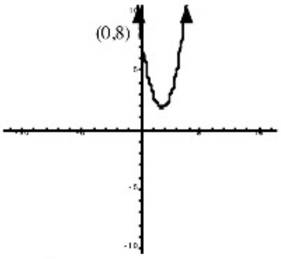
7. a) x-int $= -1$ and 6, y-int $= -6$



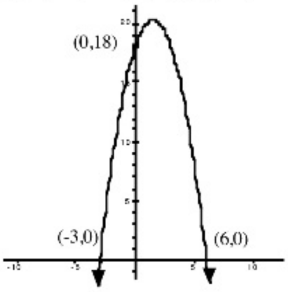
b) x-int $= -4$ and 4, y-int $= -16$



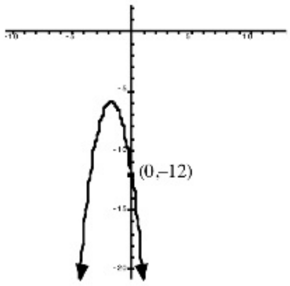
c) no x-int, y-int $= 8$



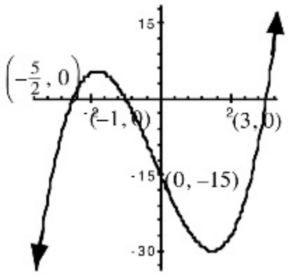
d) x-int $= -3$ and 6, y-int $= 18$



e) no x-int, y-int $= -12$



f) x-int $= -\frac{5}{2}, -1$ and 3 y-int $= -15$



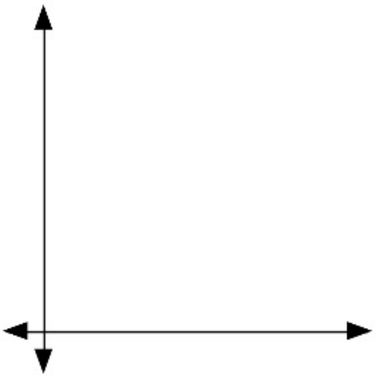
8. a) $\left(\frac{25}{3}, \frac{65}{6}\right)$ **b)** $\left(25, \frac{139}{2}\right)$ **c)** $\left(-\frac{10}{3}, 200\right)$

9. a) $(-1, 34)$ **b)** $\left(\frac{20}{3}, -\frac{44}{3}\right)$ **c)** $(-2, -18)$

Relations Lesson #6:
Interpreting Relations Using a Graphing Calculator

Consider the scenario from Class Ex. #3 from Lesson #3 of this unit.

"Lisa purchases a new car for $\$20\ 000$.
The value of the car can be represented by the formula $V = 20\ 000 - 1250t$, where V is the value of the car in dollars and t is the age of the car in years."



a)
- Graph $V = 20\ 000 - 1250t$ using a graphing calculator.
- Use the ZoomFit feature as a guide to adjust the graphing window. Then use the **Window** key to align the graph appropriately for this scenario. Write the window setting below.
- Sketch the graph on the grid provided.

b) Calculate the t-intercept of the graph using a graphing calculator, and label it on the sketch. Describe what this value represents in the context of the question.

c) Calculate the V-intercept of the graph using a graphing calculator, and label it on the sketch. Describe what this value represents in the context of the question.

d) Use the trace feature of a graphing calculator to determine what the car will be worth in 5 years.

e) Use the intersect feature of a graphing calculator to determine when the car will be worth half of the purchase price. Illustrate this on your sketch.

f) Write an appropriate domain and range for the function which describes the value of the car over time.

The height of a human cannon ball, "Cano", can be described by the formula $h = 12 + 6t - t^2$, where h is the height in metres above ground level and t is the time in seconds. Cano is projected out of a cannon from the top of a building and lands on a soft mat. The mat is placed in a hole in the ground so that the top of the mat is level with the ground.

a) Display the graph of $h = 12 + 6t - t^2$ on a graphing calculator.

b) Write down a window setting which would be appropriate for this situation.

c) Sketch $h = 12 + 6t - t^2$ on the grid provided.

d) What is the height of the cannon above the ground?

e) What is the maximum height Cano reaches?

f) How many seconds does it take Cano to reach the highest point on the path he is travelling?

g) To the nearest hundredth of a second, how long does it take Cano to land on the mat?

h) How high is Cano one second after he is launched?

i) When will Cano be at the height in **h)** again?

j) In words, describe the relation connecting height and time.

k) Write an appropriate domain and range for the relation described in **j)**.

Complete Assignment Questions #1 - #5

Assignment

In this assignment, round answers to 2 decimal places unless otherwise stated.

1. Students at a senior high school produce an art literary magazine. The cost for this magazine can be modelled by the formula $C = 2n + 30$, where C is the total cost of the magazine in dollars and n is the number of magazines produced.

a) Sketch the graph on the grid provided. Explain why the graph is not a continuous straight line.

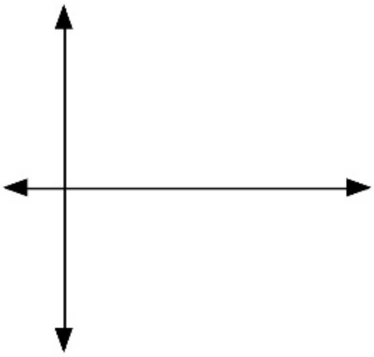
b) Is the n-intercept relevant to the graph of this relation?

c) What would be the cost for 30 magazines?

d) How many magazines are produced if the total cost is $126?

e) Describe the significance of the C-intercept.

2. For a science experiment, a pot of water is heated to a certain temperature and then put in a freezer and allowed to freeze. The rate of freezing can be estimated by the formula $T = -0.4t + 50$, where the temperature, T, in degrees Celsius is recorded for each time, t in minutes.



a) Sketch the graph on the grid using a graphing calculator.

b) What was the temperature of the water at 12 minutes?

c) How long did it take for the pot of water to cool down to 40°C?

d) Describe the significance of the t-intercept in this question.

e) Describe the relation in words.

3. During a high school football game, the height of a punt can be modelled by the relation

$$h = -5t^2 + 14t + 1$$

where

t is the number of seconds which have elapsed since the football was punted, and

h is the number of metres above the ground after t seconds

a) Sketch the graph on the grid.

b) What was the height of the football above the ground as the punter makes contact with the football?

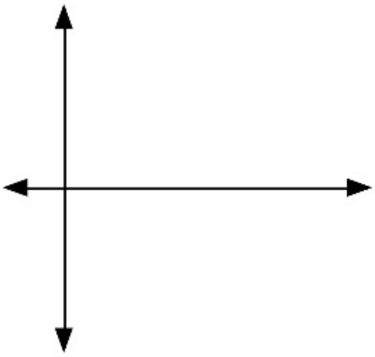
c) What was the height of the football above the ground 1 second after contact?

d) What is the maximum height of the football above the ground?

e) How many seconds had elapsed when the football reached its maximum height?

f) The punt is not caught by the opposing team, and the football hits the ground. How many seconds, to the nearest hundredth, did it take for the football to hit the ground?

4. Student Council is planning a school dance and sells tickets at $5 per student. The expenses for the dance are $800. The treasurer of the council generates a formula to determine the potential profit for the dance to be $P = 5t - 800$, where P is the profit made on the dance and t is the number of tickets sold. The maximum number of tickets which can be sold is 450.



a) Sketch the graph on the grid using a graphing calculator.

b) Determine how many tickets need to be sold to break even.

c) How many tickets need to be sold to make a $1250 profit?

d) How much profit is made if 200 tickets are sold?

e) Write the domain and range for this relation.

5. In a soccer game the ball is passed back to the goal keeper, and she kicks the ball from ground level up the field. The height, h metres, of the ball above the ground can be modelled by the equation $h = -0.033d^2 + 1.6d - 10$, where d is the distance in metres from the goal line.

a) Sketch the graph on the grid.

b) What was the distance between the goal line and the point where the ball was kicked?

c) What is the maximum height the soccer ball reaches?

d) How far from the goal line is the ball when it reaches its maximum height?

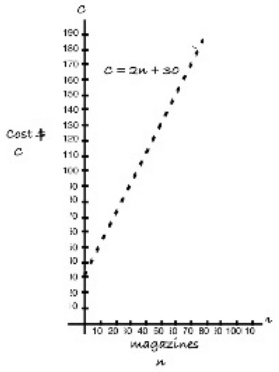
e) What horizontal distance has the ball travelled when it hits the ground for the first time?

f) Write a domain and range for this relation.

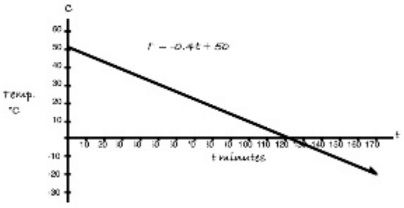
g) At what distance from the goal line is the ball 5 metres in the air?

Assignment Key

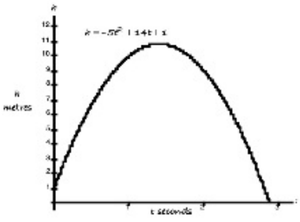
1. **a)** See grid at side. The data is discrete so there are gaps in the line.
 b) No, since the number of magazines cannot be negative.
 c) \$90
 d) 48
 e) The C-intercept of 30 represents a fixed charge of \$30 irrespective of the number of magazines produced.



2. **a)** See grid at side
 b) 45.2° C
 c) 25 min.
 d) The t intercept of 125 represents the number of minutes it takes the water to freeze.
 e) A pot of water is heated to 50°C and placed in a freezer and allowed to freeze. It cools at a constant rate and after 125 min the water is frozen.

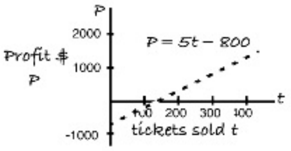


3. **a)** See grid below

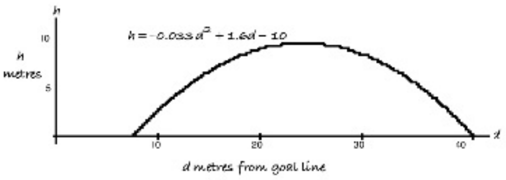


 b) 1 m
 c) 10 m
 d) 10.8 m
 e) 1.4 sec.
 f) 2.87 sec.

4. **a)** See grid below
 b) 160
 c) 410
 d) \$200
 e) domain: 0, 1, 2, 3 450
 range: –800, –795, –790, –785 1450
 (increasing in steps of 5)



5. **a)** See grid at side
 b) 7.37 m **c)** 9.39 m
 d) 24.24 m **e)** 33.74 m
 f) domain: $\{7.37 \le d \le 41.11, d \in R\}$
 range: $\{0 \le h \le 9.39, h \in R\}$
 g) 12.70 m and 35.78 m



Relations Lesson #7:
Practice Test

1. Which of the following variables is discrete?

A. The time taken to run 50 metres.
B. The distance travelled by a train.
C. The number of animals in a zoo.
D. The weight of a dog.

2. Which of the following statements is false?

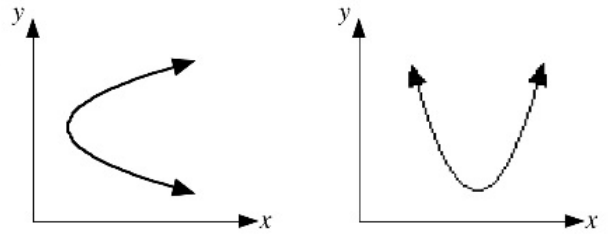
A. The domain of a relation is associated with values of the independent variable.
B. The dependent variable is represented by the first coordinate of an ordered pair.
C. The inputs of a relation are shown on the horizontal axis of a Cartesian Plane.
D. The range of a relation is represented on the vertical axis of the graph of the relation.

Numerical Response **1.** To the nearest hundredth, the positive x-intercept on the graph of the relation $2x^2 + 6y^2 = 7$ is _____ .
(Record your answer in the numerical response box from left to right)

☐☐☐☐

Use the following information to answer the next question.

The graphs of two relations are shown.



3. Which of the following statements is true?
A. The domains are the same, but the ranges are different.
B. The ranges are the same, but the domains are different.
C. The domains are the same, and the ranges are the same.
D. The domains are different, and the ranges are different.

Numerical Response **2.** The graph of the relation $4x^2 - 3y = 12$ has x-intercepts a and b, and y-intercept c. The value of the product abc is _____ .

(Record your answer in the numerical response box from left to right) ☐☐☐☐

Use the following information to answer the next two questions.

The cost (\$ C) of publishing a school newspaper is partially fixed and partially depends on the number (n) of newspapers printed. The cost is given by the formula $C = 98 + 0.12n$.

4. The cost of publishing 780 newspapers is

- **A.** \$191.60
- **B.** \$773.76
- **C.** \$94.58
- **D.** \$9 458.00

Numerical Response **3.** The domain of the relation can be expressed in the form $n \geq x$, and the range can be expressed in the form $C \geq y$. Write the value of y in the first two boxes, and the value of x in the last two boxes.

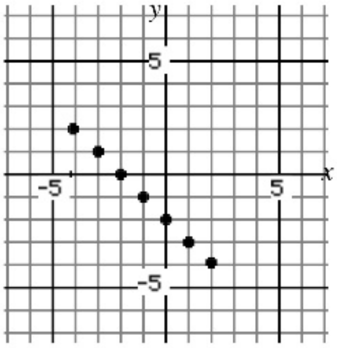
(Record your answer in the numerical response box from left to right) ☐☐☐☐

5. Dianna is driving back to Squamish Nation from Tsawwassen First Nation. Consider the relation in which the quantity of fuel in the gas tank of her truck is related to the distance she has travelled. The domain of this relation can be measured in

- **A.** km
- **B.** litres
- **C.** km/litre
- **D.** litres/km

Use the following information to answer the next two questions.

The diagram shows the graph of a relation.



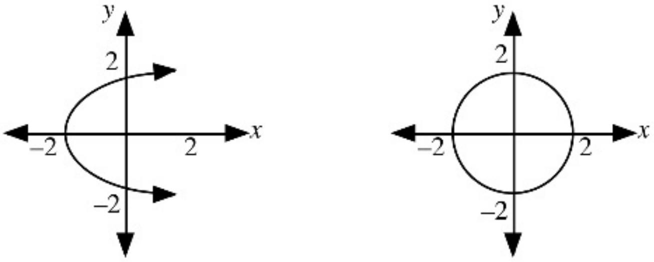
6. The domain of the relation can be written in the form

A. $\{x \mid -4 \le x \le 2, x \in R\}$
B. $\{x \mid -4 \le x \le 2, x \in W\}$
C. $\{x \mid -4 \le x \le 2, x \in I\}$
D. $\{x \mid -4 \le x \le 2, x \in N\}$

7. The range can be written in the form

A. $\{y \mid y \ge -4, y \in I\}$
B. $\{x \mid y \le 2, y \in I\}$
C. $\{y \mid -4 < y < 2, y \in I\}$
D. $\{y \mid -5 < y < 3, y \in I\}$

8. The graphs of two relations are shown. Which of the following statements is true?



A. The range of both relations is $\{y \mid -2 \le y \le 2, y \in R\}$.
B. The range of both relations is $y \in R$.
C. The domain of both relations is $\{x \mid -2 \le x \le 2, x \in R\}$.
D. None of the above.

Use the following information to answer the next four questions.

A student has graphed the relation $y = 4x^3 - 7x + 3$ on her graphing calculator.

9. Which of the following is not an x-intercept?

A. $-\frac{3}{2}$
B. $\frac{1}{3}$
C. $\frac{1}{2}$
D. 1

10. The y-intercept is

A. 0
B. 4
C. –7
D. 3

11. When $y = 2$, the number of possible values of x is

A. one
B. two
C. three
D. infinite

12. When $y = -2$, the value of x, to the nearest tenth, is

A. –1.6
B. –1.5
C. 1.5
D. 1.6

Use the following information to answer the next five questions.

The height of a soccer ball above the ground after a free kick for the Hawks is given by the equation $h = 0.02(-d^2 + 40d)$, where h is the height in metres and d is the horizontal distance in metres the ball travels.

13. The height, in metres, that the soccer ball is above the ground when the ball is kicked is

A. 0
B. 0.02
C. 0.05
D 0.2

14. The maximum height reached by the soccer ball is

A. 4 metres
B. 8 metres
C. 10 metres
D. 20 metres

15. When the ball reaches its maximum height, the horizontal distance it has travelled is

A. 8 metres
B. 16 metres
C. 20 metres
D. 40 metres

Numerical Response **4.** Before the ball strikes the ground, a defender heads the ball after it has travelled a horizontal distance of 37 m. The height of the ball above the ground, to the nearest tenth of a metre, when the defender heads the ball, is _____ .

(Record your answer in the numerical response box from left to right)

Numerical Response **5.** After the ball is kicked, it is exactly 5 m above the ground at the top of its trajectory. To the nearest tenth of a metre, the horizontal distance that the ball has travelled between the two instances when it is 5 m above the ground is _____ .

(Record your answer in the numerical response box from left to right)

Written Response - 5 marks

1. During an airshow, the path of a stunt dive of a jet can be modelled by the equation $h = t^2 - 9t + 81$, where h is the height in metres after t seconds. The end of the stunt occurs when the plane achieves its starting height.

- How high is the jet at the start of the dive?
- How high is the jet above ground level at its lowest point?
- When does the jet reach its lowest point in the dive?
- How high is the jet two seconds into its dive?
- After how many seconds does this height occur again within the jet's stunt dive?

Answer Key

1. C 2. B 3. D 4. A 5. B 6. C 7. D 8. D
9. B 10. D 11. C 12. A 13. A 14. B 15. C

Numerical Response

1. 1.87 2. 12 3. 9800

4. 2.2 5. 24.5

Written Response

1. • 81 m
• 60.75 m
• 4.5 sec
• 67 m
• 7 sec

Functions Lesson #1:
Functions

Review

We have considered six ways in which the relationship between two quantities can be represented.

- in words
- a mapping (or arrow) diagram
- function notation (this unit)
- a table of values
- an equation
- a set of ordered pairs
- a graph

In a **relation** each element of the **domain** (the **input**) is related to an element or elements of the **range** (the **output**).

In this lesson we will study a special type of relation called a **function**.

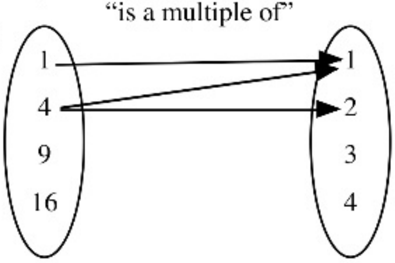
Exploration

To illustrate the concept of function, we will look at two relations described in words with domain $D = \{1, 4, 9, 16\}$ and range $R = \{1, 2, 3, 4\}$.

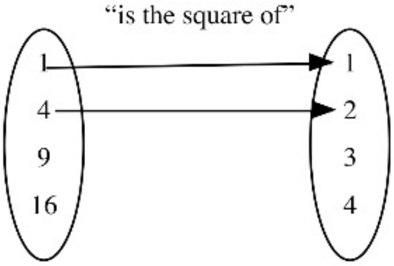
i) "is a multiple of" **ii)** "is the square of"

a) Complete the arrow diagrams.

i)



ii)



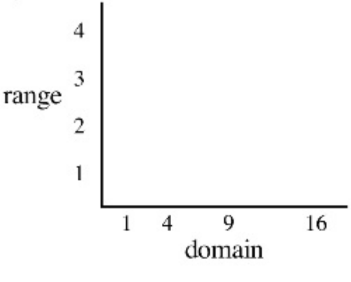
b) Complete the set of ordered pairs.

i) (1, 1), (4, 1), (

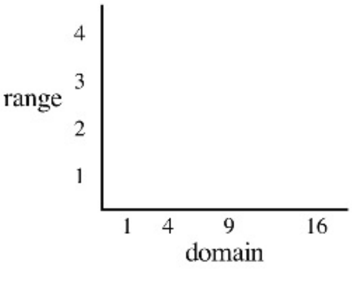
ii)

c) Plot the ordered pairs on the grid.

i)



ii)



Function

A functional relation, or **function**, is a special type of relation in which each element of the domain is related to exactly one element of the range. If any element of the domain is related to more than one element of the range, then the relation is not a function.

Class Ex. #1

In the exploration on the previous page, one of the relations is a function, and the other relation is not a function.

Explain how we can determine which relation is a function by looking at the following:

a) arrow diagrams

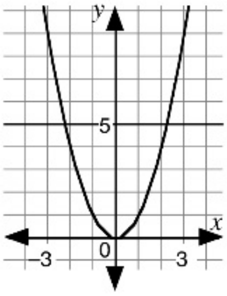
b) ordered pairs

c) graphs

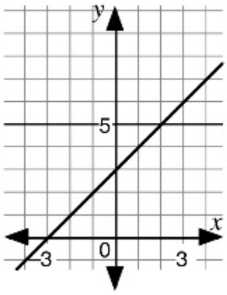
Class Ex. #2

Each of the following is the graph of a relation.

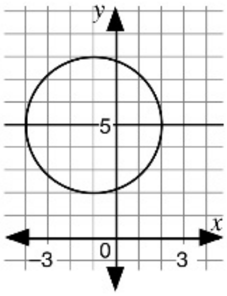
Graph A



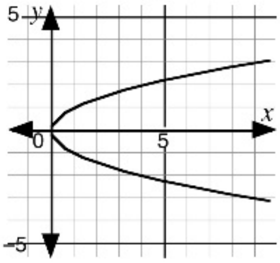
Graph B



Graph C



Graph D



a) Classify the following statements as true (T) or false (F).

	A	B	C	D
• For each input value there is only one output.				
• For each output value there is only one input.				
• The relation is a function.				

b) From graph C, write two ordered pairs which show that the relation is not a function. Draw a line joining these points.

c) From graph D, write two ordered pairs which show that the relation is not a function. Draw a line joining these points.

d) On graphs A and B draw a series of vertical lines. Do any of these lines intersect the graph of the relation at more than one point?

Vertical Line Test

The vertical line test can be used on the graph of a relation to determine whether the relation is a function or not.

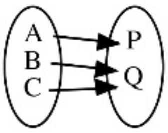
- If every vertical line, drawn on the domain of the relation, intersects the graph exactly once, then the relation is a function.
- If any vertical line intersects the graph more than once, then it is **not** a function.

Determine which of the following are functions. Explain your answers.

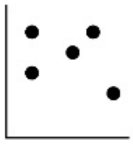
a) (5, 8), (6, 7), (–5, 3), (2, 3), (6, 8)

b) (3, 3), (2, 3), (4, 5), (–3, 2)

c)

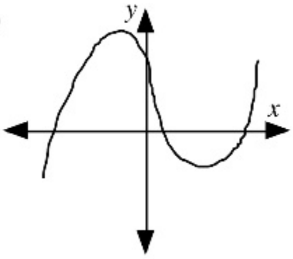


d)

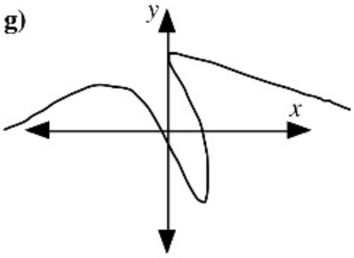


e) The relation connecting the provinces and territories of Canada with their capital cities.

f)



g)



A Function as a Mapping

A function from a set D, the domain, to a set R, the range, is a relation in which each element of D is related to exactly one element of R.

If the function f maps an element x in the domain to an element y in the range, we write $f: x \rightarrow y$.

Complete the following for the function "is the square of" on the first page of this lesson.

$1 \rightarrow$ $\quad$ $4 \rightarrow$ $\quad$ $9 \rightarrow$ $\quad$ $\rightarrow$

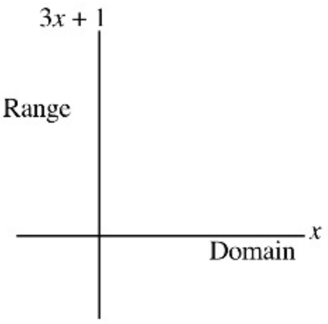
Consider the function $f: x \rightarrow 3x + 1$, for domain $\{-1, 0, 1, 2\}$.

a) Complete $\quad -1 \rightarrow \quad 0 \rightarrow \quad \rightarrow \quad \rightarrow$

b) List the elements of the range of the function.

c) Show the function as:

i) an arrow diagram **ii)** a set of ordered pairs **iii)** a Cartesian graph.



At this time we label the top of the vertical axis with $3x + 1$. In the next lesson we will learn function notation which is more commonly used.

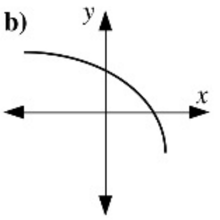
Complete Assignment Questions #1 - #12

Assignment

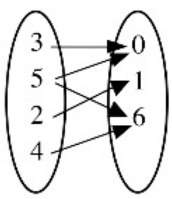
1. Determine which of the following relations are functions. Give reasons for your answers.

a) (–1, 3), (–2, 1), (5, 2), (7, 3)

b)



c)



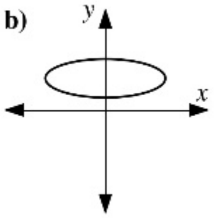
d)

Input (*x*)	Output (*y*)
2	3
0	4
–3	5
2	6

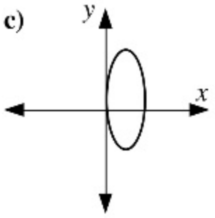
2. State which of the following relations are functions.

a) (0, 0), (1, 2), (2, 3), (3, 4), (4, 3)

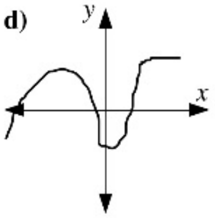
b)



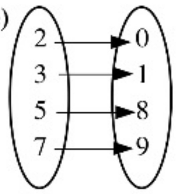
c)



d)



e)

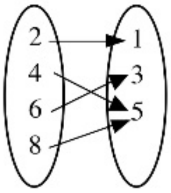


3. State which of the following relations are functions.

a)

Input (*x*)	Output (*y*)
0	3
2	4
4	5
6	3

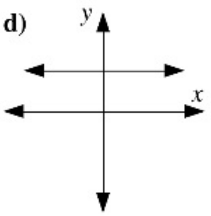
b)



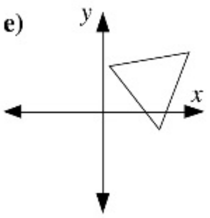
c)

Input (*x*)	Output (*y*)
1	5
–1	5
3	5
7	5

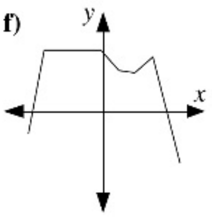
d)



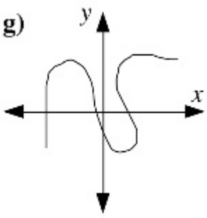
e)



f)



g)



4. Mr. A has a son Jim and a daughter Kristen. Mr. B has three daughters, Lauren, Melanie, and Noreen.

a) Draw an arrow diagram to illustrate the relation “is the father of” from the set of fathers to the set of children. Is the relation “is the father of” a function?

b) Draw an arrow diagram to illustrate the relation “is the child of” from the set of children to the set of fathers. Is the relation “is the child of” a function?

5. The function $f: x \to 2x + 5$ has domain $\{0, 1, 2, 3\}$.

a) List the elements of the range of the function.

b) Show the function f in a Cartesian graph.

6. The function $g: x \to x^3$ has domain $\{-2, -1, 0, 1, 2\}$.

a) List the elements of the range of the function.

b) Show the function g in a Cartesian graph.

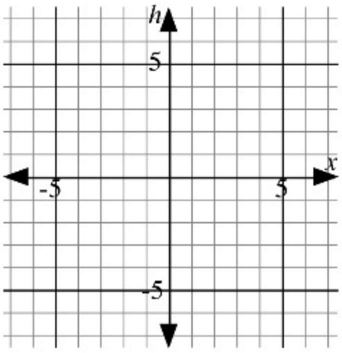
7. Consider the function $f: x \rightarrow x^2 - 4$.

a) Complete the following table of values.

Elements of Domain	3	2	1	–1	–2	–3
Elements of Range						

b) Plot the ordered pairs on a Cartesian graph.

c) Draw a smooth curve through the points to illustrate the function $f: x \rightarrow x^2 - 4,\ x \in R$.



8. The domain of the function $h: x \rightarrow 6$ is $\{ 0, 10, 20\}$.

a) List the ordered pairs of the graph of the function.

b) Show the function h in a Cartesian graph.

Multiple Choice

9. The function $f: x \rightarrow 6 - 2x$ has domain $\{0, 2, 4, 6, 8\}$.
Which of the following is **not** an element of the range of the function?

A. −10
B. 2
C. 4
D. –6

10. Which of the following statements is not always true for a function?

A. A function is a set of ordered pairs (x, y) in which for every x there is only one y.
B. A vertical line must not intersect the graph of a function in more than one point.
C. For every output there is only one input.
D. For every element in the domain, there is only one element in the range.

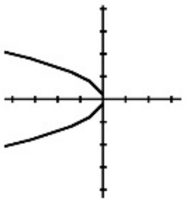
11. Which of the following represents a function?

1 "multiply the number by 3 and add 5."

2 $y = -x^2$

3 (9, 3), (4, 2), (1, 1), (0, 0). (1, –1), (4,–2), (9,–3)

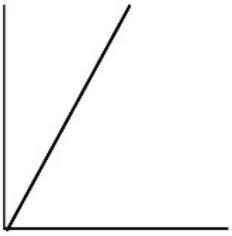
4



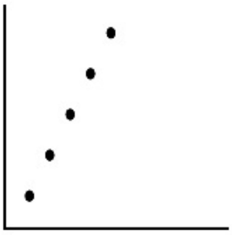
A. 1 only
B. 1 and 2 only
C. 1 and 3 only
D. some other combination of 1 - 4

Numerical Response **12.** Partial graphs of four functions are shown.

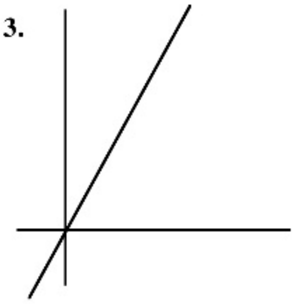
1.



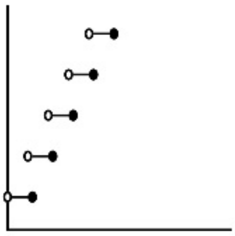
2.



3.



4.



The functions are described as follows:

A: Coffee costs \$8 per jar. Graph cost as a function of the number of jars purchased.

B: Distance cycled at a constant speed of 8 km/h. Graph distance as a function of time.

C: Parking costs \$8 per hour (or part of an hour). Graph cost as a function of time.

D: Set of ordered pairs which satisfy the equation $y = 8x, x \in R$.
Graph y as a function of x.

Place the graph number for function A in the first box.
Place the graph number for function B in the second box.
Place the graph number for function C in the third box.
Place the graph number for function D in the last box.

(Record your answer in the numerical response box from left to right)

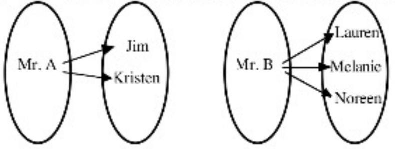
Answer Key

1. a) function: each first coordinate has only one second coordinate
b) function: vertical lines intersects the graph exactly once
c) not a function: the input 5 has two outputs
d) not a function: the input 2 has two outputs

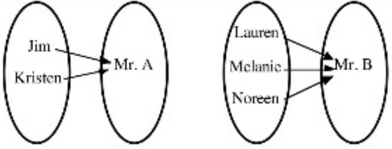
2. a) function **b)** not a function **c)** not a function **d)** not a function **e)** function

3. a) function **b)** function **c)** function **d)** function **e)** not a function
f) function **g)** not a function

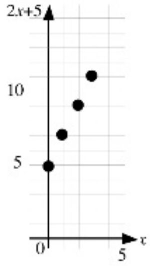
4. a) Neither mapping diagrams represents a function


Mr. A
Jim
Kristen
Mr. B
Lauren
Melanie
Noreen

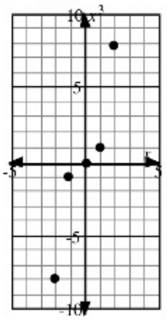
b) Both mapping diagrams represent functions


Jim
Kristen
Mr. A
Lauren
Melanie
Noreen
Mr. B

5. a) {5, 7, 9, 11}
b) see graph below

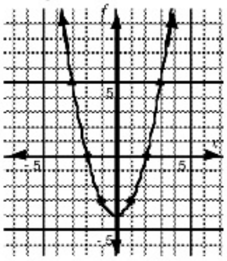


6. a) {−8, −1, 0, 1, 8}
b) see graph below

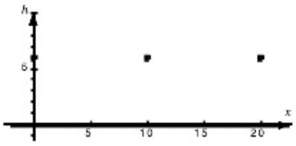


7. a) see table **b)** see grid **c)** see grid

Elements of Domain	3	2	1	−1	−2	−3
Elements of Range	5	0	−3	−3	0	5



8. a) {(0, 6), (10, 6), (20, 6)} **b)**



9. C **10.** C **11.** B **12.**

2	1	4	3

Functions Lesson #2:
Function Notation - Part One

Mapping Notation

In the previous lesson we discovered some ways in which functions can be represented:

- in words
- a table of values
- a set of ordered pairs
- a mapping (or arrow) diagram
- an equation
- a graph
- function notation (this unit)

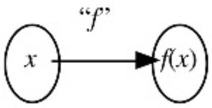
A **function** was defined in mapping notation as follows:

> "A function from a set D, the domain, to a set R, the range, is a relation in which each element of D is related to exactly one element of R.
> If the function f maps an element x in the domain to an element y in the range, we write $f: x \rightarrow y$."

Consider the function $f: x \rightarrow 2x + 3$ defined on the set of real numbers.
Under this function we know that $5 \rightarrow 2(5) + 3$ ie $5 \rightarrow 13$.
We say that under the function f, the **image** of 5 is 13.
We also say that the **value of the function** is 13 when $x = 5$.

Function Notation

In most math courses, function notation is used to replace the mapping notation $f: x \rightarrow 2x+3$. Under a function f, the image of an element x in the domain is denoted by $f(x)$, which is read "f of x".



In the example above, the function f can be defined by the formula $f(x) = 2x + 3$.
The notation $f(x) = 2x + 3$ is called **function notation**.

We showed above, that, under the function f, the image of 5 is 13. We write $f(5) = 13$.

mapping notation	function notation	equation of graph of function
$f: x \rightarrow 2x+3$	$f(x) = 2x + 3$	$y = 2x + 3$
$f: 5 \rightarrow 2(5)+3$	$f(5) = 2(5) + 3$	$y = 2(5) + 3$
$f: 5 \rightarrow 13$	$f(5) = 13$	$y = 13$

The symbol $f(x)$ is read as "f at x" or "f of x".
$f(x)$ provides a formula for the function f, and also represents the value of the function for a given value of x.

In function notation:

- $f(x)$ does not mean f times x.
- Values of the independent variable represent the **inputs** of a function and are shown on the **horizontal axis**.
- The "name" of the function is f.
- Values of the dependent variable represent the **outputs** of a function and are shown on the **vertical axis**.

Class Ex. #1

Consider the function $f(x) = x^2 + 5$ and $g(x) = 4 - x$. Evaluate:

a) $f(3)$

b) $g(1)$

c) $f(-2)$

d) $g(-2)$

e) $f(0) - g(0)$

Class Ex. #2

Consider the function f defined by $f(x) = 5x^3 - 2x, x \in R$. Determine:

a) $f(-3)$

b) the value of f when $x = 2$

c) the image of 7 under f

e) an expression for $f(a)$

f) an expression for $f(2x)$

Class Ex. #3

If $P(x) = 4x^2 - 6x + 1$, determine a simplified expression for $P(x - 3)$.

Complete Assignment Questions #1 - #7

Class Ex. #4

Consider the function $f(x) = 10x - 3$, $x \in R$.

a) Determine the value of x if $f(x) = 47$.

b) Solve the equation $f(x) = -23$.

Class Ex. #5

Consider the function $f(x) = x^2 - 5, x \in R$.

a) Evaluate $f(4)$.

b) Solve the equation $f(x) = 4$.

c) Solve the equation $f(t) = 75$, where $t > 0$. Answer as an exact value and as a decimal to the nearest hundredth.

Complete Assignment Questions #8 - #13

Assignment

1. Each statement refers to the function f whose graph has equation $y = f(x)$.
Circle the correct choice.

a) f is the *name / value* of the function.
b) The values of x represent the *inputs / outputs* of the function.
c) The values of $f(x)$ represent the *inputs / outputs* of the function.
d) The values of y represent the *inputs / outputs* of the function.
e) x represents the *independent / dependent* variable of the function.
f) $f(x)$ represents the *independent / dependent* variable of the function.
g) y represents the *independent / dependent* variable of the function.

2. If $f(x) = 5x - 7$, determine:

a) $f(2)$

$f(2) = 5(2) - 7$

$f(2) =$

b) $f(-3)$

c) $f(0)$

3. Function g is defined by $g(x) = 6 - x^2$. Evaluate

a) $g(4)$ **b)** $g(-6)$ **c)** $g\left(\sqrt{3}\right)$

4. A function f is defined by the formula $f(x) = x^3 + 1$. Find

a) the image of 2 under f **b)** the value of f at -7. **c)** an expression for $f(a)$

5. If $f(x) = x^3 - 2x^2 - x - 5$, evaluate

a) $f(5)$ **b)** $f(-3)$

6. Consider the function f defined by $f(x) = 8 - 2x,\ x \in R$. Determine

a) $f(4)$ **b)** the value of f when $x = -4$ **c)** the image of 0.5 under f

d) an expression for $f(2t)$ **e)** an expression for $f(a + 3)$

7. If $F(x) = 3x^2 - 2x - 9$, determine a simplified expression for

a) $F(-x)$ **b)** $F(x - 5)$

8. **a)** If $f(x) = 5x - 7$, then determine the value of x if $f(x) = 43$.

b) If $g(x) = 6x + 3$, then determine the value of x if $g(x) = -24$.

c) If $g(t) = 56 - 3t$, then determine the value of t if $g(t) = 11$.

d) If $h(x) = -3x + 1$, then determine the value of x if $h(x) = 22$.

e) If $P(x) = 50 - 3x^2$, then determine the values of x if $P(x) = -25$.

9. Consider the function f defined by $f(x) = 6x - 15$. Find

a) $f(0)$ **b)** an expression for $f(2x + 1)$ **c)** the solution to the equation $f(x) = 27$

10. A function C is defined by $C(x) = \sqrt{x}$ where $x \geq 0$.

a) Evaluate

i) $C(16)$ **ii)** $C\left(\frac{1}{36}\right)$ **iii)** $\frac{C(100)}{C(4)}$

b) If $C(x) = 9$, find x.

11. A function g is defined by the formula $g(t) = t + 12$.

a) Calculate the value of $g(4) + g(-2)$.

b) If $g(a^2) = 48$, determine all possible values of a.

Multiple Choice **12.** If $f(x) = 3x - 1$ and $f(t) = 8$, then $t =$

A. $\frac{7}{3}$

B. 3

C. $\frac{11}{3}$

D. 23

Numerical Response **13.** A function f is defined by the formula $f(x) = 8\sqrt{x}$, $x \in R$. The value of $f(144)$ is _____ .

(Record your answer in the numerical response box from left to right)

☐☐☐☐

Further assignment questions on Function Notation - Part One will appear in the assignment of the next lesson, Function Notation - Part Two.

Answer Key

1. a) name **b)** inputs **c)** outputs **d)** outputs
e) independent **f)** dependent **g)** dependent

2. a) 3 **b)** -22 **c)** -7 **3. a)** -10 **b)** -30 **c)** 3

4. a) 9 **b)** -342 **c)** $a^3 + 1$ **5. a)** 65 **b)** -47

6. a) 0 **b)** 16 **c)** 7 **d)** $8 - 4t$ **e)** $2 - 2a$

7. a) $3x^2 + 2x - 9$ **b)** $3x^2 - 32x + 76$

8. a) 10 **b)** $-\frac{9}{2}$ **c)** 15 **d)** -7 **e)** ± 5

9. a) -15 **b)** $12x - 9$ **c)** $x = 7$ **10. a) i)** 4 **ii)** $\frac{1}{6}$ **iii)** 5 **b)** 81

11. a) 26 **b)** ± 6 **12.** B **13.** 9 6

Functions Lesson #3:
Function Notation - Part Two

Graphing a Function

Consider the function $f(x) = 3x + 1$. The values of x represent the inputs and make up the domain of the function. The values of $f(x)$ represent the outputs and make up the range of the function.

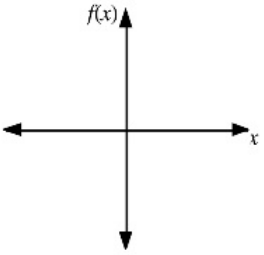
In previous lessons, we have used y to represent the outputs and the range of a relation. We can therefore write the function $f(x) = 3x + 1$ in x-y notation as $y = 3x + 1$.

The function $f(x) = 3x + 1$ can be written in x-y notation as shown.

Function notation	x-y notation
$f(x) = 3x + 1$	$y = 3x + 1$

- Values of the independent variable represent the **inputs** of a function and are shown on the **horizontal axis**.
- Values of the dependent variable represent the **outputs** of a function and are shown on the **vertical axis**.

Use a graphing calculator to sketch the graph of the function $f(x) = 3x + 1$.



a) In each case, express the relation given in function notation as an equation in two variables.

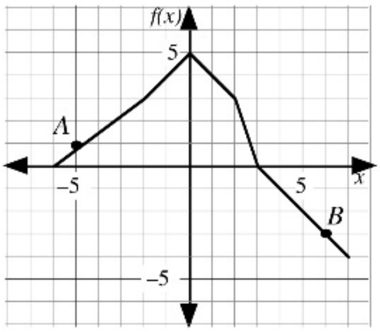
i) $f(x) = 7x - 23$ **ii)** $g(t) = t^2 - 2t + 35$

b) Express the relation $y = 11x - 15$ in function notation.

c) The graph of the function defined by $y = f(x)$ has equation $y = 4 - 3x$. Express the equation in function notation.

Class Ex. #3

The graph of a function f is shown.



a) Complete

i) $f(5) =$ **ii)** $f(-2) =$ **iii)** $f(4) =$

b) Write the ordered pairs associated in **a)**.

i) **ii)** **iii)**

c) State the value(s) of x if

i) $f(x) = -1$ **ii)** $f(x) = 3$ **iii)** $f(x) = 4$

d) Use the notation in **a)** to make a statement about the points A and B on the graph.

e) Write the x- and y- intercepts of the graph using function notation.

f) Complete the following statements.

- The domain of f is $\{x \mid$ _____ $\leq x \leq$ _____ $, x \in R\}$
- The range of f is $\{f(x) \mid$ _____ $\leq f(x) \leq$ _____ $, f(x) \in R\}$

Complete Assignment Questions #1 - #12

Assignment

1. In each case, express the relation given in function notation as an equation in two variables.

a) $f(x) = 10 - 3x$ **b)** $g(x) = 12x^2 - 5$ **c)** $P(t) = 2t + 9$

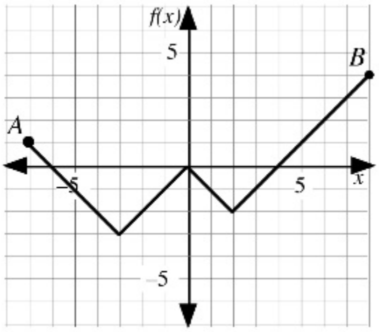
2. Express the following relations in function notation.

a) $y = 17x - 9$ **b)** $y = 4v + 25$ **c)** $x + 2y + 6 = 0$

3. a) The graph of the function defined by $y = f(x)$ has equation $y = 0.5x - 0.25$
Express the equation in function notation.

b) The graph of the velocity function defined by $v = f(t)$ has equation $v = 4.9t^2$.
Express the equation in function notation.

4. The graph of a function f is shown.



a) Complete
i) $f(3) =$ **ii)** $f(-3) =$ **iii)** $f(-6) =$

b) Write the ordered pairs associated with **a)**.
i) **ii)** **iii)**

c) State the value(s) of x if
i) $f(x) = 3$ **ii)** $f(x) = -2$ **iii)** $f(x) = -4$

d) Use the notation in a) to make a statement about the points A and B on the graph.

e) Write the x- and y- intercepts of the graph using function notation.

f) Complete the following statements.

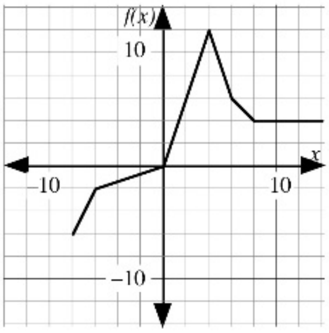
- The domain of f is $\{x \mid$ _____ $\le x \le$ _____ $, x \in R\}$
- The range of f is $\{f(x) \mid$ _____ $\le f(x) \le$ _____ $, f(x) \in R\}$

5. The function $g(x) = 3x^2 - 4$ has a domain $\{-2, -1, 0, 1, 2\}$.
a) State the range of g. **b)** Solve the equation $g(x) = -1$.

6. Consider the graph of the function f shown below.

a) Complete the table.

b) Explain why the solution to the equation $f(x) = 4$ has an infinite number of solutions.



x	$f(x)$	Ordered Pair
		(2,)
	0	
–6		
8		
	–6	
10		

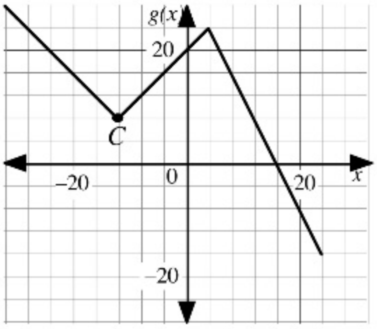
7. Given that $f(x) = 9 - 2x$

a) evaluate $f(-3)$

b) find the value of $f(t) + f(-t)$

c) calculate the x-intercept and the y-intercept on the graph of f.

8. The graph of a function is shown.



a) A student is asked to make a statement about point C on the graph. The student states that $f(-3) = 2$.

i) Explain **two** errors in the student's statement.

ii) Write a correct statement using function notation about point C.

b) Give the solution to the following equations.

i) $g(x) = -8$ **ii)** $g(x) = 16$.

c) State the value of **i)** $g(-8)$ **ii)** $g(16)$

d) State the domain and range of the function.

e) The equation $g(a) = b$ has **exactly two** solutions. Explain clearly how to use the graph to determine values of a and b, and provide two sets of answers to the problem.

9. Consider the function $f(x) = 1 - x^2$, where x is an integer.
a) Evaluate $f(2) - f(-1)$ **b)** Given that $f(a) = -8$, calculate all possible values of a.

Multiple Choice **10.** The graph of the function $f(x) = 4^x$, $x \in R$, intersects the y-axis at

A. $(0, 0)$
B. $(0, 1)$
C. $(0, 4)$
D. no point

Use the following information to answer the next question.

Function P is such that $P(5) = -1$.
Two students each make a statement about the function P.

- Rose states "When the domain value is 5, the related range value is -1."
- Susan states "The point $(-1, 5)$ is on the graph of $y = P(x)$."

11. Which of the following is true?

A. Both statements are correct.
B. Both statements are incorrect.
C. Rose is correct and Susan is incorrect.
D. Susan is correct and Rose is incorrect.

Numerical Response **12.** Consider the graph of the function $f(x) = 5x - 11$. The x-intercept of the graph of f is located at $(a, 0)$. The value of a is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $y = 10 - 3x$ **b)** $y = 12x^2 - 5$ **c)** $y = 2t + 9$

2. **a)** $f(x) = 17x - 9$ **b)** $f(v) = 4v + 25$ **c)** $f(x) = -\frac{1}{2}x - 3$

3. **a)** $f(x) = 0.5x - 0.25$ **b)** $f(t) = 4.9t^2$

4. **a)** **i)** -1 **ii)** -3 **iii)** 0
- **b)** **i)** $(3, -1)$ **ii)** $(-3, -3)$ **iii)** $(-6, 0)$
- **c)** **i)** 7 **ii)** $-4, -2, 2$ **iii)** no solution **d)** A is $f(-7) = 1$, B is $f(8) = 4$
- **e)** x-intercepts can be represented in function notation by; $f(-6) = 0, f(0) = 0, f(4) = 0$
  y-intercept can be represented in function notation by $f(0) = 0$
- **f)** $-7 \leq x \leq 8$, $-3 \leq f(x) \leq 4$

5. **a)** Range $= \{-4, -1, 8\}$ **b)** $x = \pm 1$

6. See table below.

x	$f(x)$	Ordered Pair
2	6	(2, 6)
0	0	(0, 0)
−6	−2	(−6, −2)
8	4	(8, 4)
−8	−6	(−8, −6)
10	4	(10, 4)

b) The horizontal line where $f(x) = 4$ has an infinite number of input values between 8 and 14.

7. **a)** 15 **b)** 18 **c)** x-int $= \frac{9}{2}$, y-int $= 9$

8. **a)** **i)** The name of the function is g not f. The scale is 4 units per box, not 1 unit per box.
- **ii)** $g(-12) = 8$
- **b)** **i)** $x = 20$ **ii)** $x = -20, -4, 8$
- **c)** **i)** 12 **ii)** 0
- **d)** Domain $= \{x \mid -32 \leq x \leq 24, x \in R\}$, $\{g(x) \mid -16 \leq g(x) \leq 28\}, g(x) \in R$
- **e)** A horizontal line must intersect the graph at exactly two points.
  This occurs when $g(x) = 24$ and when $g(x) = 8$.
  Solution 1: $b = 24$ when $a = -28$ or 4.
  Solution 2: $b = 8$ when $a = -12$ or 12

9. **a)** -3 **b)** ± 3 **10.** B **11.** C **12.**

2	.	2	

Functions Lesson #4:
Function Notation and Problem Solving

Using Function Notation

In the previous unit we solved problems about relations defined by an equation.
In this lesson we will solve problems where function notation is used to define the relation.

On page 256, assignment question #6, we had the following scenario.

" A candle manufacturer found that their "Long-Last" candles melted according to the formula $h = -2t + 12$, where h is the height of the candle, in cm, after t hours."

The relation between height and time is described by an **equation.**

The relation is a function because for each input there is only one output, and so it can be described using the **function notation** below.

" A candle manufacturer found that their "Long-Last" candles melted according to the formula $h(t) = -2t + 12$, where h is the height of the candle, in cm, after t hours."

In this example, the notation $h(4)$ is a simplified way of representing the height of the candle after four hours.

A candle manufacturer found that their "Long-Last" candles melted according to the formula $h(t) = -2t + 12$, where h is the height of the candle, in cm, after t hours.

a) Use a graphing calculator to sketch the graph of the function and show the graph on the grid

b) Determine the value of $h(5)$.

c) Write in words the meaning of $h(5)$.

d) Evaluate the following, and explain the significance of each.

i) $h(0)$ **ii)** $h(6)$ **iii)** $h(8)$

e) How long will it take for the candle to burn down to a height of 7 cm?

f) Suggest an appropriate domain and range for the function.

Complete Assignment Questions #1 - #4

Assignment

1. Ivory the botanist treated a 2 cm plant with a special growth fertilizer. With this fertilizer, the plant grew at a rate modelled by the function $H(t) = \frac{5}{3}t + 2$, where $H(t)$ represents the height of the plant in cm after t days.

a) Use a graphing calculator to sketch the graph of the function and show the graph on the grid.

b) Determine the value of $H(3)$.

c) Write in words the meaning of $H(3)$.

d) Evaluate the following.

i) $H(0)$ **ii)** $H(6)$ **iii)** $H(21)$

e) How long will it take for the plant to reach a height of 21 cm?

f) It takes 27 days for the plant to mature (to reach maximum height). State the domain and range of the function $H(t)$.

2. The cost to Inner Technology of producing IT graphing calculators can be modelled by the function $C(n) = 11750 + 32n$, where $C(n)$ represents the cost in dollars of producing n calculators.

a) Sketch the graph of the function for a maximum of 4000 calculators.

b) Determine the value of $C(30)$.

c) Write in words the meaning of $C(30)$.

d) Evaluate $C(0)$ and explain its significance.

e) How many calculators can be produced for $31 270?

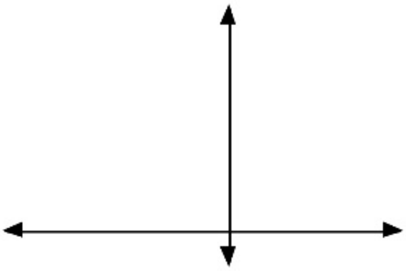
f) Last month IT produced 2 600 calculators and spent $14 000 on advertising. If there are other fixed monthly costs of $24 500, and each calculator sells for $165, how much profit would be made if all the calculators are sold?

3. Over the last 10 years, data was recorded for the number of cups of hot chocolate sold at BGB Senior High School. It was found from the data that the warmer the weather, the less cups of hot chocolate were sold. The data can be modelled by the formula $N(t) = 150 - 10t$, where $N(t)$ is the daily number of cups of hot chocolates sold when the average daily temperature is t °C.

a) Sketch the graph of the function on the grid provided.

b) Determine the value of $N(-5)$.

c) Write in words the meaning of $N(-5)$.



d) What was the average temperature if 190 cups of hot chocolate were sold?

e) Explain how to estimate the lower limit of the domain of the relation.

f) Suggest an appropriate domain and range for the function $N(t)$ if BGB High School is located in the Okanagan, British Columbia.

4. A special type of weather balloon follows a path which can be represented by the formula $h(t) = -9t^2 + 900t$, where $h(t)$ is the height in cm after t minutes.

a) Sketch the graph of the function on the grid.

b) Determine the value of $h(30)$ and $h(70)$.

c) Does $h(30) = h(70)$? Do they mean the same thing? Explain.

d) Evaluate the following, and explain their significance in the context of the question.

i) $h(0)$ **ii)** $h(100)$ **iii)** $h(110)$

e) What is the highest point the balloon will reach?

f) When will the balloon land?

g) Suggest an appropriate domain and range for the function $h(t)$?

Answer Key

1. **b)** 7 **c)** After 3 days the height is 7 cm. **d)** i) 2 ii) 12 iii) 37 **e)** 11.4 days
f) domain $\{t \mid 0 \le t \le 27, t \in R\}$ range $\{H(t) \mid 2 \le H(t) \le 47, H(t) \in R\}$

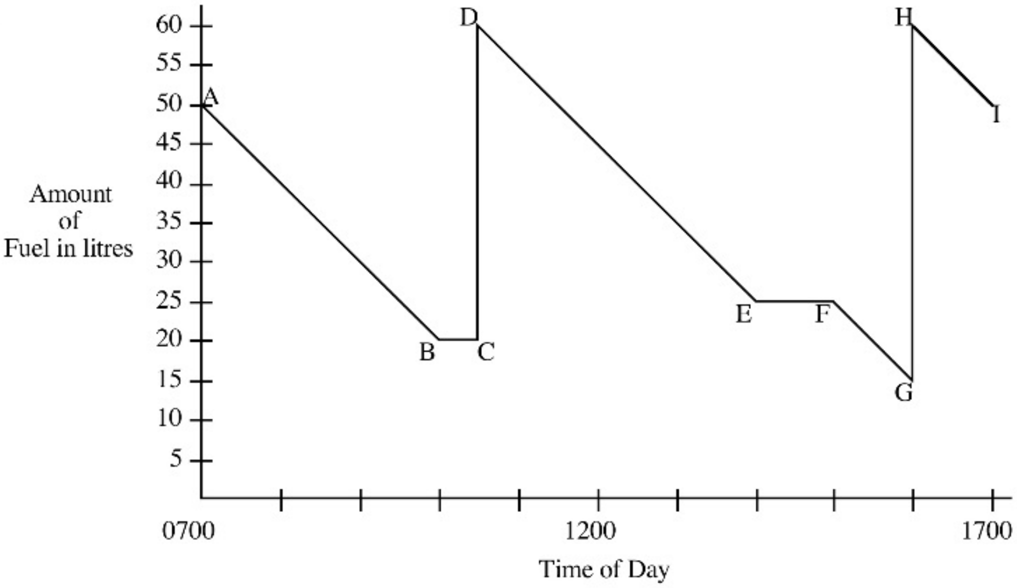
2. **b)** 12710 **c)** It costs \$12 710 to produce 30 calculators.
d) $C(0) = 11\ 750$. There are fixed costs of \$11750 before any calculators are produced.
e) 610 **f)** \$295 550

3. **b)** 200 **c)** 200 cups are sold when the average temperature is –5°C. **d)** –4°C
e) Estimate the minimum average daily temperature.
f) Answers may vary. domain $\{t \mid -20 \le t \le 15, t \in R\}$ range $\{N(t) \mid 0 \le N(t) \le 350, N(t) \in W\}$

4. **b)** both = 18 900
c) They are equal but do not represent the same thing. $h(30)$ is the height after 30 minutes. and $h(70)$ is the height after 70 minutes
d) i) 0 Initial height = 0 m ii) 0 After 100 min the balloon has landed on the ground.
iii) –9900 this has no meaning since the balloon has already landed
e) 22 500 cm = 225 m **f)** after 100 min
g) domain $\{t \mid 0 \le t \le 100, t \in R\}$ range $\{h(t) \mid 0 \le h(t) \le 22\ 500, h(t) \in R\}$

Functions Lesson #5:
Interpreting Graphs of Functions

The Carter Family are driving to the Yukon for a family vacation. The graph represents the amount of fuel (in litres) in the gas tank of their car on the first day of their journey.



The graph of the journey is divided into eight line segments.

a) With reference to the journey, explain what is happening between:

i) A and B

ii) B and C

iii) C and D

b) What is the rate of fuel consumption (in litres per hour) between D and E?

c) Which line segment represents the car being refueled for the second time?

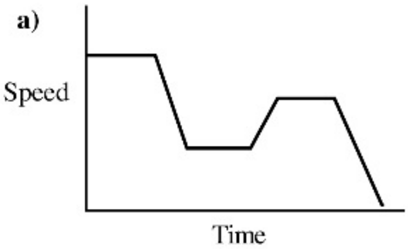
d) Calculate the total time when the car was driven.

e) If fuel costs 85¢ per litre, calculate the cost of the fuel used for the first day of the journey.

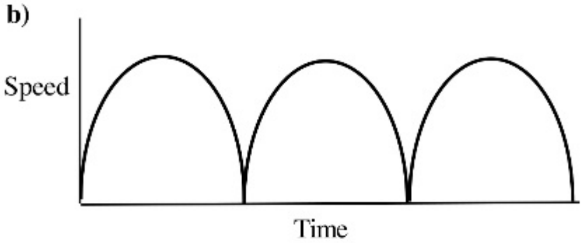
Class Ex. #2

Suggest a possible scenario for each of the following graphs:

a)



b)



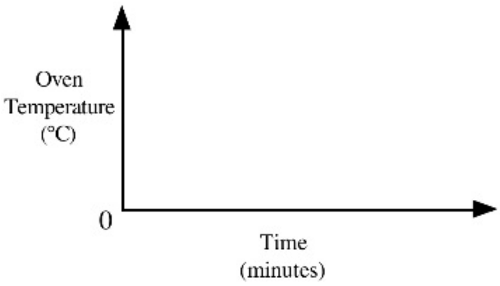
Sketching a Graph

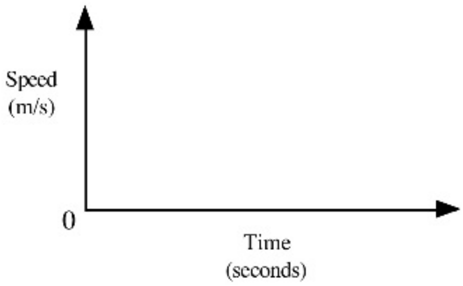
Class Ex. #3

Sketch a graph with no scale for each of the following

a) the oven temperature when baking a pie

b) Ben taking part in a 100 m sprint

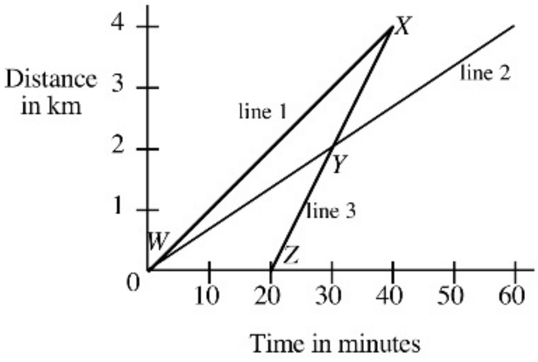




Complete Assignment Questions #1 - #12

Assignment

1. Amanda, Brittany, and Chelsea, each follow the same route to school. One morning Amanda cycles to school, Brittany walks to school, and Chelsea runs to school.
Lines 1, 2, and 3, on the graph represent the three routes.



a) Complete the table below.

	Line 1	Line 2	Line 3
Distance (km)	4		
Time (hrs)	$\frac{2}{3}$		
Rate (km/hr)	6		
Student			

b) Explain what is happening at the following points.

i) *W*

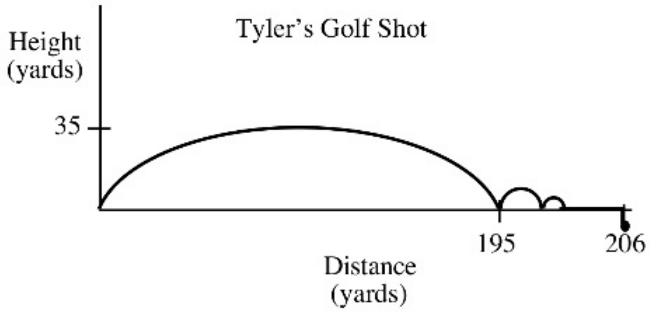
ii) *X*

iii) *Y*

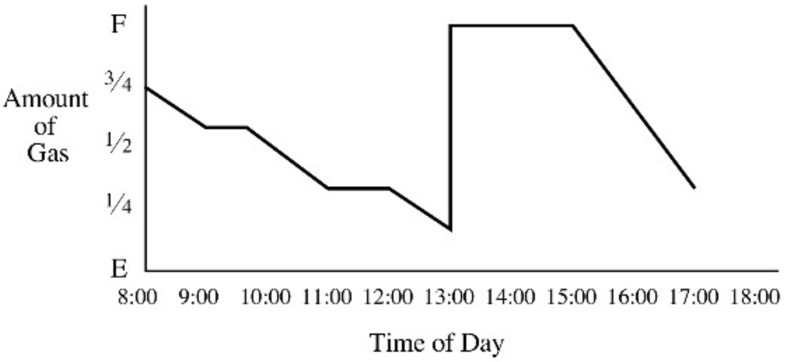
iv) *Z*

c) How can you tell from the steepness of the lines which line represents the route of each student?

2. Tyler, a member of St. Andrews High School golf team, hits a golf ball. The graph shows the path of the ball. Describe Tyler's golf shot.

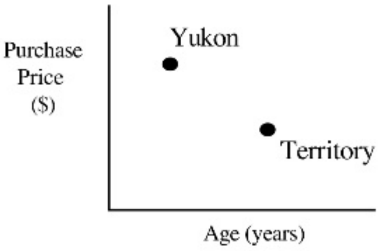


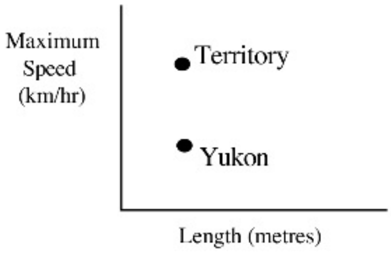
3. Dar sells medical supplies. The graph shows the amount of gasoline in his car during a particular day.



Describe how Dar may have spent the day.

4. The two graphs shown compare two yachts: the Yukon and the Territory. The first graph compares the yachts by age and cost. The second graph compares the boats by speed and length. Describe the comparison between the two yachts.

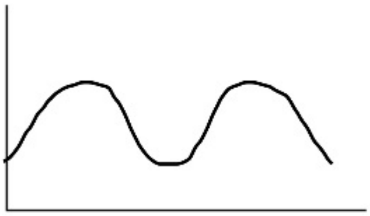




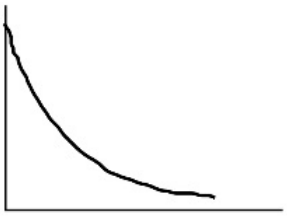
5. A super ball is dropped from a 10 m building. On each bounce, it bounces back to 80% of its previous height. Create a graph of height as a function of time.

6. Suggest a possible scenario for each of the following graphs.

a)

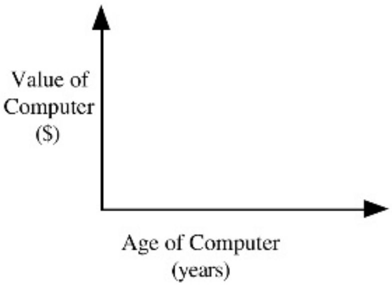


b)

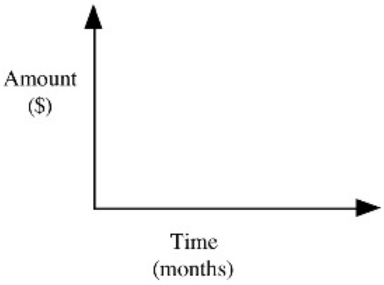


7. Sketch a graph with no scale to represent each of the following.

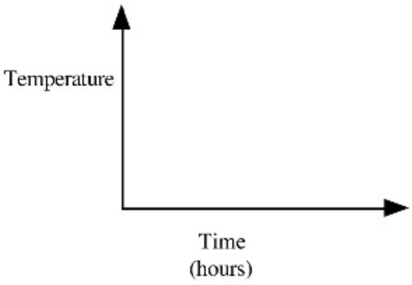
a) A computer's value compared to its age in years.


Value of
Computer
($)
Age of Computer
(years)

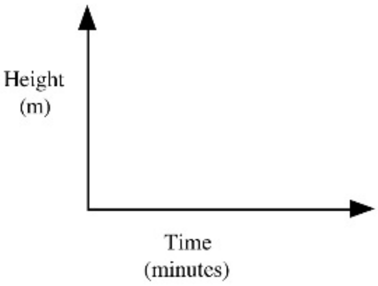
b) The amount of your savings if you save $10 every month for a period of six months.


Amount
($)
Time
(months)

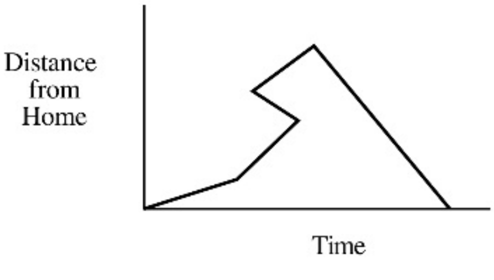
c) The air temperature during a spring day from 6:00 a.m. to 6:00 p.m.


Temperature
Time
(hours)

d) You are sitting in the bottom chair of a ferris wheel. Graph your height above the ground during two rotations of the wheel.


Height
(m)
Time
(minutes)

8. A student drew the following graph to represent a journey. Explain why the graph must be incorrect.



Matching Match each description on the left with the best graph on the right. Each graph may be used once, more than once, or not at all.

Description

9. Sketch a graph of a person's height as a function of their age.

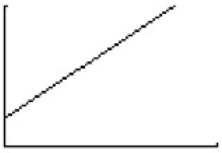
10. The number of hours of daylight in a given town in northern BC depends on the day of the year. Sketch a graph of the number of hours of daylight as a function of day of the year.

11. Sketch a graph of the area of a circle as a function of its radius.

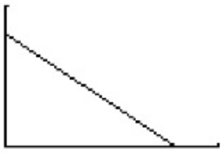
12. You start driving at a constant speed with a full tank of gas. Sketch a graph of litres of gas in the tank as a function of distance travelled.

Graph

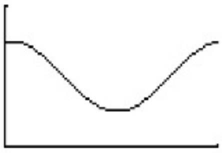
A.



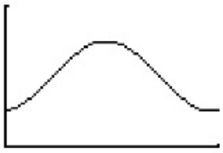
B.



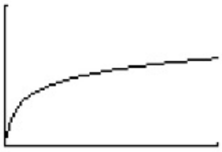
C.



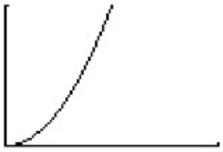
D.



E.



F.



Answer Key (Answers may vary)

1. a) see table below

	Line 1	Line 2	Line 3
Distance (km)	4	4	4
Time (hrs)	$2/3$	1	$\frac{1}{3}$
Rate (km/hr)	6	4	12
Student	Chelsea	Brittany	Amanda

b) i) Chelsea and Brittany leave home at the same time.

ii) Chelsea and Amanda arrive at school.

iii) Amanda overtakes Brittany.

iv) Amanda leaves home 20 minutes after Brittany and Chelsea.

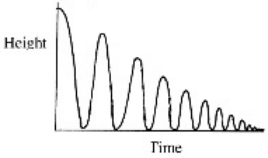
c) The steeper the graph, the less time is taken to travel to school. The steepest slope represents the cyclist Amanda, the next steepest slope represents the runner Chelsea, and the remaining line represents the walker Brittany.

2. Tyler hits the ball through the air for a distance of 195 yards. The ball bounces twice and rolls into the hole. The golf shot travelled a total of 206 yards, and had a maximum height of 35 yards.

3. Dar left home at 8:00 AM with $\frac{3}{4}$ tank of gas in his car. He drove for about one hour, had a meeting for about $\frac{1}{2}$ hour, drove for about $1\frac{1}{2}$ hours, had a second meeting for one hour, and drove for about one hour. He refueled at 1 pm and had a lunch meeting for about 2 hours. He then drove home and arrived about 5 p.m. with a quarter tank of gas left.

4. The Territory is older than the Yukon, and its purchase price was less. Both the Territory and the Yukon are the same length, but the Territory can achieve a greater maximum speed.

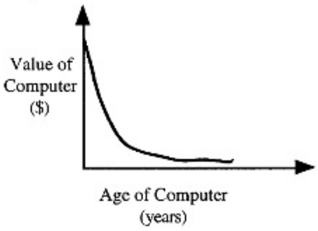
5. see graph below



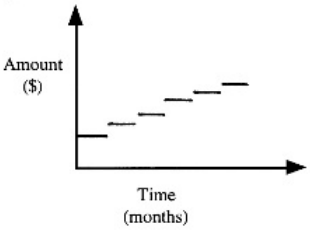
6. a) The number of hours of daylight per day over a period of two years for a location in the northern hemisphere.

b) The value of a car depreciating over time.

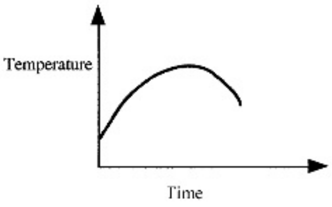
7. a)



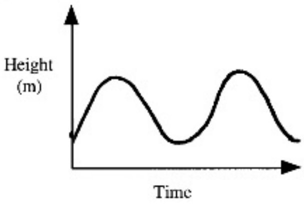
b)



c)



d)



8. The graph is not a function. The person cannot be at three different places at the same time.

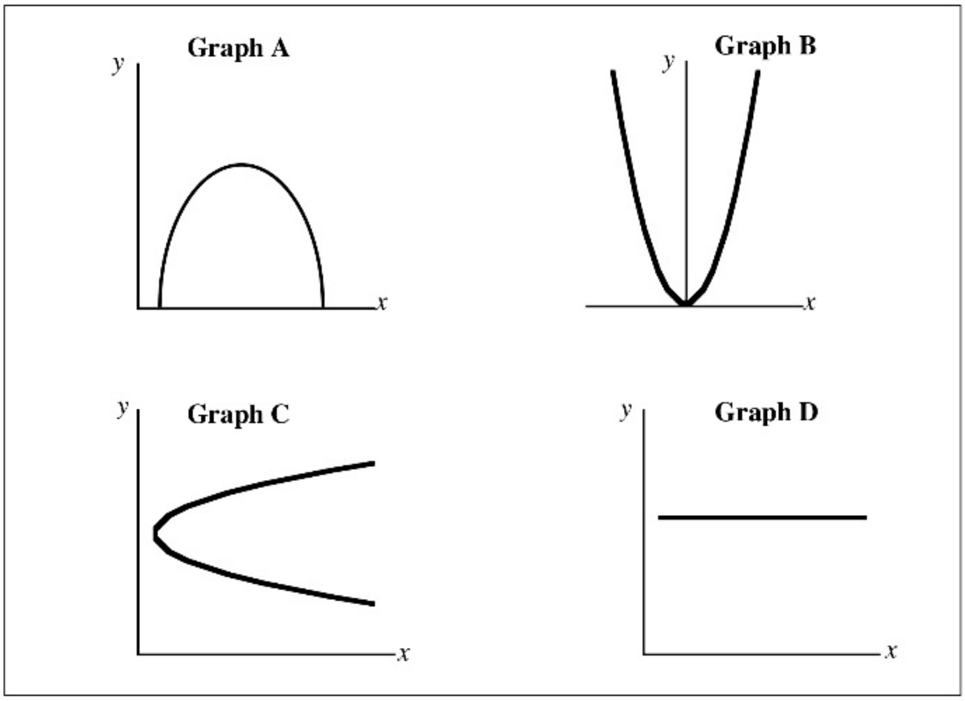
9. E **10.** D **11.** F **12.** B

Functions Lesson #6:
Practice Test

1. Which of the following cannot be used to represent a function?

- **A.** Graph
- **B.** Table of Values
- **C.** Ordered Pairs
- **D.** Coordinate

Use the following information to answer the next question.


Graph A
y
x
Graph B
y
x
Graph C
y
x
Graph D
y
x

2. Which of the graphs show a relation which is not a function?

- **A.** Graph A
- **B.** Graph B
- **C.** Graph C
- **D.** Graph D

Numerical Response **1.** The function $f(x) = 2 + x^2$ has domain $\{-3, -2, -1, 0, 1, 2\}$.

The difference between the largest element of the range and the smallest element of the range is _____ .

(Record your answer in the numerical response box from left to right)

Use the following information to answer the next question.

Four relations are represented by the sets of ordered pairs shown.

I. $\{(1,1),(2,2),(3,3)\}$

II. $\{(-1,0),(0,-1)\}$

III. $\{(2,6),(3,-8),(-2,6),(0,0)\}$

IV. $\{(3,4),(5,7),(5,9),(7,10)\}$

3. The relations which are functions are

A. I only
B. I and II only
C. I, II, III only
D. some other combination of I, II, III, IV

4. The graph of a relation results in a straight line. The relation is not a function if the line

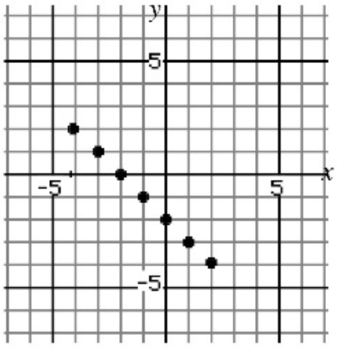
A. is horizontal
B. is vertical
C. increases from left to right
D. decreases from left to right

5. The function $f(x) = 3 - 2x^2$ has domain $\{-6, -4, 0, 2, 5\}$.
Which of the following is an element of the range of the function?

A. 35
B. 27
C. 1
D. -29

Use the following information to answer the next question.

The diagram shows the graph of the function $y = f(x)$.



6. The value of $f(-2) + f(2)$ is

A. -4
B. -2
C. 0
D. $f(0)$

Use the following information to answer questions #7 and #8.

Consider the function $P(x) = 5x + 2$.

7. The value of $P(6)$ is

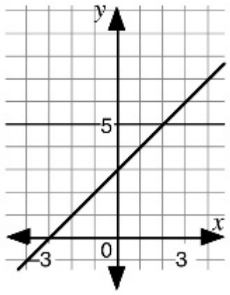
A. $\frac{8}{5}$
B. $\frac{4}{5}$
C. 32
D. 58

8. If $P(b) = 6$, then $b =$

A. $\frac{8}{5}$
B. $\frac{4}{5}$
C. $\frac{5}{4}$
D. 32

Use the following information to answer the next two questions.

The graph of a function $y = f(x)$ is shown.



Numerical Response **2.** The value of $f(-3) + f(0)$ is _____ .

(Record your answer in the numerical response box from left to right)

9. If $f(a) = 1$ and $f(b) = 4$, then the value of $b - 2a$ is

A. -3
B. -1
C. 2
D. 5

10. Consider the graph of the function $f(x) = 4x - 10$. The x-intercept of the graph of f is

A. -10
B. -2.5
C. 2.5
D. 10

Numerical Response **3.** Consider the graph of the function $P(x) = 2x^2 - 16$.
The x-intercepts of the graph of P are located at $(m, 0)$ and $(-m, 0)$.
The value of m, to the nearest tenth, is _____ .

(Record your answer in the numerical response box from left to right)

Use the following information to answer the next question.

Consider the following functions.

1. $p(x) = x^2 - 4x - 2$

2. $p(x) = \frac{1}{3}x + 14$

3. $p(x) = 3x^2 + x$

4. $p(x) = 7 - 5x$

11. For each function evaluate $p(-3)$, and put the expressions in order from greatest to least. The order is

A. 4312
B. 3412
C. 3124
D. none of the above

12. Given a function g defined by $g(x) = px + q$ with $g(0) = 2$ and $g(1) = 3$ then

A. $p = 1,\ q = 0$
B. $p = 3,\ q = 2$
C. $p = 1,\ q = 2$
D. $p = 3,\ q = 0$

13. If $f(x) = 3^x$ and $f(-a) = \frac{1}{81}$, then $a =$

A. 4
B. –4
C. –27
D. 27

14. If $g(x) = \frac{2}{3}x + 6$, an expression for $g(2x - 1)$ is

A. $\frac{4}{3}x + 5$

B. $\frac{4}{3}x + \frac{16}{3}$

C. $\frac{4}{3}x + 11$

D $\frac{8}{3}x + 5$

Numerical Response **4.** If $f(x) = 1 - 2x - 5x^2$, and if $f(x + 2)$ is written in the form $ax^2 + bx + c$, the value of $a - b - c$ is _____ .

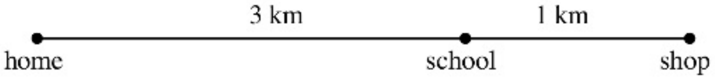
(Record your answer in the numerical response box from left to right)

Numerical Response **5.** $f(a) = \frac{a}{a+4}$. The exact value of $f(5) - f(5^{-1})$ written as a rational number in simplest form is $\frac{p}{q}$. The value of p is _____ .

(Record your answer in the numerical response box from left to right)

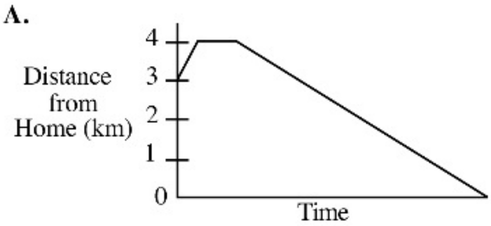
Use the following information to answer the next question.

Melanie leaves school and runs to the shop. She spends some time in the shop and walks home.

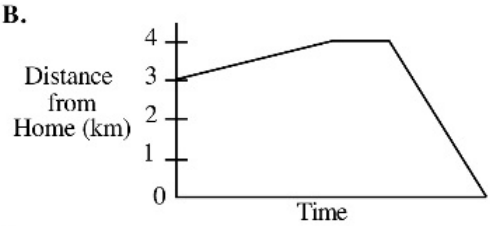


15. Which graph best describes Melanie's distance from home starting from when she left school?

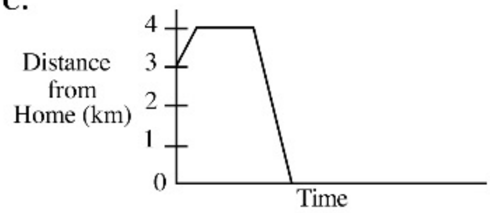
A.



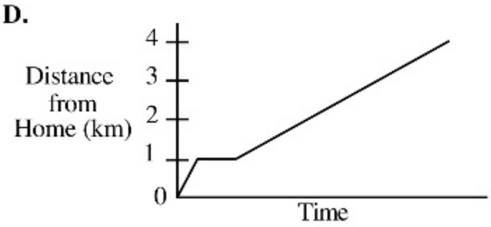
B.



C.



D.



Written Response - 5 marks

1. As part of an experiment, a 50 kg steel ball is dropped from a Canadian Air Force jet.

The height of the steel ball above the ground can be described by part of the graph of the function $h(t) = 2575 - 4.9t^2$, where $h(t)$ is the height, in metres, of the steel ball after t seconds.

- Sketch the graph of the function on the grid.
- Determine the value of $h(30)$ and explain why $h(30)$ does not represent the height of the ball after 30 seconds?

- At what height is the ball dropped from the jet?

- How long (to the nearest second) will it take the ball to make contact with the ground?

- Suggest an appropriate domain and range for the function $h(t)$.

Answer Key

1. D	**2.** C	**3.** C	**4.** B	**5.** D	**6.** A	**7.** C	**8.** B
9. D	**10.** D	**11.** B	**12.** C	**13.** A	**14.** B	**15.** A	

Numerical Response

1.	9			
2.	3			
3.	2	.	8	
4.	4	0		
5.	3	2		

Written Response

1.
- -1835. The ball has already hit the ground, so the function no longer represents the height of the ball.
- 2575 m
- 23 seconds
- domain $\{t \mid 0 \le t \le 23, t \in R\}$ range $\{h(t) \mid 0 \le h(t) \le 2575, h(t) \in R\}$

Characteristics of Linear Relations Lesson #1:
Line Segments on a Cartesian Plane

Lesson 1 and Lesson 2 of this unit are not required for this curriculum, but are included because

1. they are important characteristics of linear relations not covered elsewhere
2. this information is required in higher level math courses such as Calculus

Unit Overview

The graph of a linear relation is represented by a straight line. The line can be infinite or finite depending on the domain and range of the linear relation. In some cases we are only interested in a portion of a line. This portion is called a **line segment**.

We have already studied some of the characteristics of the graph of a linear relation: intercepts, domain, and range. In this unit we study some characteristics of line segments: namely, length, midpoint, distance, and slope. We demonstrate an understanding of slope with respect to rise and run, the slope formula, and rate of change. We then discuss the slopes of parallel and perpendicular lines.

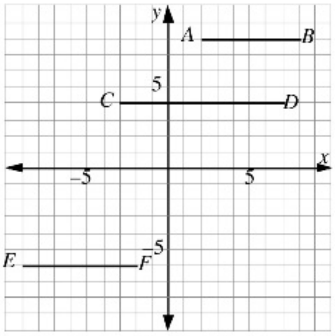
Line Segment

A line segment is the portion of a line between two points on the line.

If the endpoints of a line segment are A and B, we refer to it as line segment AB.

NOTE: Line segment AB may also be written as $\overline{AB}$.

Length of a Horizontal Line Segment



Consider the line segments shown on the grid.

a) Find the length of each line segment by counting.
- length of AB is _____ units.
- length of CD is _____ units.
- length of EF is _____ units.

b) Determine the coordinates of the endpoints of each line segment.
- $AB \rightarrow A(\quad , \quad) \quad B(\quad , \quad)$
- $CD \rightarrow C(\quad , \quad) \quad D(\quad , \quad)$
- $EF \rightarrow E(\quad , \quad) \quad F(\quad , \quad)$

c) Complete the following.
- The difference in the x-coordinates, $x_B - x_A$, is _____ .
- The difference in the x-coordinates, $x_D - x_C$, is _____ .
- The difference in the x-coordinates, $x_F - x_E$, is _____ .

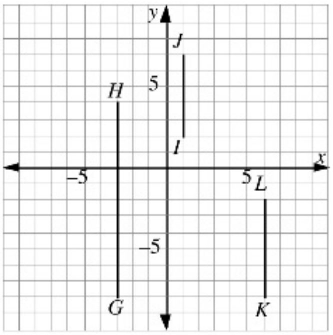
d) How can the coordinates of the end points of a horizontal line segment be used to find the length of the line segment?

Class Ex. #1

a) Line segment AB has endpoints $A(2, 8)$ to $B(-5, 8)$. Determine the length of $\overline{AB}$.

b) Determine the length of the line segment from $P(a-2, b)$ to $Q(a+4, b)$.

Length of a Vertical Line Segment



Consider the line segments shown on the grid.

a) Find the lengths of each line segment by counting.
- length of GH is_____ units.
- length of IJ is _____ units.
- length of KL is _____ units.

b) Determine the coordinates of the endpoints of each line segment.
- $GH \rightarrow G(\quad,\quad)\quad H(\quad,\quad)$
- $IJ \rightarrow I(\quad,\quad)\quad J(\quad,\quad)$
- $KL \rightarrow K(\quad,\quad)\quad L(\quad,\quad)$

c) Complete the following.
- The difference in the y-coordinates, $y_H - y_G$, is _____ .
- The difference in the y-coordinates, $y_J - y_I$, is _____ .
- The difference in the y-coordinates, $y_L - y_K$, is _____ .

d) How can the coordinates of the end points of a vertical line segment be used to find the length of the line segment?

Class Ex. #2

a) Line segment RS has endpoints $R(1, -4)$ to $S(1, -9)$. Determine the length of $\overline{RS}$.

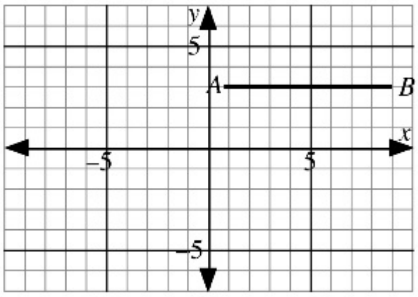
b) Determine the length of the line segment from $P(a, b)$ to $Q(a, b+10)$.

Complete Assignment Questions #1 - #5

Midpoint

The **midpoint, M,** of a line segment on the graph of a linear relation is the point at the centre of the line segment.

Midpoint of a Horizontal Line Segment

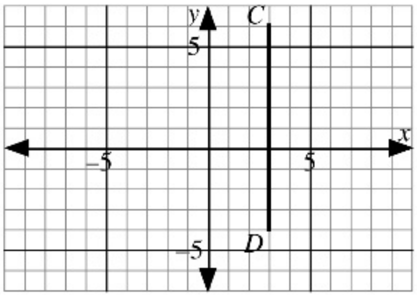


Consider the line segment AB shown on the grid.

a) Determine the coordinates of the midpoint by counting. Label the midpoint, M, on the grid and list the coordinates beside it.

b) List the coordinates of point A and point B on the grid. How can the x-coordinates of points A and B be used to find the coordinates of the midpoint of a horizontal line?

Midpoint of a Vertical Line Segment

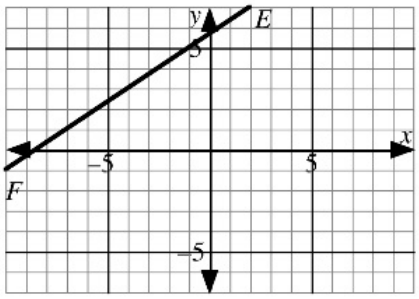


Consider the line segment CD shown on the grid.

a) Determine the coordinates of the midpoint by counting. Label the midpoint, M, on the grid and list the coordinates beside it.

b) List the coordinates of point C and point D on the grid. How can the y-coordinates of points C and D be used to find the coordinates of the midpoint?

Midpoint of an Oblique (Diagonal) Line Segment



Consider the line segment EF shown on the grid.

a) Use the results from above to determine the midpoint of EF.

b) Express in words how to find the midpoint, M, of the line segment joining the points (x_1, y_1) and (x_2, y_2).

c) Complete the formula to express the relationship in **b)**. $x_M =$ $y_M =$

Midpoint of a Line Segment

Consider line segment PQ with endpoints $P(x_1, y_1)$ and $Q(x_2, y_2)$.

The midpoint, M, of the line segment has coordinates.

$$\boxed{M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$

Note Line segment PQ can also be written as $\overline{PQ}$.

Class Ex. #3

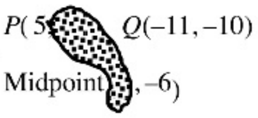
Determine the coordinates of the midpoint of the line segment with the given pair of endpoints.

a) $P(4, 7)$, $Q(12, 3)$ **b)** $E(-5, 7)$, $F(-11, -2)$ **c)** $A(w + 3, 2w)$, $C(5w - 1, 7w + 1)$

Class Ex. #4

Ruby was doing a question in her coordinate geometry homework, and her little brother Max wrote over part of the question as a prank.

Calculate the missing coordinates.



Complete Assignment Questions #6 - #17

Assignment

1. Determine the length of each line segment.

a) $A(2,7)$ to $B(5,7)$

b) $C(-5,3)$ to $D(-5,12)$

c) $I(-3,-8)$ to $J(-3,-3)$

d) $K(7,-10)$ to $L(-35,-10)$

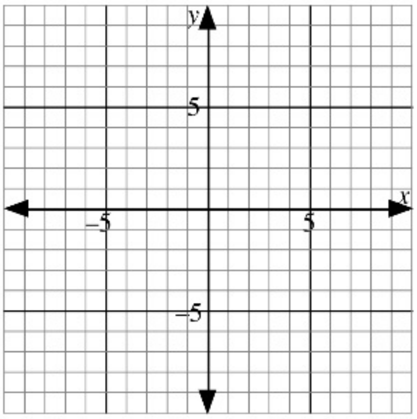
2. Determine whether each line segment is horizontal or vertical, and write an expression for its length.

a) $A(p,q)$ to $B(p-4,q)$

b) $C(m-3,n+5)$ to $D(m-3,n+12)$

3. A triangle has vertices $P(-4,-3)$, $Q(9,-3)$, and $R(1,5)$.

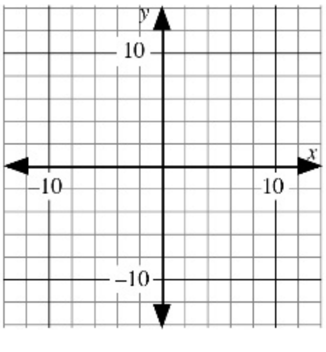
- Sketch the triangle on the grid.
- Calculate the area of the triangle.



4. On the grid, plot the points $P(-6,6)$, $Q(-6,-10)$, and $R(6,-10)$.

a) Determine the distance from P to R using the Pythagorean Theorem.

b) Calculate the area and perimeter of $\triangle PQR$.



5. Rebecca uses quadrant I in a Cartesian plane to describe the location of the bases in a game of high school softball. The four bases form a square. The origin is at home plate. First base is at $(18, 0)$, and the distance between each base is 18 m. The pitcher's mound is located between home plate and second base.

a) State the coordinates of second base.

b) The pitcher stands on the mound 12 m from home plate. If she has to throw a ball to second base, what distance to the nearest tenth of a metre, would she throw the ball?

6. Determine the coordinates of the midpoint of the line segment with the given pair of endpoints.

a) $A(2, 6),\ C(4, 16)$ **b)** $X(-3, -8),\ Y(-11, 0)$ **c)** $K(15, -17),\ L(-11, 3)$

7. Determine the coordinates of the midpoint of the line segment with the given pair of endpoints.

a) $C(3x, 8y),\ D(7x, -4y)$ **b)** $S(a + b, a + 7b),\ T(a + b, a - 3b)$

8. Otto was given two points: $A(-6, 4)$ and $B(12, -8)$. He was asked to divide $\overline{AB}$ into four equal parts. State the coordinates of the points which will divide $\overline{AB}$ into four equal parts.

9. In each case M is the midpoint of $\overline{AB}$. Determine the value of x.

a) $A(2,6)$, $B(6,x)$, $M(4,-1)$ **b)** $A(3,6)$, $B(x,0)$, $M(0,3)$

Multiple Choice

10. $ABCD$ is a square with vertices $\left(\sqrt{5},0\right)$, $\left(0,\sqrt{5}\right)$, $\left(-\sqrt{5},0\right)$, and $\left(0,-\sqrt{5}\right)$ respectively. The area of the square, in unit^2, is

A. 5
B. 10
C. 20
D. 100

11. $P(4,-8)$ and $Q(-2,10)$ are the endpoints of a diameter of a circle.
The coordinates of the centre of the circle are

A. $(-3,9)$
B. $(2,2)$
C. $(3,-9)$
D. $(1,1)$

12. AB is a diameter of a circle; the centre is C.
If $A(8,-6)$ and $C(5,-2)$ then B is the point

A. $(2,2)$
B. $(6.5,-4)$
C. $(11,-10)$
D. $(13,-8)$

13. Which statement is always true?

A. Two line segments of equal length have the same midpoint.
B. Two line segments with the same midpoint are of equal length.
C. A point equidistant from the endpoints of a line segment is the midpoint.
D. None of the above statements is always true.

Numerical Response

14. To the nearest tenth, the perimeter of $\triangle PQR$ with vertices, $P(3,8)$, $Q(3,0)$, and $R(-1,8)$ is _____.

(Record your answer in the numerical response box from left to right)

15. The midpoint of line segment ST is $M\left(\frac{1}{2},-4\right)$. If the coordinates of T are $(-3,3)$, and the coordinates of S are (x,y), the value of x is _____.

(Record your answer in the numerical response box from left to right)

16. The point $M(a,6)$ is the midpoint of $\overline{GH}$ with $G(22,b)$ and $H(6,-8)$. The value of $a+b$ is _____.

(Record your answer in the numerical response box from left to right)

17. The midpoint of line segment AB lies on the y-axis. A lies on the x-axis, and B has coordinates $(-4,5)$. The length of AB, to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** 3 **b)** 9 **c)** 5 **d)** 42 **2.** **a)** horizontal, 4 **b)** vertical, 7

3. 52 units^2 **4.** **a)** 20 units **b)** area = 96 units^2, perimeter = 48 units

5. **a)** (18,18) **b)** 13.5 m **6.** **a)** (3, 11) **b)** (−7, −4) **c)** (2, −7)

7. **a)** $(5x, 2y)$ **b)** $(a+b, a+2b)$ **8.** $\left(-\frac{3}{2}, 1\right)$, $(3, -2)$, $\left(\frac{15}{2}, -5\right)$

9. **a)** −8 **b)** −3 **10.** B **11.** D **12.** A **13.** D

14.

2	0	.	9

15.

4			

16.

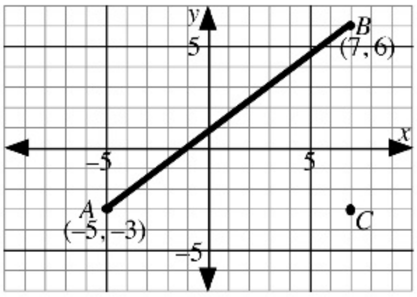
3	4		

17.

9	.	4	

Characteristics of Linear Relations Lesson #2:
The Distance Formula

Investigation



Consider line segment AB shown on the grid.

a) Use the Pythagorean theorem to show that the length of AB is 15 units.

b) Complete the following:

length of $AC = x_B - x_A = 7 - \_\_\_ = \_\_\_\_$

length of $CB = y_B - y_A = \_\_\_ - \_\_\_ = \_\_\_\_$

c) Complete the following to verify the length of AB.

(length of $AB)^2 =$ (difference in x-coordinates of B and $A)^2$ + (difference in y-coordinates of B and $A)^2$

length of $AB = \sqrt{(\text{difference in } x\text{-coordinates of } B \text{ and } A)^2 + (\text{difference in } y\text{-coordinates of } B \text{ and } A)^2}$

length of $AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

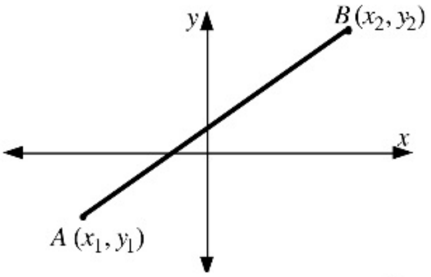
length of $AB = \sqrt{(7-(-5))^2 \ + \ (\qquad\qquad)^2}$

length of $AB = \sqrt{12^2 + \qquad}$

length of $AB = \sqrt{\qquad}$

length of $AB =$

d) Use the same procedure to make a rule for finding the distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$.



(length of $AB)^2 =$ (difference in x-coordinates of B and $A)^2$ + (difference in y-coordinates of B and $A)^2$

length of $AB = \sqrt{(\text{difference in } x\text{-coordinates of } B \text{ and } A)^2 + (\text{difference in } y\text{-coordinates of } B \text{ and } A)^2}$

length of $AB = \sqrt{(x_2 - \qquad)^2 \ + \ (\qquad\qquad)^2}$

The Distance Formula

To find the length of a line segment on the graph of a linear relation, we can use the distance formula.

To find the distance, d, between points $P(x_1, y_1)$ and $Q(x_2, y_2)$, use

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{or} \quad d_{PQ} = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2}}$$

Class Ex. #1

Find the exact length of the following line segments.

a) $P(2, 3)$ to $Q(10, -3)$ **b)** $G(-25, 3)$ to $H(-17, -5)$

Class Ex. #2

Sometimes grids are superimposed on maps to find the distance between two locations. For example, to calculate the distance between two craters, Copernicus and Plato, on the Earth's moon, a coordinate system could be superimposed on a map of the moon. Ordered pairs would then be assigned to Copernicus and Plato to find the distance between the two craters. If the location of Copernicus on the coordinate grid is $(-89, 226)$ and the location of Plato is $(136, 179)$, calculate the distance between the two craters to the nearest unit.

a) Explain how we can determine if a triangle is right-angled if we know the length of each side.

b) L is the point $(0, 1)$, M is $(-3, -3)$ and N is $(-7, 0)$.
Prove that $\triangle LMN$ is right-angled.

Complete Assignment Questions #1 - #10

Assignment

1. Determine the distance between each pair of points.

a) $A(2, 0)$ and $B(7, 12)$ **b)** $C(3, 7)$ and $D(6, 11)$

2. Determine the distance, to the nearest hundredth, between each pair of points.

a) $P(4, 0)$ and $Q(-2, -7)$ **b)** $R(-2.3, 8.9)$ and $S(-3.4, -6.8)$

3. Consider the points $P(-2, 2)$, $Q(1, 6)$ and $R(7, 14)$.

a) Calculate the lengths of PQ, QR, and PR. What do you notice?

b) What does this mean with regard to the points P, Q, and R?

4. A is the point $(6, -2)$, B is $(4, 4)$ and C is $(-3, -5)$.

a) Show that the exact length of AB is $\sqrt{40}$.

b) Determine the exact lengths of BC and AC and prove that $\angle BAC$ is a right angle.

5. A refinery is to be built halfway between the rural towns of Branton and Deer Bridge. A railway is to be built connecting the towns to the refinery.
On a Cartesian plane, Branton is located at $(1232, 3421)$ and Deer Bridge is located at $(1548, 3753)$.

a) What are the coordinates of the refinery?

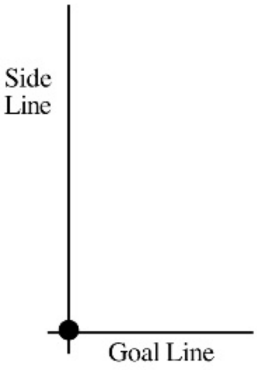
b) Determine the length of the railway, to the nearest kilometre, if the grid scale is 1 unit represents 100 metres.

6. In a high school football game, the Chiefs' quarterback scrambles to the Chiefs' 7 yard line, 15 yards from the left sideline. From that position he throws the ball upfield. The pass is caught by a wide receiver who is on the Chiefs' 38 yard line, 4 yards from the left sideline.

The first quadrant of a coordinate grid is superimposed on the football field, with the origin located at the intersection of the Chiefs' goal line and the left side line.

a) With reference to this origin, state the ordered pairs for the location of the quarterback (when he throws the ball) and the wide receiver (when he catches the ball). Mark these points on the grid shown.

b) Determine the length of the pass (to the nearest yard).



7. At the end of a high school soccer game, Jonas walks 6 blocks west and 5 blocks north from the corner of the soccer field to his house. Beverly walks 3 blocks east and 4 blocks south to reach her house from the same corner of the soccer field.

a) Taking the corner of the soccer field as the origin, list the coordinates of each home.

b) If a block represents 135 metres, determine the direct distance, to the nearest metre, between their homes.

Multiple Choice

8. The distance between the points $(2, -1)$ and $(6, 2)$ is

A. 5
B. $\sqrt{7}$
C. $\sqrt{17}$
D. 25

9. A circle with its centre at the origin passes through the point $(-6, -8)$.
The radius of the circle is

A. 6
B. 8
C. 10
D. 100

Numerical Response

10. The diameter, to the nearest tenth, of the circle with centre $(-5, 9)$ which passes through $(4, -3)$ is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** 13 **b)** 5 **2.** **a)** 9.22 **b)** 15.74
3. **a)** $PQ = 5$, $QR = 10$, $PR = 15$. $PQ + QR = PR$ **b)** The points P,Q and R lie on a straight line.
4. **b)** $BC = \sqrt{130}$, $AC = \sqrt{90}$.
Since $BC^2 = AB^2 + AC^2$, triangle ABC must be right angled at A so angle BAC is a right angle.
5. **a)** (1390, 3587) **b)** 46 km
6. **a)** $Q(15, 7)$ $W(4, 38)$ **b)** 33 yards
7. **a)** $J(-6, 5)$ $B(3, -4)$ **b)** $\sqrt{162}$ blocks ~ 1718 m.
8. A **9.** C **10.**

3	0	.	0

Characteristics of Linear Relations Lesson #3:
Slope of a Line Segment

A trucker driving up a hill with a heavy load may be concerned with the steepness of the hill. When building a roof, a builder may be concerned with the steepness (or pitch) of the roof. A skier going down a hill may be concerned with the steepness of the ski hill.

In mathematics, the term **slope** is used to describe the steepness of a line segment.

Slope of a Line Segment

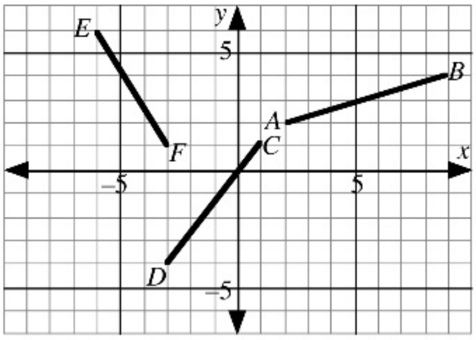
The **slope** of a line segment is a measure of the steepness of the line segment.

It is the ratio of **rise** (the change in vertical height between the endpoints) over **run** (the change in horizontal length between the endpoints).

$$\textbf{Slope} = \frac{\textbf{rise}}{\textbf{run}}$$

- the **rise** is POSITIVE if we count UP, and NEGATIVE if we count DOWN.
- the **run** is POSITIVE if we count RIGHT, and NEGATIVE if we count LEFT.

Each line segment on the grid has endpoints with integer coordinates.
Complete the table below.

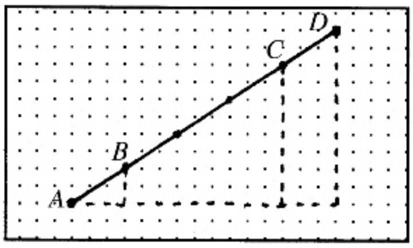


Line Segment	Rise	Run	Slope = $\frac{\text{Rise}}{\text{Run}}$
AB			
CD			
EF			

Investigation #1 *Investigating the Slope of Line Segments*

a) Complete the chart. Write the slopes in simplest form.

Line Segment	Rise	Run	Slope = $\frac{\text{Rise}}{\text{Run}}$
AB			
AC			
AD			
BC			



b) How are the slopes of the line segments related?

Slope of a Line

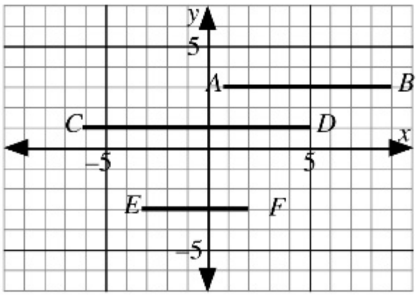
The slopes of all line segments on a line are equal.
The slope of a line representing the graph of a linear relation can be found using

$$\text{slope} = \frac{\text{rise}}{\text{run}} \quad \text{for any two points on the line.}$$

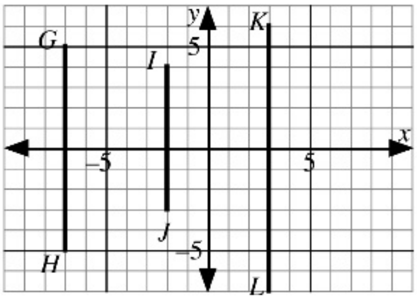
Investigation #2 *Slopes of Horizontal and Vertical Line Segments*

Consider the line segments in Grid 1 and Grid 2 below.

Grid 1



Grid 2



a) Determine the slopes of all the line segments in Grid 1.

b) Determine the slopes of all the line segments in Grid 2.

c) Complete the following statements.

- Horizontal line segments have a slope of __________ .
- Vertical line segments have an _______________ slope.

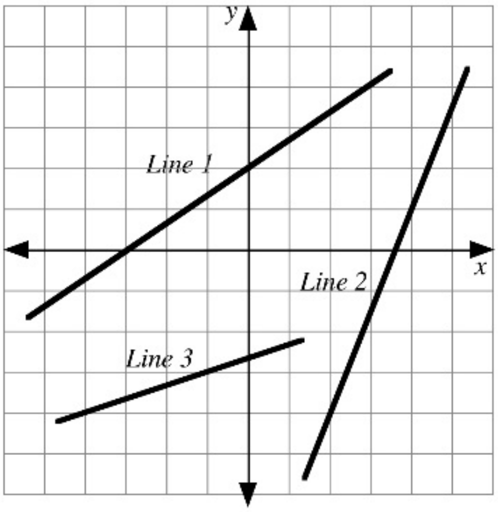
Investigation #3 *Positive and Negative Slopes*

a) Each line on the grids passes through at least two points with integer coordinates. Calculate the slope of each of the lines.

Remember on a Cartesian Plane

- the **rise** is POSITIVE if we count UP, and NEGATIVE if we count DOWN
- the **run** is POSITIVE if we count RIGHT, and NEGATIVE if we count LEFT

Grid 1



Grid 2

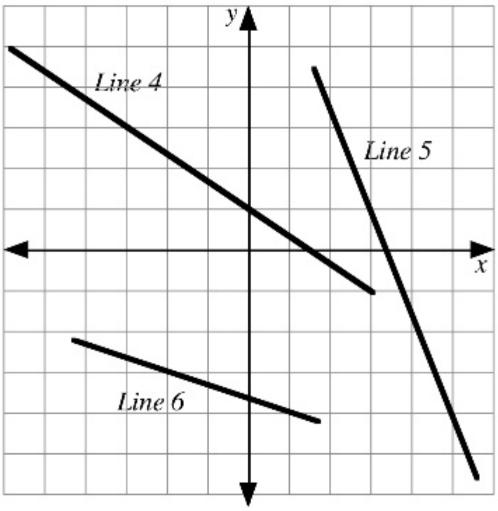


Table For Grid 1

Line	**Slope**
1	
2	
3	

Table For Grid 2

Line	**Slope**
4	
5	
6	

b) Compare the slopes of:

- Line 1 and Line 4
- Line 2 and Line 5
- Line 3 and Line 6

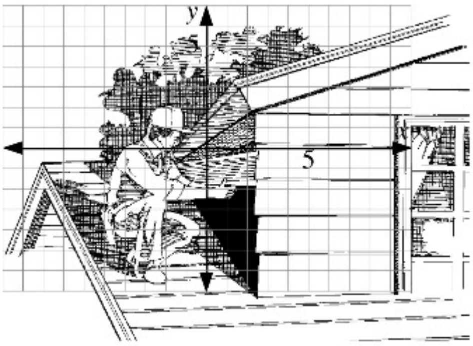
c) Complete the following statements.

- A line which rises from left to right has a _______________ slope.
- A line which falls from left to right has a _______________ slope.

Complete Assignment Questions #1 and #2

Class Ex. #2

A grid has been superimposed on the sketch.



a) Estimate the pitch (slope) of the roof to the right of the worker's head.

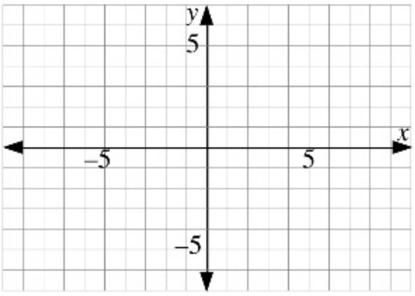
b) Could the grid be used to estimate the pitch of the roof the worker is standing on? Explain.

Class Ex. #3

Draw a line segment on the grid which passes through the point (–4, 2) and has a slope of $-\frac{2}{3}$.

The line segment must be long enough to cross both the x-axis and the y-axis.

Write the coordinates of three other points on the line segment which have integer coordinates.



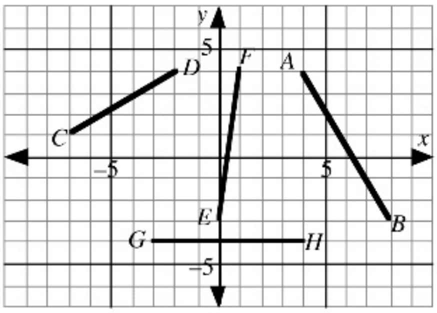
Class Ex. #4

A line segment has a slope of $-\frac{5}{7}$ and a rise of 12. Calculate the run as an exact value.

Complete Assignment Questions #3 - #13

Assignment

1. Each line segment on the grid has endpoints with integer coordinates. Complete the table.



Line Segment	**Rise**	**Run**	**Slope =** $\frac{\textbf{Rise}}{\textbf{Run}}$
AB			
CD			
EF			
GH			

2. Every line on the grid passes through at least two points with integer coordinates. Calculate the slope of each of the lines.

slope of Line 1 :

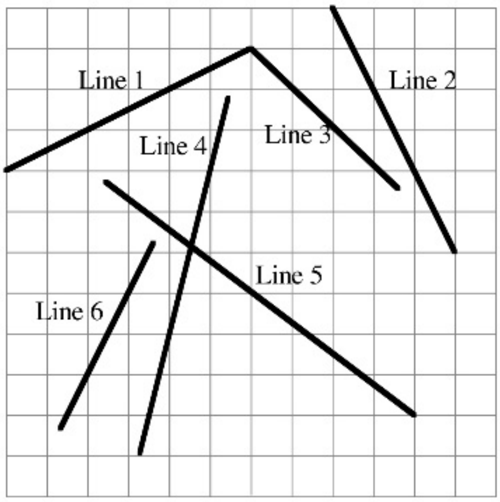
slope of Line 2:

slope of Line 3:

slope of Line 4:

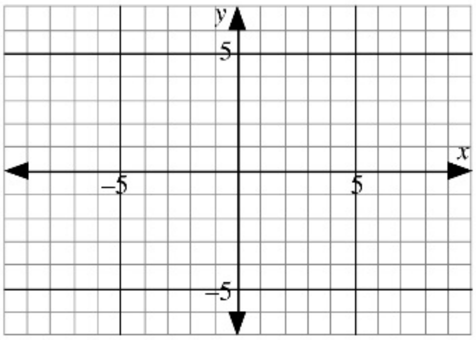
slope of Line 5:

slope of Line 6:



3. Draw a line segment on the grid which passes through the point (–5, –2) and has a slope of $\frac{2}{3}$. The line segment must be long enough to cross both the x-axis and the y-axis.

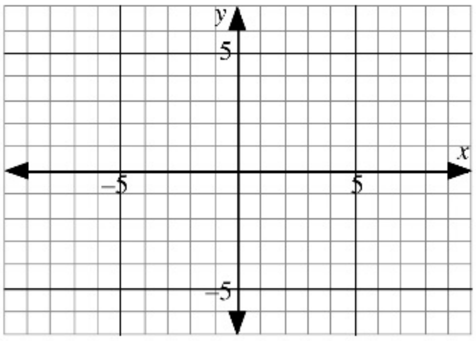
Write the coordinates of three other points on the line segment which have integer coordinates.



4. Repeat question #3 for line segments with the given slope passing through the given point.

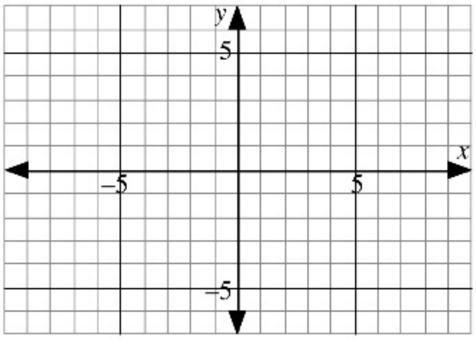
a) slope $= \frac{2}{5}$, $(2, 1)$

b) slope $= -\frac{1}{3}$, $(6, -3)$



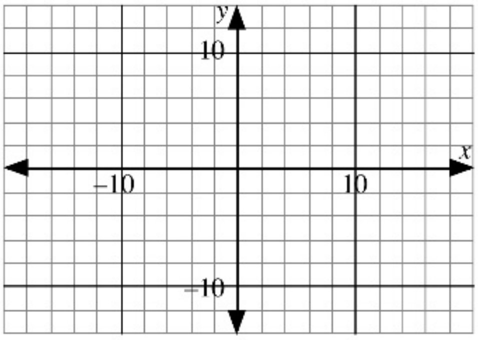
c) slope $= -\frac{4}{3}$, $(-9, 6)$

d) slope $= 4$, $(0, -7)$



e) slope $= -2$, $(4, -12)$

f) slope $= 0$, $(0, 6)$



5. P has coordinates $(-1, 2)$. Find two positions for point Q so that the slope of PQ is

a) 2 **b)** -3 **c)** $\frac{1}{3}$

d) $-\frac{2}{5}$ **e)** 0 **f)** undefined

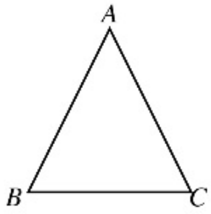
6. Two of three measures are given for rise, run, and slope.
Calculate the value of the third measure in each of the following.

a) slope = $\frac{5}{7}$ and run = 49 **b)** slope = $-\frac{3}{8}$ and rise = 15

c) slope = $-\frac{6}{11}$ and run = 33 **d)** slope = $\frac{3}{4}$ and rise = 15

7. A ramp which has been set up by skateboarders has a slope of $\frac{2}{3}$. Calculate the height of the ramp if the it has a base length of 1.5 metres.

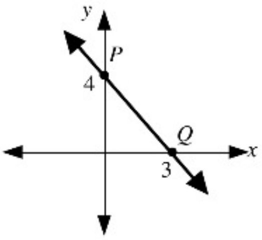
8. Triangle ABC is isosceles with $AB = AC$ and $BC = 6.8$ cm. Calculate the area of the triangle if the slope of $AC = -\frac{5}{4}$.



Multiple Choice

9. The slope of $\overline{PQ}$ is

A. $\frac{3}{4}$
B. $-\frac{3}{4}$
C. $\frac{4}{3}$
D. $-\frac{4}{3}$



10. The point $(-4, 0)$ is on a line which has a slope of $-\frac{2}{5}$. The next point with integer coordinates on the line to the right of $(-4, 0)$ is

A. $(-9, -2)$
B. $(-9, 2)$
C. $(1, -2)$
D. $(-2, -5)$

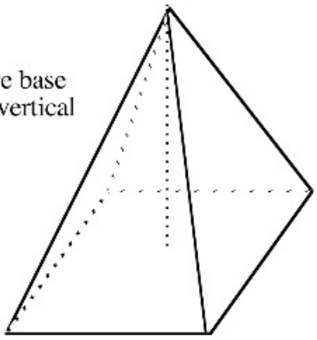
11. P is a point in quadrant I, Q is a point in quadrant II, R is a point in quadrant III, and S is a point in quadrant IV.

Which one of the following statements must be true?

A. Line segment PQ has a positive slope.
B. Line segment QR has a positive slope.
C. Line segment PR has a positive slope.
D. Line segment QS has a positive slope.

Use the following information to answer questions #12 and #13.

A pyramid has a square base of length 12 m and a vertical height of 15 m.



Numerical Response **12.** A beetle starts to climb the pyramid starting from the midpoint of one of the faces. To the nearest tenth, the slope of the beetle's climb is _____.

(Record your answer in the numerical response box from left to right)

13. A fly starts to climb the pyramid along one of the edges. To the nearest tenth, the slope of the fly's climb is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

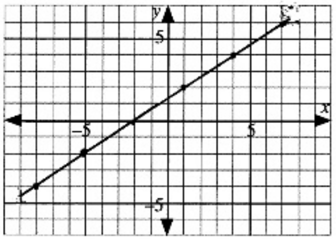
1.

Line Segment	Rise	Run	Slope = $\frac{\text{Rise}}{\text{Run}}$
AB	-7	4	$-\frac{7}{4}$
CD	3	5	$\frac{3}{5}$
EF	7	1	$\frac{7}{1} = 7$
GH	0	7	$\frac{0}{7} = 0$

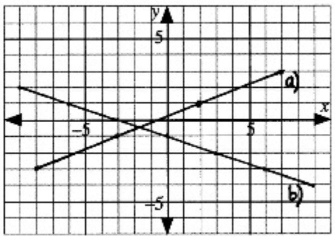
2. slope of line 1 = $\frac{1}{2}$, slope of line 2 = -2, slope of line 3 = -1

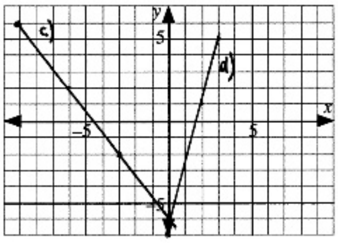
slope of line 4 = 4, slope of line 5 = $-\frac{3}{4}$, slope of line 6 = 2

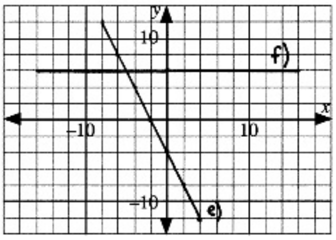
3. Any three of $(-8, -4)$, $(-2, 0)$, $(1, 2)$, $(4, 4)$



4.







a) $(-8, -3)$, $(-3, -1)$, $(7, 3)$

b) Any 3 of $(-9, 2)$, $(-6, 1)$, $(-3, 0)$, $(0, -1)$, $(3, -2)$, $(9, -4)$

c) $(-6, 2)$, $(-3, -2)$, $(0, -6)$

d) $(1, -3)$, $(2, 1)$, $(3, 5)$

e) Many possible answers including $(2, -8)$, $(0, -4)$, $(-2, 0)$

f) Many possible answers including $(1, 6)$, $(2, 6)$, $(3, 6)$

5. Many possible answers, including any two from:

a) $(-3, -2)$, $(-2, 0)$, $(0, 4)$, $(1, 6)$ **b)** $(-3, 8)$, $(-2, 5)$, $(0, -1)$, $(1, -4)$,
c) $(2, 3)$, $(5, 4)$, $(-4, 1)$, $(-7, 0)$ **d)** $(-11, 6)$, $(-6, 4)$, $(4, 0)$, $(9, -2)$
e) $(-3, 2)$, $(-2, 2)$, $(0, 2)$, $(1, 2)$ **f)** $(-1, 1)$, $(-1, 0)$, $(-1, -1)$, $(-1, 3)$

6. **a)** rise = 35 **b)** run = -40 **c)** rise = -18 **d)** run = 20

7. 1 metre **8.** 14.45 cm^2 **9.** D **10.** C **11.** C

12.

2	.	5	

13.

1	.	8	

Characteristics of Linear Relations Lesson #4:
The Slope Formula

Review

Complete the following statements.

a) Slope is the measure of the _______________ of a line.

b) Slope is the ratio of the vertical change (called the __________) over the horizontal change (called the __________) .

c) A line segment which rises from left to right has a ______________ slope.

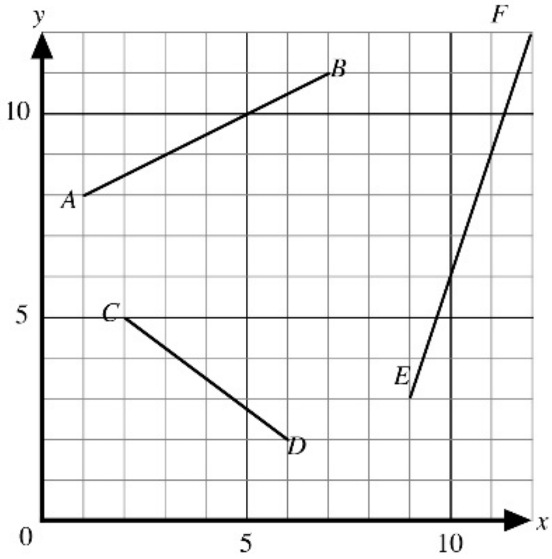
d) A line segment which falls from left to right has a ______________ slope.

e) A horizontal line segment has a slope of _____.

f) A vertical line segment has an ______________ slope.

g) The slopes of all line segments on a line are ________ .

Developing the Slope Formula



a) Calculate the slope of line segment AB using slope = $\frac{\text{rise}}{\text{run}}$.

b) List the coordinates of the endpoints of line segment AB.

$A($, $)$ $B($, $)$

c) How can the rise of line segment AB be determined using y_B and y_A?

d) How can the run of line segment AB be determined using x_A and x_B?

e) Use your results from **c)** and **d)** to write a formula which describes how the slope of line segment AB can be calculated using its endpoints.

f) Calculate the slope of line segment AB using the formula in **e)**.

g) Calculate the slope of the line segments CD and EF using the method in **a)** and verify using the formula from **e)**.

The Slope Formula

In mathematics the letter "m" is used to represent slope. If the graph of a linear relation passes through the points $P(x_1, y_1)$ and $Q(x_2, y_2)$, then the slope of this line can be calculated using

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{or} \qquad m_{PQ} = \frac{y_Q - y_P}{x_Q - x_P}$$

Find the slope of a line which passes through the points $G(-3, 8)$ and $H(7, -2)$.

$$m_{GH} = \frac{y_H - y_G}{x_H - x_G} =$$

Eleanor, Bonnie, and Carl are calculating the slope of a line segment with endpoints $E(15, 8)$ and $F(-10, 6)$. Their work is shown below.

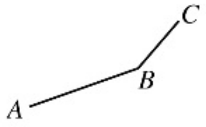
	Eleanor	Bonnie	Carl
Step 1:	$m_{\overline{EF}} = \frac{-10 - 15}{6 - 8}$	$m_{\overline{EF}} = \frac{6 - 8}{15 - (-10)}$	$m_{\overline{EF}} = \frac{8 - 6}{15 - 10}$
Step 2:	$= \frac{-25}{-2}$	$= \frac{-2}{25}$	$= \frac{2}{5}$
Step 3:	$m_{\overline{EF}} = \frac{25}{2}$	$m_{\overline{EF}} = -\frac{2}{25}$	$m_{\overline{EF}} = \frac{2}{5}$

Since their answers are all different, at least two of the students have made errors in their calculations. Describe all the errors which have been made and determine the correct slope.

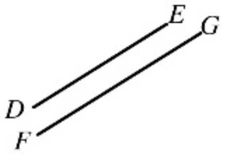
Complete Assignment Questions #1 - #5

Collinear Points

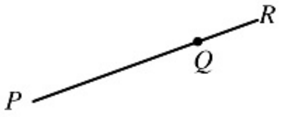
Two lines in a plane can either be



at an angle



parallel and distinct



parallel and form a straight line.

Points that lie on the same straight line are said to be **collinear**, i.e. P, Q, and R are collinear.

If three points P, Q, and R are collinear then $m_{PQ} = m_{QR} = m_{PR}$.
Proving that any two of these three slopes are equal is sufficient for the third to be equal and for the points to be collinear.

Consider points $A(5, -3)$, $B(2, 6)$, and $C(-7, 33)$.

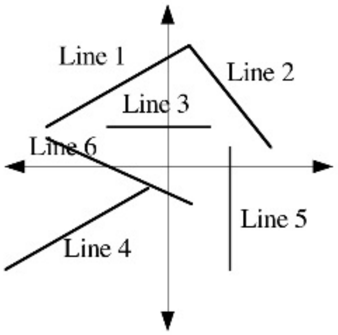
a) Prove that the points A, B, and C are collinear.

b) Find the value of y if the point $D(-4, y)$ lies on line segment AC.

Complete Assignment Questions #6 - #12

Assignment

1. State whether the slope of each line is positive, negative, zero, or undefined.



2. Use the slope formula to calculate the slope of the line segment with the given endpoints.

a) $A(12,-2)$ and $B(0,3)$

b) $C(-2,3)$ and $D(2,-2)$

$m_{AB} = \frac{y_B - y_A}{x_B - x_A} =$

c) $P(-15,-2)$ and $O(0,0)$

d) $S(36,-41)$ and $T(-20,-27)$

e) $U(-172,-56)$ and $V(-172,32)$

f) $K(8,-41)$ and $L(397,-41)$

3. Use the slope formula to calculate the slope of the line passing through the given points.

a) $(3,-6)$ and $(8,4)$

b) $(-12,7)$ and $(0,-2)$

$m = \frac{y_2 - y_1}{x_2 - x_1} =$

c) $(-3,-8)$ and $(1,5)$

d) $(21,1)$ and $(-4,-9)$

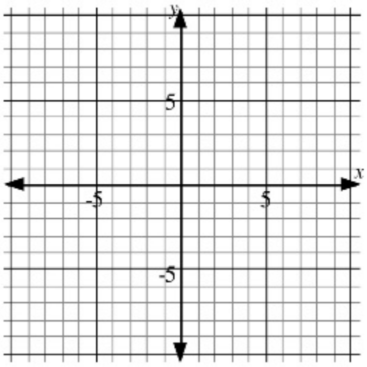
4. A coordinate grid is superimposed on a cross-section of a hill. The coordinates of the bottom and the top of a straight path up the hill are, respectively, $(3, 2)$ and $(15, 47)$, where the units are in metres.

a) Calculate the slope of the hill.

b) Calculate the coordinates of the midpoint of the path up the hill.

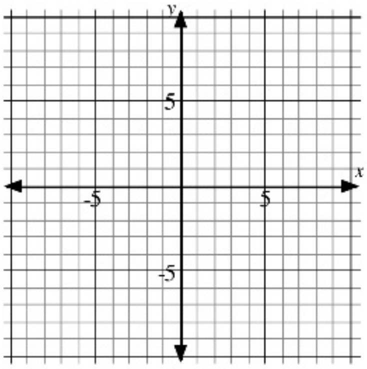
c) Calculate the length of the path to the nearest tenth of a metre.

5. The line segment joining each pair of points has the given slope. Determine each value of k and draw the line segment on the grid.

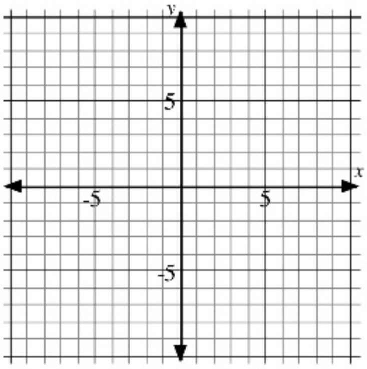


a) $S(4, 6)$ and $T(5, k)$ slope $= 3$

b) $L(k, -2)$ and $M(3, -7)$ slope $= -\frac{1}{2}$



c) $U(2, 5)$ and $V(k, 3)$ slope $= \frac{2}{7}$



6. Consider points $P(4,-9)$, $Q(-1,-7)$, and $R(-11,-3)$.

a) Use the slope formula to prove that the points P, Q, and R are collinear.

b) Use the distance formula to prove that the points P, Q, and R are collinear.

7. Consider points $A(8,-7)$, $B(-8,-3)$, and $C(-24,1)$.

a) Prove that the points A, B, and C are collinear.

b) Does the point $D(-2,-4)$ lie on line segment AC? Explain.

c) Find the value of k if the point $E(k,k)$ lies on line segment AC.

8. A private jet has crashed in the desert at the point $P(-10, 17)$. A search party sets out in an all terrain vehicle from A_1, passing in a straight line through A_2. A helicopter sets out from B_1 and flies in a straight line through B_2.
If the search parties continue in these directions, will either of them discover the crashed plane?

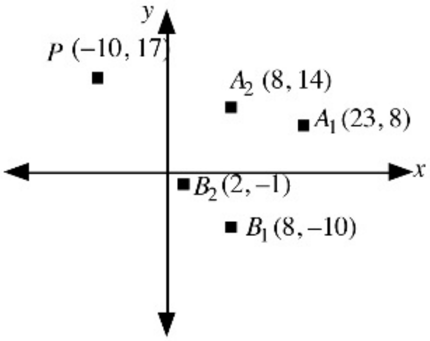


Diagram not to scale

Multiple Choice

9. The slope of the line segment joining $E(5,-1)$ and $F(3, 7)$, is

A. -3
B. -4
C. $-\frac{1}{3}$
D. $-\frac{1}{4}$

10. If the line segment joining $(2, 3)$ and $(8, k)$ has slope $-\frac{2}{3}$, then $k =$

A. -1
B. -3
C. -6
D. 7

11. One endpoint of a line segment is $(1, 6)$. The other endpoint is on the x-axis. If the slope of the line segment is -3, then the midpoint of the line segment is

A. $(4, 6)$
B. $(2, 3)$
C. $(-10, 3)$
D. $\left(\frac{1}{2}, \frac{15}{2}\right)$

Numerical Response **12.** $P(3, 6)$, $Q(8, -2)$, and $R(-6, 0)$, are the vertices of a triangle. T is the midpoint of QR. The slope of the line PT, to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

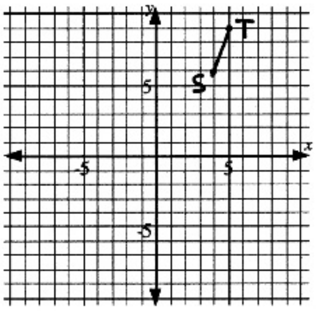
1. Line 1 - positive, Line 2 - negative, Line 3 - zero, Line 4 - positive, Line 5 - undefined, Line 6 - negative

2. **a)** $-\frac{5}{12}$ **b)** $-\frac{5}{4}$ **c)** $\frac{2}{15}$ **d)** $-\frac{1}{4}$ **e)** undefined **f)** 0

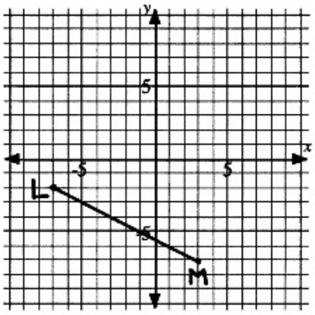
3. **a)** 2 **b)** $-\frac{3}{4}$ **c)** $\frac{13}{4}$ **d)** $\frac{2}{5}$

4. **a)** $\frac{15}{4}$ **b)** $\left(9, \frac{49}{2}\right)$ **c)** 46.6 m.

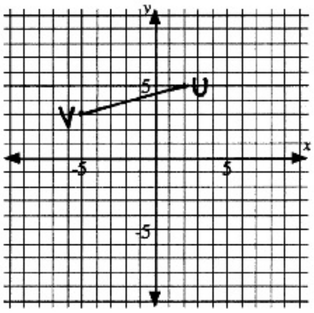
5. **a)** $k = 9$



b) $k = -7$



c) $k = -5$



6. **a)** $m_{PQ} = -\frac{2}{5}$, $m_{QR} = -\frac{2}{5}$. Since $m_{PQ} = m_{QR}$, the points P, Q and R are collinear.

b) $PQ = \sqrt{29}$, $QR = 2\sqrt{29}$, $PR = 3\sqrt{29}$. Since $PQ + QR = PR$, the points P, Q and R are collinear.

7. **a)** $m_{AB} = -\frac{1}{4}$, $m_{BC} = -\frac{1}{4}$. Since $m_{AB} = m_{BC}$, the points A, B and C are collinear.

b) $m_{AD} = -\frac{3}{10}$ Since $m_{AD} \neq m_{AB}$, the point D does not lie on line segment AC. **c)** $k = -4$

8. $m_{A_1A_2} = -\frac{2}{5}$, $m_{A_2P} = -\frac{1}{6}$. Since $m_{A_1A_2} \neq m_{A_2P}$, the search party in the all terrain vehicle will not discover the plane.

$m_{B_1B_2} = -\frac{3}{2}$, $m_{B_2P} = -\frac{3}{2}$. Since $m_{B_1B_2} = m_{B_2P}$, the search party in the helicopter will discover the plane.

9. B **10.** A **11.** B **12.**

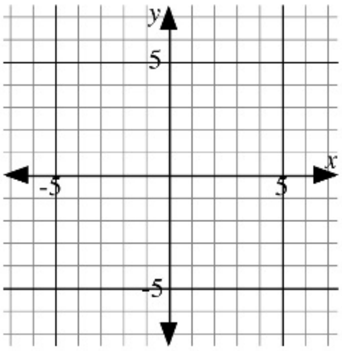
3	.	5	

Characteristics of Linear Relations Lesson #5:
Parallel and Perpendicular Lines

Review of Transformations

In earlier mathematics courses we studied transformations: translations, reflections, and rotations. In order to investigate parallel and perpendicular line segments, we will review translations and rotations.

On the grid, show the image of the point $A(2, 5)$ after the following transformations. In each case write the coordinates of the image.



a) A translation 3 units right and 2 units up.

$A(2, 5) \rightarrow \quad B(\quad , \quad)$

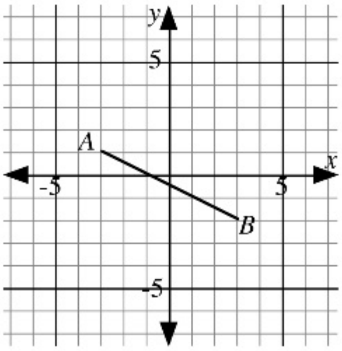
b) A 90° clockwise rotation about the origin.

$A(2, 5) \rightarrow \quad C(\quad , \quad)$

c) A 90° counterclockwise rotation about the origin.

$A(2, 5) \rightarrow \quad D(\quad , \quad)$

Investigating Parallel Line Segments



a) On the grid, show the image of line segment AB after the following transformations.

- **i)** A translation 4 units right and 1 unit down to form line segment CD.
- **ii)** A translation 3 units left and 6 units up to form line segment EF.
- **iii)** A translation 3 units down to form line segment GH.

b) Calculate the slope of each of the line segments.

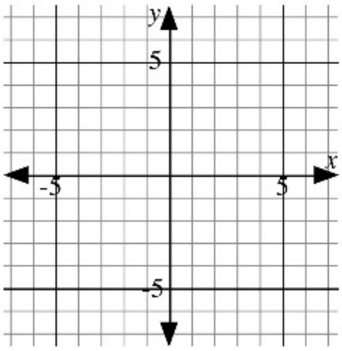
c) The four line segments are parallel. Make a conjecture about the slopes of parallel line segments.

Investigating Perpendicular Line Segments

a) i) On the grid, plot the point $A(5, 2)$ and draw the line joining the point to the origin, O.

ii) Rotate the line through an angle of 90° clockwise about O and show the image on the grid.

iii) Find the slopes of the two perpendicular lines and multiply them together.



b) Repeat part **a)** for the point $B(-6, 1)$.

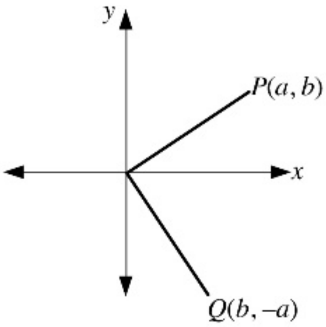
c) Make a conjecture about the slopes of perpendicular line segments.

d) Complete the following to prove the conjecture in **c)**.
Under a rotation of 90° clockwise about O, $P(a, b) \rightarrow Q(b, -a)$.

$$m_{OP} = \frac{y_P - y_O}{x_P - x_O} = \frac{b - 0}{a - 0} =$$

$$m_{OQ} = \frac{y_Q - y_O}{x_Q - x_O} = \frac{-a - 0}{b - 0} =$$

$$m_{OP} \times m_{OQ} =$$



Parallel Lines and Perpendicular Lines

Recall that the slope of any line segment within a line represents the slope of the line.

Consider then two lines with slopes m_1 and m_2.

- The lines are **parallel** if they have the same slope, i.e. $m_1 = m_2$.
- The lines are **perpendicular** if the product of the slopes is -1,

  i.e. $m_1 \times m_2 = -1$ or $m_1 m_2 = -1$ or $m_1 = -\dfrac{1}{m_2}$
- For perpendicular lines, each slope is the negative reciprocal of the other provided neither slope is equal to zero.

Class Ex. #1

Consider line segment AC with a slope of $\dfrac{3}{4}$.

a) Write the slope of line segment GH which is parallel to AC.

b) Write the slope of line segment BF which is perpendicular to AC.

Class Ex. #2

The slopes of two lines are given.
Determine if the lines are parallel, perpendicular, or neither.

a) $m_1 = \dfrac{1}{4},\ m_2 = \dfrac{3}{12}$

b) $m_1 = \dfrac{5}{7},\ m_2 = \dfrac{14}{10}$

Complete Assignment Questions #1 - #6

Class Ex. #3

If P is the point $(4, 7)$ and Q is the point $(6, -2)$, find the slope of a line segment

a) parallel to line segment PQ

b) perpendicular to line segment PQ

Class Ex. #4

$\triangle LMN$ has coordinates $L(-4, 2)$, $M(-2, 7)$, and $N(1, 0)$. Use slopes to show that the triangle is right-angled at L.

Class Ex. #5

Two lines have slopes of $-\frac{3}{4}$ and $\frac{k}{5}$ respectively. Find the value of k if the lines are

a) parallel **b)** perpendicular

Complete Assignment Questions #7 - #16

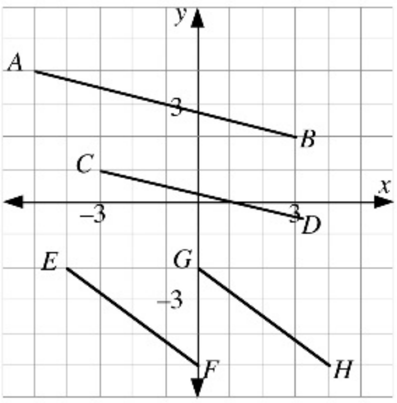
Assignment

1. AB is parallel to CD. EF is parallel to GH.

a) Determine the slopes of the following pairs of parallel line segments using $m = \frac{\text{rise}}{\text{run}}$.

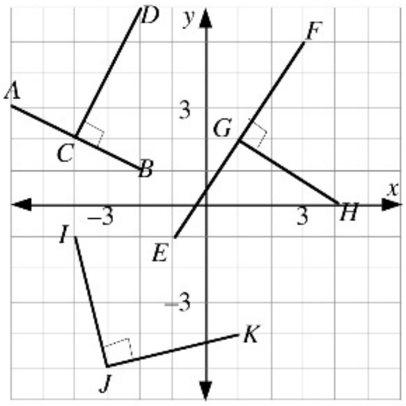
Line Segment	Slope
AB	
CD	

Line Segment	Slope
EF	
GH	



b) Observe the slopes of the pairs of parallel lines in **a)**.
Write a rule in reference to the slopes of parallel lines.

2.a) Determine the slopes of the following pairs of perpendicular line segments using $m = \dfrac{\text{rise}}{\text{run}}$.



Line Segment	Slope
AB	
CD	

Line Segment	Slope
EF	
GH	

Line Segment	Slope
IJ	
JK	

b) Multiply the slopes of the pairs of perpendicular line segments.

$m_{AB} \times m_{CD}$

$-\dfrac{1}{2} \times 2 =$

$m_{EF} \times m_{GH}$

$m_{IJ} \times m_{JK}$

c) Write a rule in reference to the slope of two lines which are perpendicular to each other.

3. The slopes of two line segments are given. Determine if the lines are parallel, perpendicular, or neither.

a) $m_{AB} = \dfrac{8}{20},\ m_{PQ} = \dfrac{2}{5}$ **b)** $m_{AB} = \dfrac{3}{2},\ m_{PQ} = -\dfrac{2}{3}$ **c)** $m_{AB} = \dfrac{1}{6},\ m_{PQ} = \dfrac{2}{12}$

d) $m_{AB} = \dfrac{7}{8},\ m_{PQ} = \dfrac{8}{7}$ **e)** $m_{AB} = \dfrac{9}{3},\ m_{PQ} = -\dfrac{1}{3}$ **f)** $m_{AB} = -5,\ m_{PQ} = \dfrac{1}{5}$

g) $m_{AB} = \dfrac{4}{8},\ m_{PQ} = 2$ **h)** $m_{AB} = -\dfrac{12}{2},\ m_{PQ} = -6$ **i)** $m_{AB} = -\dfrac{5}{2},\ m_{PQ} = -\dfrac{2}{5}$

4. The slopes of some line segments are given.

$m_{AB} = 6$ $m_{CD} = \frac{1}{6}$ $m_{EF} = -6$ $m_{GH} = 6$ $m_{IJ} = -6$ $m_{KL} = \frac{1}{6}$

Which pairs of lines are parallel to each other?

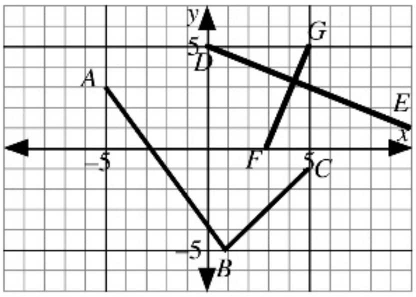
5. The slopes of some line segments are given.

$m_{RS} = -2$ $\quad m_{UV} = \frac{1}{4}$ $\quad m_{EF} = 0.5$ $\quad m_{ZT} = 2$

$m_{PQ} = -4$ $\quad m_{KL} = -\frac{1}{2}$ $\quad m_{MN} = 4$ $\quad m_{XY} = -\frac{1}{4}$

Which pairs of lines are perpendicular to each other?

6. The four line segments have endpoints with integer coordinates. In each case determine whether the two intersecting line segments are perpendicular.



7. A, B, and C are the points $(0, 4)$, $(-3, 1)$, and $(5, -2)$ respectively. Determine the slope of a line

a) parallel to line segment AB

b) perpendicular to line segment AB

c) parallel to line segment BC

d) perpendicular to line segment AC

8. ΔABC has vertices $A(3, 5)$, $B(-2, -5)$, $C(-5, 1)$.

a) Explain how we can determine if $\triangle ABC$ is a right triangle.

b) Determine if $\triangle ABC$ is a right triangle.

9. The vertices of two triangles are given.
Determine if either of the triangles is right-angled.

a) $\Delta PQR \rightarrow P(-3,3),\ Q(-1,1),\ R(-5,-1)$ **b)** $\Delta ABC \rightarrow A(-7,9),\ B(3,13),\ C(7,3)$

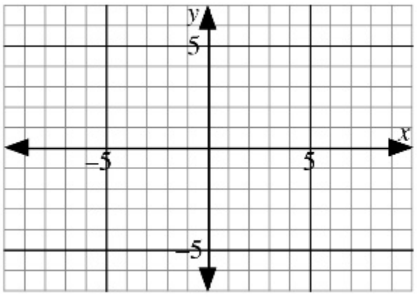
10. The slopes of parallel lines are given.
Determine the value of the variable.

a) $4, \frac{k}{3}$ **b)** $-2, \frac{2}{n}$ **c)** $\frac{5}{6}, 3m$ **d)** $\frac{3}{4}, -\frac{w}{6}$

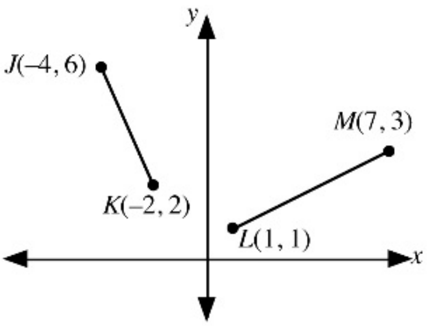
11. The slopes of perpendicular lines are given.
Determine the value of the variable.

a) $\frac{1}{3}, 3h$ **b)** $4, \frac{8}{p}$ **c)** $-5, \frac{s}{2}$ **d)** $-\frac{3}{4}, -\frac{q}{6}$

12. $P(-4,0)$ and $R(1,-3)$ are opposite vertices of a rhombus $PQRS$. Find the slope of diagonal QS.



13. a) Show that when line segments JK and ML are extended until they intersect, they will not meet at right angles.



b) If the y-coordinate of M is changed, the line segments, when extended, will meet at right angles. To what value should the y-coordinate of M be changed?

14. Given that $A, B,$ and C are the points $(-3, 3)$, $(0, 6)$, and $(5, 1)$ respectively, prove that triangle ABC is right angled by using

a) the slope formula

b) the distance formula

Multiple Choice

15. A and B are the points $(1, 2)$ and $(-2, 3)$ respectively. A line perpendicular to AB will have slope

A. -3
B. $-\frac{1}{3}$
C. 3
D. $\frac{1}{3}$

Numerical Response **16.** The line segment joining $U(-3, p)$ and $V(-6, 5)$ is perpendicular to the line segment joining $X(4, 2)$ and $Y(9, 0)$. The value of p, to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** slope $AB = -\frac{1}{4}$ slope $CD = -\frac{1}{4}$ slope $EF = -\frac{3}{4}$ slope $GH = -\frac{3}{4}$

b) Lines which are parallel have the same slope

2. **a)** slope $AB = -\frac{1}{2}$ slope $EF = \frac{3}{2}$ slope $IJ = -4$

slope $CD = 2$ slope $GH = -\frac{2}{3}$ slope $JK = \frac{1}{4}$

b) All the products are -1. **c)** The product of the slopes is -1.

3. **a)** parallel **b)** perpendicular **c)** parallel

d) neither **e)** perpendicular **f)** perpendicular

g) neither **h)** parallel **i)** neither

4. AB and GH, CD and KL, EF and IJ.

5. RS and EF, UV and PQ, ZT and KL, MN and XY.

6. AB and BC are not perpendicular. DE and FG are perpendicular.

7. **a)** 1 **b)** -1 **c)** $-\frac{3}{8}$ **d)** $\frac{5}{6}$

8. **a)** Determine the slope of each side of the triangle. If two of the slopes are negative reciprocals of each, then the triangle is a right triangle.

b) $m_{BC} = -2$, $m_{AC} = \frac{1}{2}$. Since the slopes are negative reciprocals, the triangle is a right triangle.

9. **a)** ΔPQR is not a right triangle **b)** ΔABC is a right triangle

10.a) $k = 12$ **b)** $n = -1$ **c)** $m = \frac{5}{18}$ **d)** $w = -\frac{9}{2}$

11.a) $h = -1$ **b)** $p = -32$ **c)** $s = \frac{2}{5}$ **d)** $q = -8$

12. $m_{QS} = \frac{5}{3}$

13.a) $M_{JK} = -2$, $M_L = \frac{1}{3}$. The product of the slopes does not equal -1. **b)** $y_M = 4$

14.a) $m_{AB} = 1$, $m_{BC} = -1$ Since the product of the slopes $= -1$, AB and BC are perpendicular. Triangle ABC is right angled at B.

b) $AB = \sqrt{18}$, $BC = \sqrt{50}$, $AC = \sqrt{68}$. $AC^2 = 68$. $AB^2 + BC^2 = 68$.
$AC^2 = AB^2 + BC^2$ so the Pythagorean theorem is satisfied and the triangle is right angled at B.

15. C **16.**

1	2	.	5

Characteristics of Linear Relations Lesson #6:
Practice Test

1. Which of the following horizontal or vertical line segments has the greatest length?

A. PQ with $P(2,9)$ and $Q(7,9)$.
B. RS with $R(4,11)$ and $S(4,4)$.
C. TV with $T(-6,1)$ and $V(2,1)$.
D. WZ with $W(-5,-5)$ and $Z(-5,-11)$.

2. The exact distance between the points $(8,3)$ and $(5,-2)$ is

A. 5.83
B. $\sqrt{34}$
C. 8
D. 34

Numerical Response 1. To the nearest hundredth, the distance between $A(-2,3)$ and $B(-5,-6)$ is _____.
(Record your answer in the numerical response box from left to right)

Use the following information to answer the next two questions.

The points $A(4,2)$ and $B(-2,6)$ are on the circumference of a circle.
The line segment AB passes through the centre of the circle.

3. The centre of the circle is the point

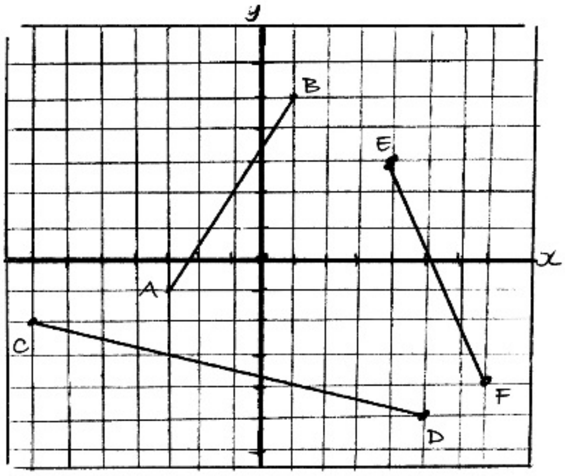
A. $(2,8)$ **B.** $(3,-2)$
C. $(1,4)$ **D.** $(6,-4)$

Numerical Response 2. The area of the circle, to the nearest whole number, is _____.
(Record your answer in the numerical response box from left to right)

Numerical Response 3. PQ is the diameter of a circle, the centre is R. If $Q(6.7, 4.5)$ and $R(8.5, 7.9)$ then the x–coordinate of P is _____.
(Record your answer in the numerical response box from left to right)

Use the following information to answer questions #4 - #6.

Nadine practiced for a math quiz by drawing line segments on a grid and then determining their slopes. Her line segments and grid are shown.



Matching Match each line segment on the left with the slope on the right.
Each slope may be used once, more than once, or not at all.

Line Segment	**Slope**	
4. *AB*	**A.** -4	**D.** $\frac{3}{7}$
5. *CD*	**B.** $-\frac{3}{2}$	**E.** $\frac{3}{2}$
6. *EF*	**C.** $-\frac{7}{3}$	**F.** $-\frac{1}{4}$

7. Consider $\overline{AB}$ joining $A(6,-4)$ and $B(-4,-4)$, and $\overline{CD}$ joining $C(1,-9)$ and $D(1,1)$. Which one of the following statements about these line segments is true?

A. $\overline{AB}$ and $\overline{CD}$ have the same slope and are equal in length.
B. $\overline{AB}$ and $\overline{CD}$ have the same slope and are unequal in length.
C. $\overline{CD}$ has a length of 10 units and a slope of zero.
D. $\overline{AB}$ and $\overline{CD}$ have the same midpoint and are equal in length.

Use the following information to answer questions #8 - #10.

Line Segment AB	Line Segment PQ
$A(-2, 4)$ $B(2, -6)$	$P(7, 1)$ $Q(-3, -3)$

8. Which of the following statements is correct about the line segments?

A. The length of line segment AB is greater than the length of line segment PQ.
B. The length of line segment AB is less than the length of line segment PQ.
C. The length of line segment AB is equal to the length of line segment PQ.
D. Not enough information is given to calculate the lengths of the line segments.

9. Which of the following statements is correct about the line segments?

A. The slope of line segment AB is positive and the slope of line segment PQ is negative.
B. The slope of line segment AB is positive and the slope of line segment PQ is positive.
C. Line segment AB is parallel to line segment PQ.
D. Line segment AB is perpendicular to line segment PQ.

10. Which of the following statements is correct about the line segments?

A. The midpoint of AB has an x-coordinate greater than the midpoint of PQ.
B. The midpoint of AB has an y-coordinate greater than the midpoint of PQ.
C. The midpoints of AB and PQ are the same point.
D. The line segment joining the midpoints is horizontal.

11. K and L are the points $(4, 7)$ and $(-1, -3)$ respectively. The slope of a line perpendicular to KL is

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. 2

D. -2

Use the following information to answer questions #12–#15.

Quadrilateral $PQRS$ has vertices $P(-4,-6)$, $Q(-6,-2)$, $R(0,1)$, and $S(2,-3)$.

12. The slope and length of line segment PQ are respectively

A. -2 and $\sqrt{20}$

B. $-\frac{1}{2}$ and $\sqrt{20}$

C. -2 and $\sqrt{116}$

D. $-\frac{1}{2}$ and $\sqrt{116}$

13. The slope and length of line segment QR are respectively

A. 2 and $\sqrt{37}$

B. $\frac{1}{2}$ and $\sqrt{37}$

C. 2 and $\sqrt{45}$

D. $\frac{1}{2}$ and $\sqrt{45}$

14. Consider the following statements:

I. PQ is parallel to SR.
II. QR is perpendicular to SR.
III. The lengths of PQ and SR are the same.

Which of the following is correct?

A. Statement **I** is false.

B. Statement **II** is false.

C. Statement **III** is false.

D. None of the above statements is false.

15. Which of the following most completely describes Quadrilateral $PQRS$?

A. rectangle **B.** square **C.** parallelogram **D.** rhombus

Numerical Response **4.** Two lines have slopes of $-\frac{2}{3}$ and $\frac{15}{t}$, respectively.
If the lines are perpendicular, then the value of t must be _____ .
(Record your answer in the numerical response box from left to right)

Numerical Response **5.** The line segment joining $K(-1, y)$ and $L(-2, 8)$ is parallel to the line segment joining $M(4, 4)$ and $N(0, 5)$. The value of y, to the nearest hundredth, is _____.

(Record your answer in the numerical response box from left to right)

Written Response - 5 marks

1. Consider the points $A(-2, 6)$, $B(2, 0)$, $C(0, 2)$, and $D(6, -6)$.

- Three of these points lie on the same straight line and one point does not.
  Explain how you could algebraically determine which of these points is not collinear with the other three.

- **Determine** algebraically which of the points $A(-2, 6)$, $B(2, 0)$, $C(0, 2)$, and $D(6, -6)$ is not collinear with the other three.

- Without using the midpoint formula determine algebraically that B is the midpoint of line segment AD.

Answer Key

1. C **2.** B **3.** C **4.** E **5.** F **6.** C **7.** D **8.** C
9. D **10.** D **11.** B **12.** A **13.** D **14.** D **15.** A

Numerical Response

1.	9	.	4	9
2.	4	1		
3.	1	0	.	3
4.	1	0		
5.	7	.	7	5

Written Response

1. • Use the slope formula to determine the slopes of AB, AC, AD. If the slopes are all different, then A is the point which is not collinear with the other three. If, on the other hand, only one of the slopes is different, then the point that is connected to A in the line segment with the different slope is the point that is not collinear with the other three.
• C
• Since we know that $A, B,$ and D lie on the same straight line, B will be the midpoint of AD if we can show using the distance formula that $AB = BD$ in length.

Equations of Linear Relations Lesson #1:
The Equation of a Line in
Slope y-intercept Form → $y = mx + b$

Overview of Unit

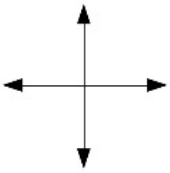
In this unit we express the equation of a linear relation in three different forms: slope y-intercept form, point-slope form, and general form. We relate linear relations expressed in these forms to their graphs.

We also determine the linear relation given: a graph, a table of data points, a point and the slope, two points, a point and the equation of a parallel or perpendicular line.

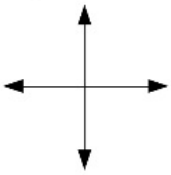
Investigating the Graphs of Linear and Non-Linear Relations

a) The equations of the graphs of some relations are given. In each case, use a graphing calculator to sketch the graph of the relation and make a rough sketch of the graph on the grid provided. Do not list any x- or y-intercepts.

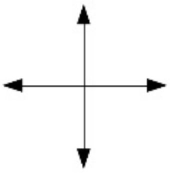
$y = 3x + 1$



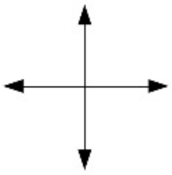
$y = -2x + 3$



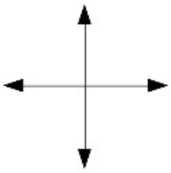
$y = -x^3 + 1$



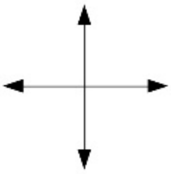
$y = x^2 - 3$



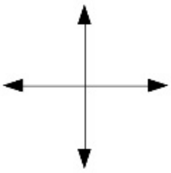
$y = \frac{1}{x}$



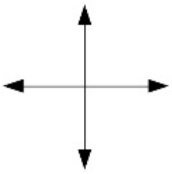
$y = \frac{1}{2}x - 2$



$y = 4x$



$y = 2^x$



b) List the equations of the graphs as linear or non-linear.

LINEAR:

NON-LINEAR:

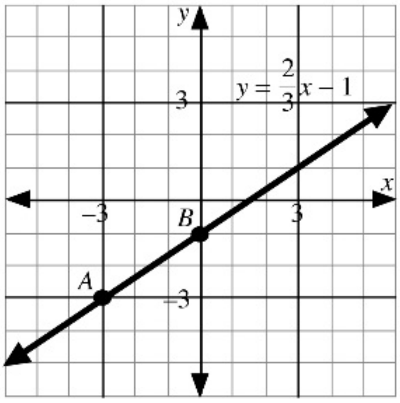
c) Compare the lists. Write a rule from the equation which can be used to determine whether the graph is a straight line or not.

Linear Equation

A **linear equation** is an equation of the form $y = mx + b$, where $m, b \in R$.
The graph of a linear equation is a straight line.

Investigating m and b in the equation $y = mx + b$

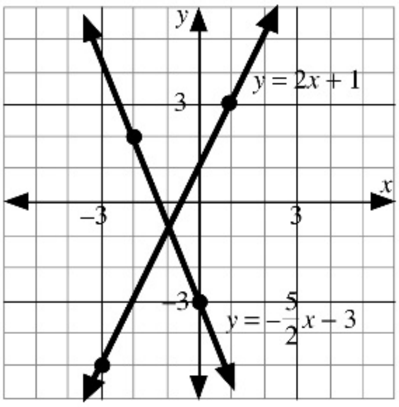
Part One



Jenine used a graphing calculator to sketch the graph of the linear equation $y = \frac{2}{3}x - 1$.
Her sketch is shown on the grid.

a) Use the sketch and points A and B to find the slope and y-intercept of the graph of $y = \frac{2}{3}x - 1$.

b) Compare the values found in **a)** with the coefficient of x and the constant term in the equation $y = \frac{2}{3}x - 1$.



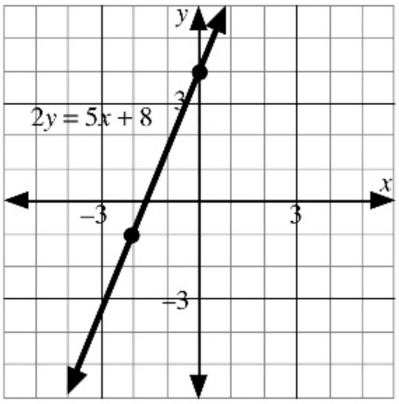
c) Jenine sketched the graphs of two more linear equations. Use the grid to determine the slope and y-intercept of each graph.

equation	slope	y-intercept
$y = 2x + 1$		
$y = -\frac{5}{2}x - 3$		

d) Make a conjecture about the slope and y-intercept of the graph of the linear equation $y = mx + b$.

Part Two

Hashib used a graphing calculator to graph the linear equation $2y = 5x + 8$.
The graph is shown on the grid.



a) Use the sketch to determine the slope and y-intercept of the graph of $2y = 5x + 8$.

b) Explain why, in this case, the slope is not 5 (the coefficient of x) and the y-intercept is not 8 (the constant term).

Slope y-intercept Form of the Equation of a Line → y = mx + b

The graph of an equation in the form $\boldsymbol{y = mx + b}$ (or a function in the form $f(x) = mx + b$) is a straight line with **slope m** and **y-intercept b**.

The equation $y = mx + b$ is known as the **slope y-intercept form** of the equation of a line.

The graph of an equation in this form can be drawn without making a table of values.

Determine the slope and y-intercept of the graph of each linear equation listed below:

a) $y = 3x + 2$ **b)** $y = 7 - \frac{2}{3}x$ **c)** $6y = 8x + 1$

Graphing an Equation of the Form $y = mx + b$

In this section, we will look at two ways of sketching the graph of a linear equation without using a graphing calculator or a table of values.

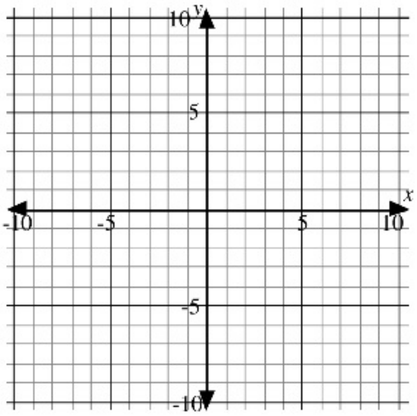
Consider the equation $y = 2x - 5$.

a) State the slope and y-intercept.

b) Mark the y-intercept on the grid.

c) Use the y-intercept and the formula slope $= \frac{\text{rise}}{\text{run}}$ to mark three other points on the grid.
Join the points together, and extend the line.

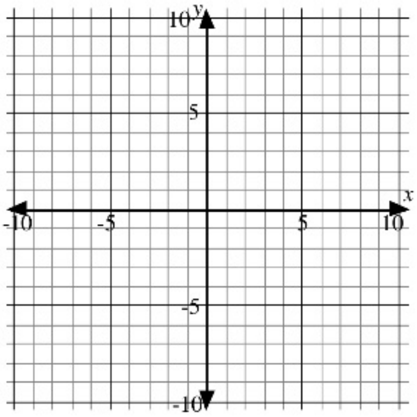
d) Verify the graph using a graphing calculator.



Consider the equation $y = \frac{2}{3}x - 6$.

a) State the y-intercept.

b) Determine the x-intercept algebraically.



c) Mark the x- and y-intercepts on the grid.
Join the points together, and extend the line.

d) Verify the graph and the intercepts using a graphing calculator.

Complete Assignment Questions #1 - #14

Assignment

1. Each equation represents a relation.

a) $y = 6x + 1$ **b)** $y = x^2$ **c)** $y = 3x^4 + 5$ **d)** $y = -\frac{1}{4}x - 8$

e) $y = 1 - x$ **f)** $y = \frac{2}{1 - x}$ **g)** $y = 4x$ **h)** $y = 4^x$

Without sketching the graph of the relation, list the letters a) through h) as linear or non-linear

LINEAR:

NON-LINEAR:

2. State the slope and y-intercept of the graph of each linear equation.

a) $y = 7x - 2$ **b)** $y = \frac{4}{3}x + 3$ **c)** $y = 6 - \frac{1}{6}x$ **d)** $4y = 6x + 8$ **e)** $y = ax + b$

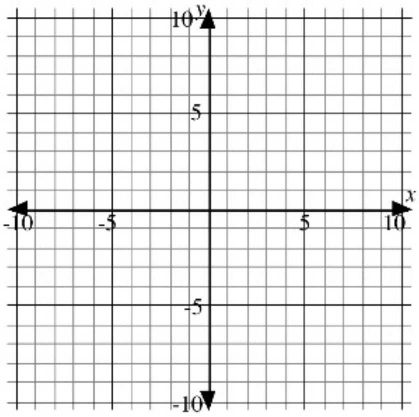
3. Write the equation of each line with the given slope and y-intercept.

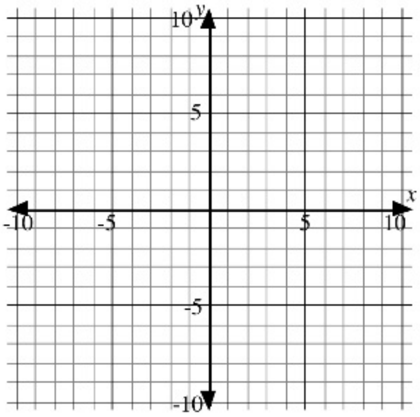
a) slope $= 4$ **b)** slope $= \frac{1}{5}$ **c)** slope $= -3$ **d)** slope $= m$

y-intercept $= -9$ y-intercept $= \frac{1}{2}$ y-intercept $= 0$ y-intercept $= b$

4. For each line, state the slope and the y-intercept. Graph the equation without using a graphing calculator.

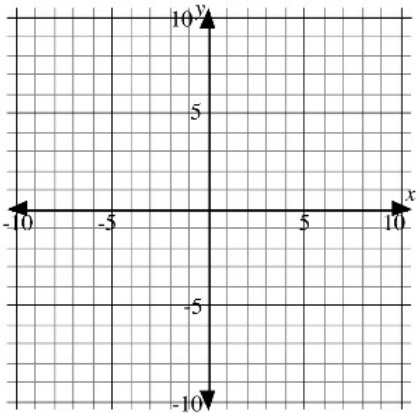
a) $y = \frac{1}{4}x + 2$ **b)** $y = -x - 1$ **c)** $y = -\frac{4}{3}x$ **d)** $y = 5$

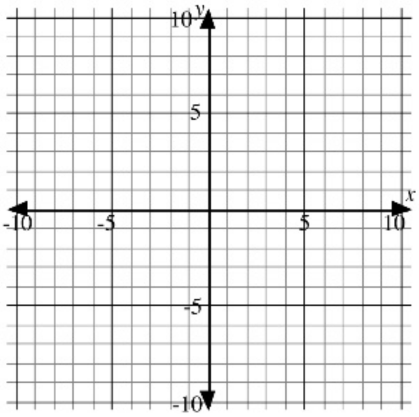




5. For each line, state the y-intercept. Determine the x-intercept algebraically, and graph the equation without using a graphing calculator.

a) $y = 2x + 6$ **b)** $y = -x - 4$ **c)** $y = \frac{6}{7}x - 6$ **d)** $y = -\frac{1}{2}x + 1$





6. Explain why the linear equation $y = 5x$ can be graphed using the method in question 4 but not by the method in question 5.

7. Consider the graph of the function with equation $y = x$.

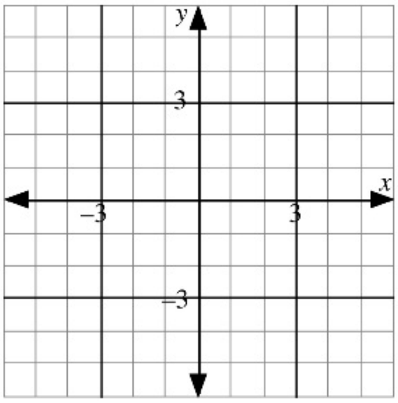
a) State the values of m and b.

b) Determine the x- and y-intercepts.

c) Sketch the graph on the grid provided without using a graphing calculator.

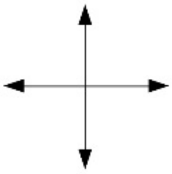
d) Determine the domain and range of the function.

e) Use a graphing calculator to graph the line $y = -x$, and sketch the graph on the grid.



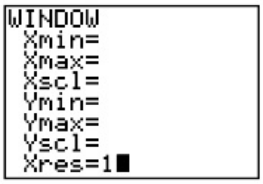
8. Use a graphing calculator to sketch the graph of each of the following linear equations. Complete the table giving the x-intercept to the nearest hundredth.

i) $y = 7x - 8$



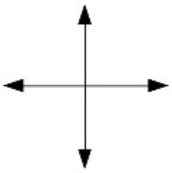
slope	
x-intercept	
y-intercept	

Graphing Window
which includes
both intercepts:



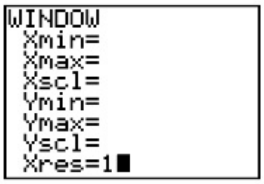
```
WINDOW
 Xmin=
 Xmax=
 Xscl=
 Ymin=
 Ymax=
 Yscl=
 Xres=1■
```

ii) $y = -\frac{31}{2}x - 25$



slope	
x-intercept	
y-intercept	

Graphing Window
which includes
both intercepts:



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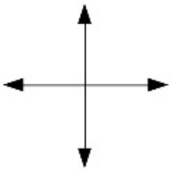
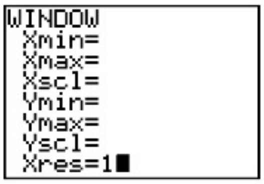
WINDOW
 Xmin=
 Xmax=
 Xscl=
 Ymin=
 Ymax=
 Yscl=
 Xres=1■
```

iii) $y = 75 - \frac{5}{3}x$



slope	
x-intercept	
y-intercept	

Graphing Window
which includes
both intercepts:



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WINDOW
 Xmin=
 Xmax=
 Xscl=
 Ymin=
 Ymax=
 Yscl=
 Xres=1■
```

Multiple Choice

9. Which of the following does not represent the equation of a straight line?

A. $y = 3x$
B. $y = 11 - 3x$
C. $y = \frac{x}{3}$
D. All of the above represent the equation of a straight line.

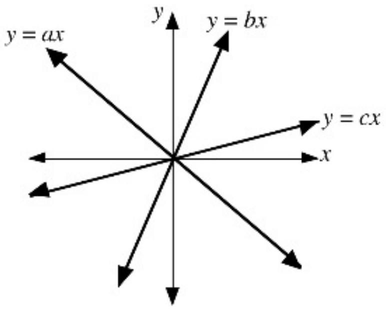
10. Which of the following statements is false for the line $y = -\frac{1}{2}x + 1$?

A. The graph of the line falls from left to right.
B. The x-intercept is 2.
C. The graph passes through the point $(8, -3)$.
D. The line is perpendicular to the line $y = -2x + 4$.

11. Which of the following statements is true for the line $2y = \frac{1}{4}x + 6$?

- **A.** The x-intercept is 24.
- **B.** The y-intercept is 6.
- **C.** The slope is $\frac{1}{8}$.
- **D.** The graph passes through the point $(-4, 5)$.

12. The lines $y = ax$, $y = bx$, and $y = cx$ are shown. Which of the following statements is true?



- **A.** $a < b < c$
- **B.** $a < c < b$
- **C.** $c < a < b$
- **D.** $c < b < a$

Use the following information to answer questions 13 and 14.

> Consider the line with equation $y = 3x + 5$. The line intersects the x-axis at P and the y-axis at Q. Triangle POQ is formed where O is the origin.

Numerical Response **13.** The area of $\angle POQ$, in square units, to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right)

☐☐☐☐

14. To the nearest tenth, the perimeter of ⊿ POQ is _____.

(Record your answer in the numerical response box from left to right)

☐☐☐☐

Answer Key

1. LINEAR a), d), e), g). NON-LINEAR b), c), f), h).

2. **a)** slope = 7, y-int = −2 **b)** slope = $\frac{4}{3}$, y-int = 3

c) slope = $-\frac{1}{6}$, y-int = 6 **d)** slope = $\frac{3}{2}$, y-int = 2 **e)** slope = a, y-int = b

3. **a)** $y = 4x - 9$ **b)** $y = \frac{1}{5}x + \frac{1}{2}$ **c)** $y = -3x$ **d)** $y = mx + b$

4. **a)** slope = $\frac{1}{4}$, y-int = 2 **b)** slope = −1, y-int = −1

c) slope = $-\frac{4}{3}$, y-int = 0 **d)** slope = 0, y-int = 5

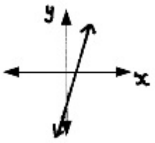
5. **a)** y-int = 6, x-int = −3 **b)** y-int = −4, x-int = −4 **c)** y-int = −6, x-int = 7 **d)** y-int = 1, x-int = 2

6. The method in #4 needs a point and a slope. We have point (0, 0) and slope = 5. The method in #5 needs two points to be joined. Since the x- and y-intercepts are the same point, the line cannot be drawn.

7. **a)** $m = 1$, $b = 0$ **b)** x-int = 0 and y-int = 0. **d)** $D = x \in R$ $R = y \in R$

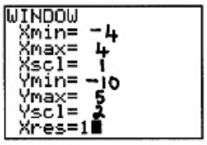
8.

i) $y = 7x - 8$



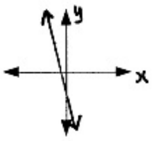
slope	7
x-intercept	1.14
y-intercept	−8

Graphing Window
which includes
both intercepts:



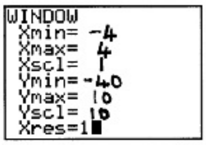
```
WINDOW
 Xmin= -4
 Xmax=  4
 Xscl=  1
 Ymin= -10
 Ymax=  5
 Yscl=  2
 Xres=1█
```

ii) $y = -\frac{31}{2}x - 25$



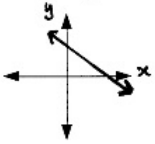
slope	$-\frac{31}{2}$
x-intercept	−1.61
y-intercept	−25

Graphing Window
which includes
both intercepts:



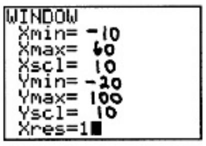
```
WINDOW
 Xmin= -4
 Xmax=  4
 Xscl=  1
 Ymin= -40
 Ymax=  10
 Yscl= 10
 Xres=1█
```

iii) $y = 75 - \frac{5}{3}x$



slope	$-\frac{5}{3}$
x-intercept	45
y-intercept	75

Graphing Window
which includes
both intercepts:



```
WINDOW
 Xmin= -10
 Xmax=  60
 Xscl=  10
 Ymin= -20
 Ymax= 100
 Yscl=  10
 Xres=1█
```

9. D **10.** D **11.** C **12.** B

13.

4	.	2	

14.

1	1	.	9

Equations of Linear Relations Lesson #2:
Writing Equations Using y = mx + b

Review

We have learned that the graph of an equation in the form $y = mx + b$ is a straight line with slope m and y-intercept b.

Using the Form y = mx + b to Write the Equation of a Line

The form $y = mx + b$ can be used to determine the equation of a line when the following information is given:

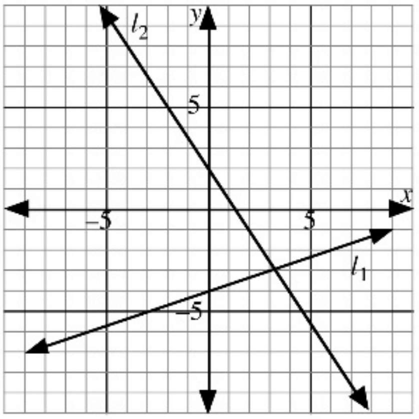
- the slope of the line
- the y-intercept of the line.

Class Ex. #1

Write the equation of a line passing through the point $(0, 2)$ with slope $\frac{5}{2}$.

Class Ex. #2

Each line on the grid passes through points with integer coordinates. In each case, state the slope and y–intercept of the line, and determine the equation of the line.



Class Ex. #3

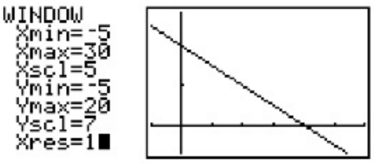
Determine the equation of the following lines.

a) The line parallel to $y = \frac{1}{3}x + 4$, and with the same y-intercept as $y = 6x - 7$.

b) The line passing through $(0, 9)$, and perpendicular to the line joining $(2, -6)$ and $(-5, 0)$.

Class Ex. #4

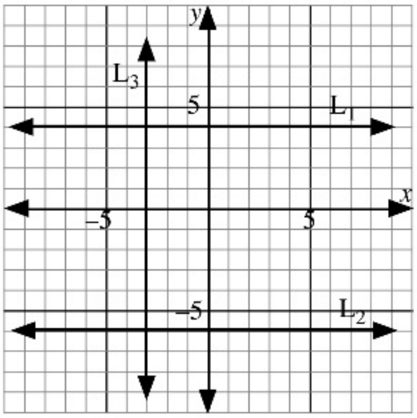
The diagram shows the display from a graphing calculator screen. The intercepts are integers. Determine the equation of the line shown.



```
WINDOW
 Xmin=-5
 Xmax=30
 Xscl=5
 Ymin=-5
 Ymax=20
 Yscl=7
 Xres=1■
```

Complete Assignment Questions #1 - #5

Horizontal and Vertical Lines



a) State the slope and y-intercept of the horizontal line L_1 shown on the grid.

b) Use the form $y = mx + b$ to determine the equation of the horizontal line L_1 .

c) Predict the equation of the horizontal line L_2 . Use a graphing calculator to verify.

d) State the slope and y-intercept of the vertical line L_3 shown on the grid.

e) Why can we not use the form $y = mx + b$ to determine the equation of the vertical line L_3?

f) Predict the equation of the vertical line L_3 .
Why can we not use a graphing calculator to verify?

The equation $y = k$ represents a horizontal line through $(0, k)$.
The equation $x = k$ represents a vertical line through $(k, 0)$.

Determine the equation of the line through the point $(-2, 8)$ and

a) parallel to the y-axis

b) parallel to the x-axis

Complete Assignment Questions #6 - #15

Assignment

1. Write the equation of each line

a) with slope 4 and y-intercept -6

b) with a y-intercept of 3 and a slope of $-\frac{4}{3}$

c) passing through the origin with a slope of $-\frac{3}{5}$

d) with y-intercept -5 and parallel to $y = x$

e) with a y-intercept of -9 and perpendicular to $y = -\frac{2}{3}x + 7$

f) with the same y-intercept as $y = x + 2$ and parallel to $y = \frac{1}{4}x - 6$

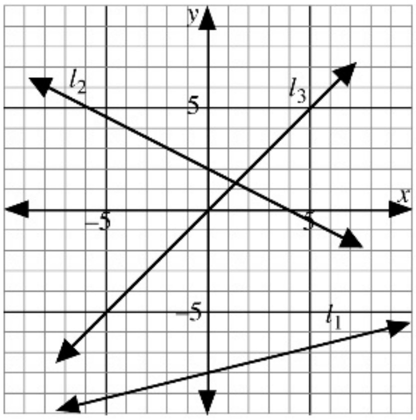
g) through the point $(0, 1)$ and perpendicular to $y = 4x - 2$

h) through the point $(0, 4)$ and parallel to $y = \frac{1}{10}x + 24$

i) with the same y-intercept as $y = 2x - 3$ and perpendicular to $y = \frac{7}{3}x - 2$

j) with the same y-intercept as $y = ax + b$ and perpendicular to $y = cx + d$

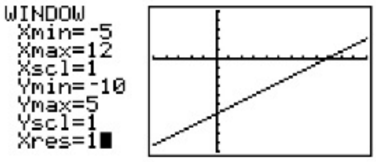
2. Every line on the grid passes through points with integer coordinates.
Determine the equation of each line.



3. Each diagram represents the image from the display of a graphing calculator and the window setting used to graph a linear equation. The x- and y-intercepts of each graph are integers.

In each case complete the table.

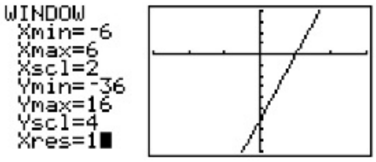
a)



```
WINDOW
 Xmin=-5
 Xmax=12
 Xscl=1
 Ymin=-10
 Ymax=5
 Yscl=1
 Xres=1
```

x-intercept	
y-intercept	
slope	
equation	

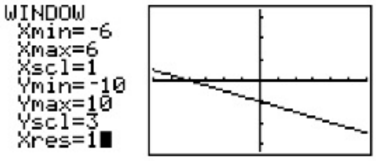
b)



```
WINDOW
 Xmin=-6
 Xmax=6
 Xscl=2
 Ymin=-36
 Ymax=16
 Yscl=4
 Xres=1
```

x-intercept	
y-intercept	
slope	
equation	

c)



```
WINDOW
 Xmin=-6
 Xmax=6
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=3
 Xres=1
```

x-intercept	
y-intercept	
slope	
equation	

4. Determine the equation of the line which passes through the point $(0, 16)$ and is parallel to the line which passes through $(1, 3)$ and $(4, -6)$.

5. Determine the equation of the line which passes through the point $(0, -1)$ and is perpendicular to the line which passes through $(7, -2)$ and $(12, -3)$.

6. State the equations of the following lines

a) through the point $(-5, 3)$ and parallel to the y-axis

b) through the point $(-5, 3)$ and parallel to the x-axis

c) through the point $(1, -1)$ and parallel to the x-axis

d) through the point (a, b) and parallel to the y-axis

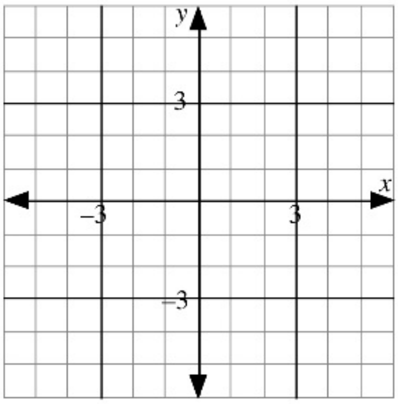
7. Consider the graph of the function with equation $y = 2$.

a) State the values of m and b.

b) Sketch the graph on the grid provided.

c) State the x- and y-intercepts of the graph.

d) Determine the domain and range of the function.



e) On the same grid draw the line with equation $y = 2x - 4$ without using a graphing calculator.

f) State the coordinates of the point of intersection of the two lines.

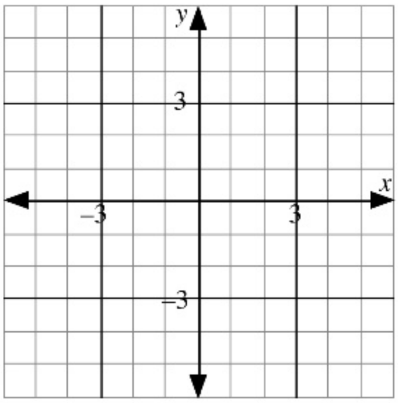
g) On the grid, draw the line with equation $y = -5$.

8. Consider the graph of the relation with equation $x = -4$.

a) Sketch the graph on the grid provided.

b) State the x- and y-intercepts of the graph.

c) Explain why the relation is not a function.



d) Explain why the Y = editor key cannot be used to graph $x = -4$.

e) Determine the domain and range of the relation with equation $x = -4$.

f) On the grid draw the line with equation $x = 2$.

9. Write the equation of each line

a) parallel to the x-axis through $(3, -9)$

b) parallel to the y-axis through $(3, -9)$

c) perpendicular to the x-axis through $(1, 4)$

d) perpendicular to the y-axis through $(1, 4)$

e) the x-axis

f) the y-axis

Multiple Choice

10. A line is parallel to the y-axis and passes through the point $(2, -7)$. The equation of the line is

A. $x = 2$
B. $x = -7$
C. $y = 2$
D. $y = -7$

11. A line is parallel to the x-axis and passes through the point (–6, 10).
The equation of the line is

A. $x = 10$
B. $x = -6$
C. $y = 10$
D. $y = -6$

12. The line through the origin, perpendicular to the line with equation $y = \frac{2}{3}x$, has equation

A. $y = \frac{2}{3}x$
B. $y = \frac{3}{2}x$
C. $y = -\frac{2}{3}x$
D. $y = -\frac{3}{2}x$

13. The point $(2, -1)$ lies on a line with slope 3. The y- intercept of the line is

A. -7
B. -5
C. 5
D. 7

Numerical Response **14.** Consider the line which is perpendicular to the line $y = \frac{1}{3}x + 4$ and has the same y-intercept as $y = 6x - 7$. If the equation of this line is written in the form $y = mx + b$, then the exact value of $m - b$ is _____.

(Record your answer in the numerical response box from left to right)

15. Two perpendicular lines intersect on the y-axis. One line has equation $y = 4x + 6$. If the equation of the other line is $y = mx + b$, then the exact value of $m + b$ is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $y = 4x - 6$ **b)** $y = -\frac{4}{3}x + 3$ **c)** $y = -\frac{3}{5}x$ **d)** $y = x - 5$ **e)** $y = \frac{3}{2}x - 9$
f) $y = \frac{1}{4}x + 2$ **g)** $y = -\frac{1}{4}x + 1$ **h)** $y = \frac{1}{10}x + 4$ **i)** $y = -\frac{3}{7}x - 3$ **j)** $y = -\frac{1}{c}x + b$

2. l_1: $y = \frac{1}{4}x - 8$ l_2: $y = -\frac{1}{2}x + 2$ l_3: $y = x$

3. **a)**

x-intercept	9
y-intercept	-6
slope	2/3
equation	$y = \frac{2}{3}x - 6$

b)

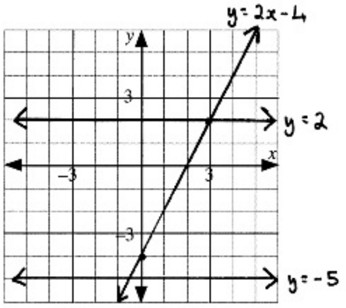
x-intercept	2
y-intercept	-24
slope	12
equation	$y = 12x - 24$

c)

x-intercept	-4
y-intercept	-3
slope	-3/4
equation	$y = -\frac{3}{4}x - 3$

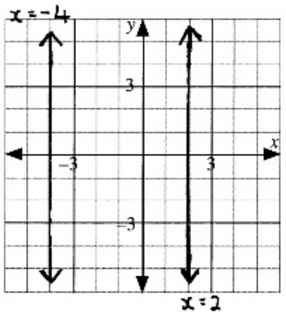
4. $y = -3x + 16$ **5.** $y = 5x - 1$ **6.** **a)** $x = -5$ **b)** $y = 3$ **c)** $y = -1$ **d)** $x = a$

7. **a)** $m = 0, b = 2$
b), e), g) see graph below



c) no x-intercept, y-intercept = 2
d) domain $x \in R$, range $y = 2$ **f)** (3, 2)

8. **a), f)** see graph below
b) x-intercept = –4, no y-intercept



c) When the input = –4, there are multiple values for the output. The graph of the relation does not pass the vertical line test.
d) The equation $x = -4$ cannot be written in the form y= . . . .
e) domain $x = -4$, range $y \in R$

9. **a)** $y = -9$ **b)** $x = 3$ **c)** $x = 1$ **d)** $y = 4$ **e)** $y = 0$ **f)** $x = 0$

10. A **11.** C **12.** D **13.** A

14.

4			

15.

5	.	7	5

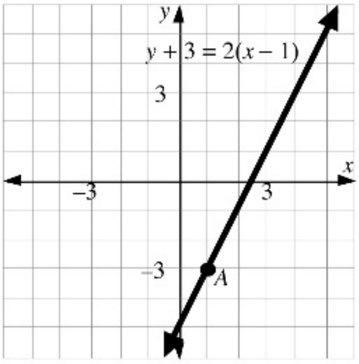
Equations of Linear Relations Lesson #3:
Writing Equations using $y - y_1 = m(x - x_1)$

Review

In the last lesson we learned how to write the equation of a straight line using slope y-intercept form, namely $y = mx + b$ where m is the slope and b is the y-intercept.

There is another equally important method exists in writing the equation of straight lines. The investigation below explores this method.

Investigation *Point-Slope Form*

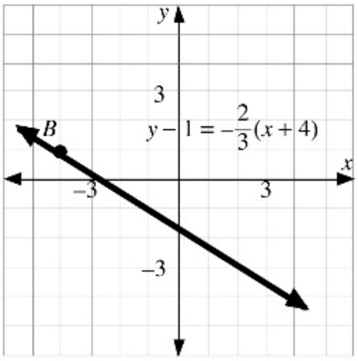


The graph of $y + 3 = 2(x - 1)$ is shown on the grid.

a) Determine the slope of the graph of $y + 3 = 2(x - 1)$.

b) List the coordinates of point A on the line.

c) Compare your answers in **a)** and **b)** with the numbers in the equation.



The graph of $y - 1 = -\frac{2}{3}(x + 4)$ is shown on the grid.

d) Determine the slope of the graph of $y - 1 = -\frac{2}{3}(x + 4)$.

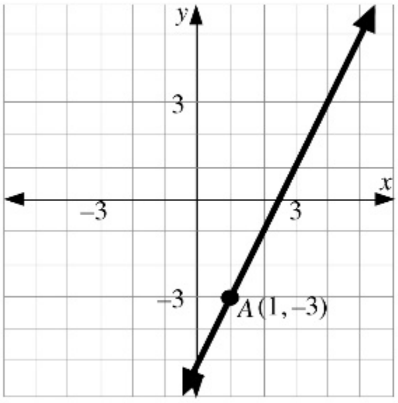
e) List the coordinates of point B on the line.

f) Compare your answers in **d)** and **e)** with the numbers in the equation.

g) Consider the graph of the linear equation $y - y_1 = m(x - x_1)$. Based on your observations in **c)** and **f)**, state the slope of the line, and write the coordinates of one point on the line.

Equation of a Line Given the Slope of the Line and a Point on the Line

Consider the line with slope 2 passing through the point $A(1, -3)$. The line is shown on the grid.



Our objective is to determine the equation of the line. In other words, to find a relation between x and y which is satisfied by every point (x, y) on the line.

Let $P(x, y)$ be any point on the line except A.

Using the slope formula we have

$$\frac{y_P - y_A}{x_P - x_A} = m_{AP} \qquad \frac{y-(-3)}{x-1} = 2$$

Cross multiply and solve for y to determine the equation of the line in the form $y = mx + b$.

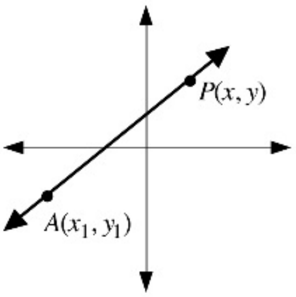
At this point in the exploration, the equation above is valid for all points on the line except A.

Note that the coordinates of A also satisfy the equation, and therefore it is the equation of all points on the line.

In the next section we will use the same procedure to develop a formula for the equation of any line, given the slope of the line and the point on the line.

The Equation of the Line with slope m through the point (x_1, y_1)

Consider the line with slope m passing through the point with coordinates (x_1, y_1).
We will use the same procedure as above to show that the equation of the line can be expressed in the form $y - y_1 = m(x - x_1)$.



Let $P(x, y)$ be any point on the line distinct from A.

Using the slope formula we have

$$m_{AP} = \frac{y_P - y_A}{x_P - x_A} \qquad \text{so} \qquad m = \frac{y - \quad}{x - \quad}$$

Point-Slope Equation of a Line → $y - y_1 = m(x - x_1)$

- The point-slope form of the equation of a line is $y - y_1 = m(x - x_1)$ where m is the slope of the line, and (x_1, y_1) represents a point on the line.
- To determine the equation of a line in future math courses, the point-slope equation, $y - y_1 = m(x - x_1)$, is used more frequently than the slope-y-intercept equation, $y = mx + b$.

- The point-slope equation is used when we have **the slope of a line and the coordinates of any point on that line**.
- It is customary to give the final equation in slope y-intercept form or in the general form, $Ax + By + C = 0$ (to be taught in the next lesson).

State the equation, in point-slope form, of the line through the given point and with the given slope.

a) $(6, 5),\ 3$ **b)** $(1, -1), -4$ **c)** $(-9, -8),\ \frac{1}{2}$

In each case the slope of a line and a point on the line are given. Determine the equation of the line in slope y-intercept form, $y = mx + b$.

a) $m = 5$, point $(-5, 2)$ **b)** $m = -7$, point $(-3, 4)$

John and Nicki were solving the following quiz question:

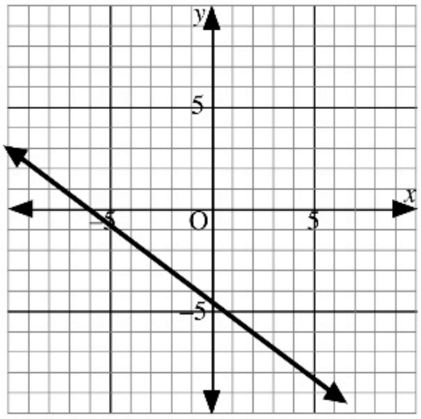
" Determine the equation, in slope y-intercept form, of a line with slope -2 passing through the point $(3, -5)$".

John could only remember the slope y-intercept form $y = mx + b$, but Nicki remembered the point-slope form $y - y_1 = m(x - x_1)$. Complete their work which is started below.

John's work	Nicki's work
$y = mx + b$	$y - y_1 = m(x - x_1)$
$y = -2x + b$	$y - y_1 = -2(x - x_1)$
$-5 = -2(3) + b$	$y - (-5) = -2(x - 3)$

Class Ex. #4

The line on the grid passes through at least two points with integer coordinates. Determine the equation of the line in slope y-intercept form.



Class Ex. #5

In each case, state the slope of the line and write the coordinates of a point on the line.

a) $y + 11 = \frac{1}{7}(x - 4)$

b) $y - 9 = -\frac{5}{3}(x - 7)$

c) $y = -3(x - 6)$

d) $y = -3x - 6$

Complete Assignment Questions #1 - #9

Assignment

1. State the equation, in point-slope form, of the line through the given point and with the given slope.

a) $(9, 3)$, 4

b) $(8, -2)$, -3

c) $(-5, 7)$, 1

d) $(0, 3)$, $\frac{1}{2}$

e) $(-7, 0)$, $\frac{1}{4}$

f) $(-\frac{1}{2}, -\frac{5}{4})$, $\frac{6}{5}$

2. Write the following equations in slope y-intercept form $y = mx + b$.

a) $y + 1 = 8(x - 2)$

b) $y - 3 = -2(x - 7)$

c) $y - 9 = -11(x + 3)$

3. Find the equation, in slope y-intercept form, of the line through the given point and with the given slope.

a) $(2,4)$, 6 **b)** $(2,-1)$, 2 **c)** $(0,4)$, -2

d) $(-6,2)$, $\frac{1}{2}$ **e)** $(-7,-7)$, 1 **f)** $(0,b)$, m

4. Find the equation, in slope y-intercept form, of the line through the given point and with the given slope.

a) $(2,-5)$, $\frac{1}{4}$ **b)** $(-4,2)$, $-\frac{1}{3}$ **c)** $(0,-8)$, $-\frac{3}{4}$

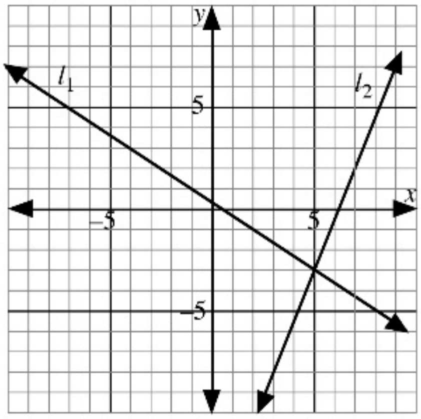
5. The point-slope equation of a line is given. State the slope and the coordinates of the point which were used to write the equation.

a) $y - 9 = -\frac{11}{3}(x+3)$ **b)** $y + 3 = \frac{1}{2}x$ **c)** $y - 8 = -2(x-6)$

d) $y = 3(x+12)$ **e)** $y - 9 = -\frac{5}{3}x$ **f)** $y = \frac{2}{5}x$

6. Two lines have been drawn on the grid. Each line passes through at least two points with integer coordinates.

Determine the equation of each line.



Multiple Choice

7. The equation of the line passing through the point $(4, 2)$ with slope -3 is

A. $y = -3x - 14$
B. $y = -3x - 10$
C. $y + 2 = -3(x + 4)$
D. $y - 2 = -3(x - 4)$

8. Which of the following linear equations is equivalent to $y - 3 = -\frac{3}{4}(x + 7)$?

A. $y = -\frac{3}{4}x + \frac{9}{4}$

B. $y = -\frac{3}{4}x - \frac{9}{4}$

C. $y = -\frac{3}{4}x + 10$

D. $y = -\frac{3}{4}x - 10$

Numerical Response **9.** The equation of the line with an x-intercept of -2 and slope 12 can be written in the form $y - A = C(x - B)$. The value of $A + B + C$ is _____.

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $y - 3 = 4(x - 9)$ **b)** $y + 2 = -3(x - 8)$ **c)** $y - 7 = 1(x + 5)$
d) $y - 3 = \frac{1}{2}x$ **e)** $y = \frac{1}{4}(x + 7)$ **f)** $y + \frac{5}{4} = \frac{6}{5}\left(x + \frac{1}{2}\right)$

2. **a)** $y = 8x - 17$ **b)** $y = -2x + 17$ **c)** $y = -11x - 24$

3. **a)** $y = 6x - 8$ **b)** $y = 2x - 5$ **c)** $y = -2x + 4$ **d)** $y = \frac{1}{2}x + 5$ **e)** $y = x$ **f)** $y = mx + b$

4. **a)** $y = \frac{1}{4}x - \frac{11}{2}$ **b)** $y = -\frac{1}{3}x + \frac{2}{3}$ **c)** $y = -\frac{3}{4}x - 8$

5. **a)** $m = -\frac{11}{3}$, $P(-3, 9)$ **b)** $m = \frac{1}{2}$, $P(0, -3)$ **c)** $m = -2$, $P(6, 8)$ **d)** $m = 3$, $P(-12, 0)$
e) $m = -\frac{5}{3}$, $P(0, 9)$ **f)** $m = \frac{2}{5}$, $P(0, 0)$

6. $l_1 \Rightarrow 2x + 3y - 1 = 0$ or $y = -\frac{2}{3}x + \frac{1}{3}$ $l_2 \Rightarrow 5x - 2y - 31 = 0$ or $y = \frac{5}{2}x - \frac{31}{2}$

7. D **8.** B **9.**

1	0		

Equations of Linear Relations Lesson #4:
The General Form Equation Ax + By + C = 0

Review

In Lesson #2 and Lesson #3 of this unit we studied two forms of the equation of a straight line. The form of these equations are

- the slope y-intercept form which can also be writen as ___________________
- the point-slope form which can also be written as _______________________

General Form of the Equation of a Line → Ax + By + C = 0

The **general form** of the equation of a line is an equation where all the terms are collected to the left side of the equation. The right side of the equation is zero.
It has the following characteristics:

- It is written as $Ax + By + C = 0$, where A, B, and C are expressed as **integers** if possible, and A is usually positive.
- It allows us to write equations for oblique lines, horizontal lines, and vertical lines.
- In some texts, the form $Ax + By + C = 0$ is referred to as **standard form**.

Convert the following equations from slope y-intercept form, $y = mx + b$, to general form, $Ax + By + C = 0$, where A, B, and C are integers.

a) $y + 2 = \frac{1}{2}(x - 3)$ **b)** $y = \frac{2}{3}x + 7$ **c)** $y = -\frac{1}{4}x + \frac{3}{5}$

Determine the equation, in general form, of the line with slope $-\frac{3}{7}$ passing through $(2, -1)$.

Determining the Slope and y-intercept from $Ax + By + C = 0$

Given the equation of a line in general form, $Ax + By + C = 0$, the slope and y-intercept can be found by converting the equation into slope y-intercept form, $y = mx + b$.

Determine the slope and y-intercept of the graph of the following lines.

a) $2x - 5y + 25 = 0$ **b)** $6x + 2y - 15 = 0$

Complete Assignment Questions #1 - #4

The lines $3x - 4y + 8 = 0$ and $5x - ky - 6 = 0$ have the same y-intercept. Determine the value of k.

Which of the following lines is/are perpendicular to the line $4x - 2y + 9 = 0$?

i) $6x + 3y - 1 = 0$ **ii)** $x + 2y - 12 = 0$ **iii)** $5x + 10y = 0$

Use the following information to answer Class Ex. #6.

A student made the following statements about the line with equation $2y = 5x + 12$.

Statement 1: The line has a slope of 5.
Statement 2: The line is parallel to $10x - 4y + 13 = 0$.
Statement 3: The line passes through $(-2, 1)$.

Which of the above statement(s) is/are true?

A. 1 and 2 only
B. 1 and 3 only
C. 2 and 3 only
D. some other combination of statements 1, 2, and 3

Complete Assignment Questions #5 - #16

Assignment

1. Convert the following equations to general form $(Ax + By + C = 0)$ where A, B, and C are integers.

a) $y - 4 = 7(x - 1)$ **b)** $y = -2x + 9$ **c)** $y = mx + b$

d) $y = -\frac{3}{4}x + 5$ **e)** $y + 8 = -\frac{3}{2}(x - 5)$ **f)** $y = \frac{5}{3}x - \frac{1}{4}$

2. Find the equation, in general form, of the line through the given point and with the given slope.

a) $(6, 1)$, 3 **b)** $(-9, -2)$, $\frac{2}{5}$ **c)** $(0, 0)$, $\frac{4}{3}$

3. Determine the slope and y-intercept of the graph of the following lines.

a) $x + y - 11 = 0$ **b)** $3x - 2y + 30 = 0$ **c)** $3x + 6y - 7 = 0$

4. Determine the slope, y-intercept, and x-intercept of the graph of the following lines.

a) $2x + y - 6 = 0$ **b)** $5x - 2y + 20 = 0$ **c)** $4x - 5y - 3 = 0$

5. Consider the lines $x - 2y + 1 = 0$ and $4x + ky - 8 = 0$.

a) Determine the value of k if the lines have the same slope.

b) Determine the value of k if the lines have the same y-intercept.

6. Consider the lines $3x - 5y - 15 = 0$ and $ax + 2y - 6 = 0$.

a) Determine the value of a if the lines have the same slope.

b) Determine the value of a if the lines have the same x-intercept.

Multiple Choice

7. The equation of the line passing through the orgin with slope $-\frac{1}{2}$ is

A. $x + 2y = 0$

B. $x - 2y = 0$

C. $2x + y = 0$

D. $2x - y = 0$

8. Match each equation on the left with the correct characteristic of the graph of the equation on the right. Each characteristic may be used once, more than once, or not at all.

Equation		Characteristic	
i)	$6x - 2y + 5 = 0$	**A.**	Slope = $-\frac{1}{3}$
ii)	$2x - 5y = 0$	**B.**	y-intercept = $-\frac{5}{2}$
iii)	$x + 3y + 6 = 0$	**C.**	Passes through $(-10, -4)$
iv)	$x - 4y + 10 = 0$	**D.**	Slope = 0
v)	$2x - y - 5 = 0$	**E.**	y-intercept = $\frac{5}{2}$
		F.	Perpendicular to $y = \frac{5}{2}x - 3$
		G.	x-intercept = $\frac{5}{2}$

9. The slope of the line with equation $6x + 5y - 1 = 0$ is

A. $-\frac{6}{5}$
B. $-\frac{5}{6}$
C. $\frac{6}{5}$
D. $\frac{1}{5}$

10. Which line has a y-intercept of 1?

A. $x + 5y + 1 = 0$
B. $x + 3y + 3 = 0$
C. $x - 2y + 2 = 0$
D. $2y = 3x + 1$

11. The slope of a line perpendicular to the line $x + 3y + 8 = 0$ is

A. -8

B. $-\frac{1}{3}$

C. $\frac{1}{3}$

D. 3

12. The line $2y + 3x + 6 = 0$ intersects the y-axis at P.
The slope of the line joining P to $Q(6, -2)$ is

A. $-\frac{5}{6}$

B. $\frac{1}{6}$

C. $-\frac{1}{6}$

D. $-\frac{2}{3}$

13. The lines with equations $ay = 4x + 9$ and $y = 5x - 7$ are perpendicular.
The value of a is

A. $\frac{4}{5}$

B. $-\frac{4}{5}$

C. $-\frac{5}{4}$

D. -20

Use the following information to answer the next question.

Consider the following statements about all the lines in the form $kx + 4y - 8 = 0$, where $k \in R$.

Statement 1: The lines have the same slope.
Statement 2: The lines have the same y-intercept.
Statement 3: The lines have the same x-intercept.

14. Which of the above statement(s) is/are true?

A. 1, 2, and 3
B. 1 only
C. 2 only
D. 3 only

15. Line L has equation $5x - 3y + 21 = 0$. A is the point $(-6, -3)$, B is $(3, -2)$, and C is $(-3, 2)$. Which of these points lie on line L?

A. A only
B. A and B only
C. A and C only
D. B and C only

Numerical Response

16. Given that the line joining the points $(2, 3)$ and $(8, -q)$, where $q \in W$, is perpendicular to the line $3x - 2y - 5 = 0$, then the value of q is ________ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $7x - y - 3 = 0$ **b)** $2x + y - 9 = 0$ **c)** $mx - y + b = 0$
d) $3x + 4y - 20 = 0$ **e)** $3x + 2y + 1 = 0$ **f)** $20x - 12y - 3 = 0$

2. a) $3x - y - 17 = 0$ **b)** $2x - 5y + 8 = 0$ **c)** $4x - 3y = 0$

3. a) slope $= -1$, y–int $= 11$ **b)** slope $= \frac{3}{2}$, y–int $= 15$ **c)** slope $= -\frac{1}{2}$, y–int $= \frac{7}{6}$

4. a) slope $= -2$, y–int $= 6$, x–int $= 3$
b) slope $= \frac{5}{2}$, y–int $= 10$, x–int $= -4$
c) slope $= \frac{4}{5}$, y–int $= -\frac{3}{5}$, x–int $= \frac{3}{4}$

5. a) -8 **b)** 16 **6. a)** $-\frac{6}{5}$ **b)** $\frac{6}{5}$

7. A
8. i) E **ii)** C **iii)** A **iv)** E **v)** G
9. A **10.** C **11.** D **12.** B
13. D **14.** C **15.** C **16.** 1

Find the equation, in general form, of the line perpendicular to the line $5x - 7y - 10 = 0$ and with the same x-intercept as $x - 2y - 12 = 0$.

Complete Assignment Questions #1 - #14

Assignment

1. Find the equation, in general form, of the line through each pair of points.

a) $(7, 5)$ and $(6, 1)$ **b)** $(3, -7)$ and $(-5, 9)$ **c)** $(-3, 4)$ and $(11, 25)$

d) $(10, -15)$ and $(-2, -12)$ **e)** $(4, -7)$ and $(3, -7)$ **f)** $(-5, -8)$ and $(-4, -10)$

2. Identify the lines in **1.** which are

i) parallel **ii)** perpendicular

3. Write the equation of each line in general form

a) with slope $\frac{2}{7}$ and an x-intercept of -6 **b)** with a y-intercept of $-\frac{8}{3}$ and a slope of 7

c) through the point $(2, 0)$ and perpendicular to $3x - 5y + 19 = 0$

d) through the point $(3, -6)$ and parallel to $5x + 3y + 9 = 0$

4. Write the equation of each line in general form

a) perpendicular to $y = x$ and with the same x-intercept as $y = 2x + 10$

b) parallel to $2x - 3y + 7 = 0$ and with the same y-intercept as $5x - 3y - 12 = 0$

5. Write the equation of each line in general form

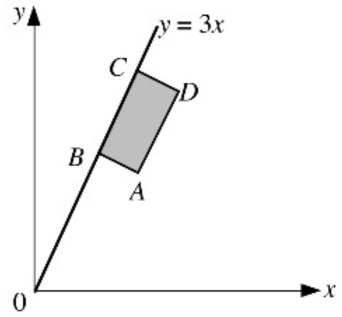
a) perpendicular to $6x - 2y + 5 = 0$ and with the same y-intercept as $x - y + 8 = 0$

b) with the same x-intercept as $9x - 2y + 18 = 0$ and through the point $(4, -5)$

6. Line l contains the point $A(7, 9)$ and is parallel to a line which contains the points $B(-4, 5)$ and $C(8, -1)$. Determine the equation of line l in the form $y = mx + b$.

7. A Cartesian plane is placed on the plan of a farm. The farmhouse is at the origin, and $ABCD$ represents a rectangular field of wheat. A farm road, with equation $y = 3x$, runs from the farmhouse along one side of the field.

a) If the point A has coordinates $(2, 4)$, determine the equation of AD.



b) Determine the equation of AB.

8. A child with a fixed amount of money can buy 2 bags of chips and 5 cans of pop, or 3 bags of chips and 2 cans of pop. A linear relationship exists between the number of bags of chips, x, and the number of cans of pop, y, which can be bought.

a) Write the coordinates of two points which lie on the graph of this linear relationship.

b) Determine the equation of the linear relationship.

Multiple Choice

9. The equation of the line through the point $(7, -4)$ and perpendicular to the line with equation $5x - 4y + 13 = 0$, can be written in the form

A. $y + 4 = \frac{5}{4}(x - 7)$

B. $y = -\frac{4}{5}(x + 7)$

C. $y + 4 = -\frac{4}{5}(x - 7)$

D. $y + 4 = \frac{4}{5}(x - 7)$

10. A line passing through the point $(0, 3)$ is perpendicular to the line $x - 2y - 5 = 0$. The equation of the line is

A. $2x + y - 3 = 0$

B. $2x + y + 3 = 0$

C. $x - 2y + 6 = 0$

D. $2x - y + 3 = 0$

11. Which of the following linear relations is not equivalent to the other three?

A. $y - 4 = -\frac{1}{3}(x + 6)$

B. $x + 3y + 2 = 0$

C. the line passing through $(0, 2)$ and $(6, 0)$

D. $y = -\frac{1}{3}x + 2$

12. A line passing through the point $(0, 3)$ is parallel to the line $x - 2y - 5 = 0$. The equation of the line is

A. $2x + y - 3 = 0$
B. $2x + y + 3 = 0$
C. $x - 2y + 6 = 0$
D. $2x - y + 3 = 0$

13. The image of $y = 2x + 7$ after a counterclockwise rotation of 90° about the origin is

A. $y = -\frac{1}{2}x + \frac{7}{2}$
B. $y = \frac{1}{2}x - \frac{7}{2}$
C. $y = -\frac{1}{2}x - \frac{7}{2}$
D. $y = -2x - 7$

Numerical Response **14.** The line through the points $(-3, 4)$ and $(-1, -2)$ has equation $y + ax + b = 0$, where a and b are integers. The value of $a + b$ is _____.
(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** $4x - y - 23 = 0$ **b)** $2x + y + 1 = 0$ **c)** $3x - 2y + 17 = 0$
d) $x + 4y + 50 = 0$ **e)** $y + 7 = 0$ **f)** $2x + y + 18 = 0$
2. **i)** b and f **ii)** a and d
3. **a)** $2x - 7y + 12 = 0$ **b)** $21x - 3y - 8 = 0$ **c)** $5x + 3y - 10 = 0$ **d)** $5x + 3y + 3 = 0$
4. **a)** $x + y + 5 = 0$ **b)** $2x - 3y - 12 = 0$
5. **a)** $x + 3y - 24 = 0$ **b)** $5x + 6y + 10 = 0$ **6.** $y = -\frac{1}{2}x + \frac{25}{2}$
7. **a)** $y = 3x - 2$ or $3x - y - 2 = 0$ **b)** $y = -\frac{1}{3}x + \frac{14}{3}$ or $x + 3y - 14 = 0$
8. **a)** $(2, 5)$ and $(3, 2)$ **b)** $3x + y - 11 = 0$
9. C **10.** A **11.** B **12.** C **13.** C **14.**

8			

Equations of Linear Relations Lesson #6:
Graphing Linear Equations

Graphing Linear Equations Without Technology

Linear equations can be written in different forms:

$Ax + By + C = 0$ → General form of a linear equation.

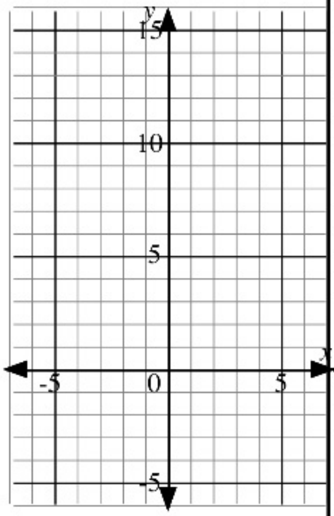
$y = mx + b$ → Slope y-intercept form of a linear equation.

$y - y_1 = m(x - x_1)$ → Point-slope form of a linear equation.

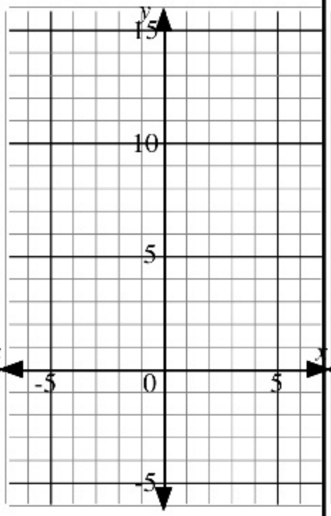
The method used to graph a linear relation without technology depends on the form in which the linear equation is written.

Without altering the form in which the linear equation is written, explain the different strategies used to graph (without technology) each of the following linear relations. Draw the graph of each linear relation on the grid provided.

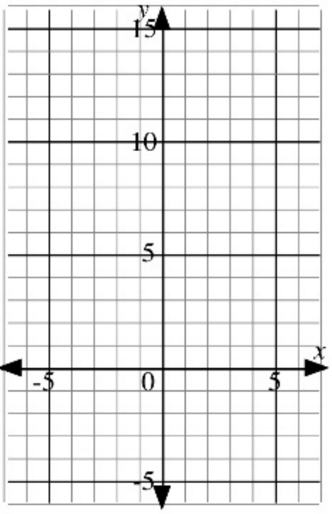
a) $y = 2x + 4$



b) $y - 2 = 2(x + 1)$



c) $2x - y + 4 = 0$



Class Ex. #2

Match each linear relation to its graph.

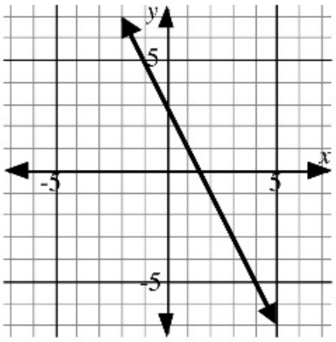
Equation 1:
$2x - y + 3 = 0$

Equation 2:
$y = -2x + 3$

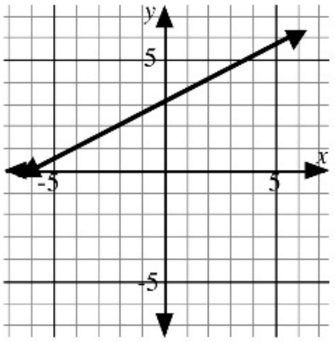
Equation 3:
$y + 1 = -\frac{1}{2}(x + 4)$

Equation 4:
$2y - x - 6 = 0$

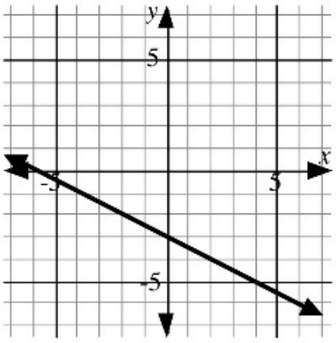
Graph A



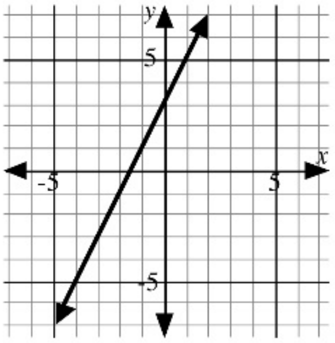
Graph B



Graph C



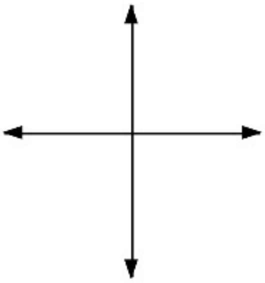
Graph D



Class Ex. #3

Graphing Linear Equations With Technology

- Explain the strategy used to graph (with technology) the linear relations.
  $y + 8 = -5(x - 2)$ and $4x - y + 9 = 0$



- State an appropriate window to show x- and y-intercepts, and draw the graph of both linear relations on the same grid.

- Determine the x and y-intercepts of $4x - y + 9 = 0$.

Complete Assignment Questions #1 - #6

Determining Linear Relationships from Tables of Data

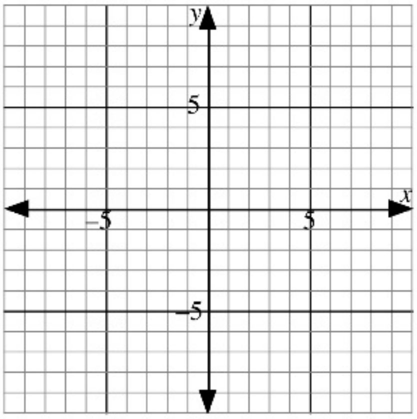
Consider the following data points expressed in a table of values

x	2	4	6	8	10
y	-1	0	1	2	3

a) Plot the data points on the grid to verify that there is a linear relationship.

b) Join the points together and determine the slope of the line.

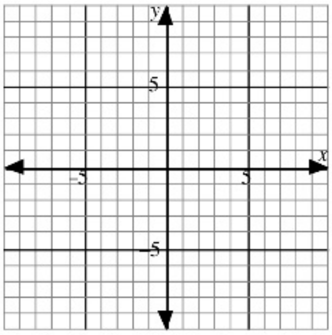
c) Determine the equation of the linear relationship in the form $y = mx + b$.



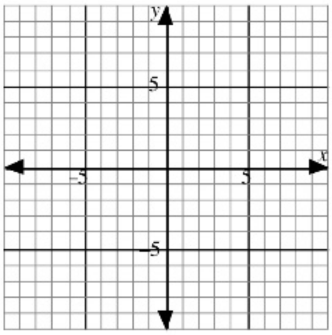
Complete Assignment Questions #7 - #10

Assignment

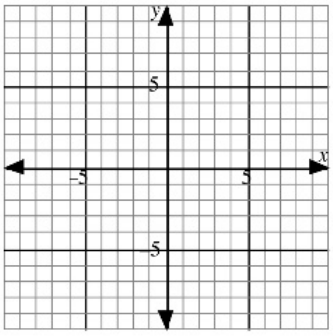
1. Without using technology or without altering the form in which the linear equation is written, explain how to graph $y = -3x - 6$ on a grid.
Draw the graph on the grid provided.



2. Without using technology or without altering the form in which the linear equation is written, explain how to graph $2x - 5y + 20 = 0$ on a grid.
Draw the graph on the grid provided.

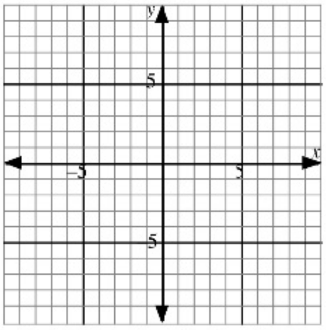


3. Without using technology or without altering the form in which the linear equation is written, explain how to graph $y + 4 = \frac{1}{2}(x - 2)$ on a grid.
Draw the graph on the grid provided.

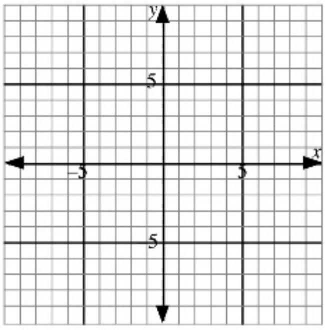


4. Without altering the form in which the linear equation is written, draw the graph (without technology) of each of the following linear relations on the grid provided.

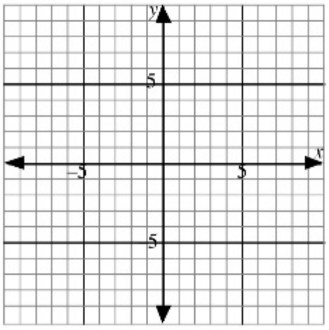
a) $y = -\frac{1}{3}x + 3$



b) $y - 5 = -2(x + 6)$



c) $x - 4y - 8 = 0$



5. Match each linear relation to its graph.

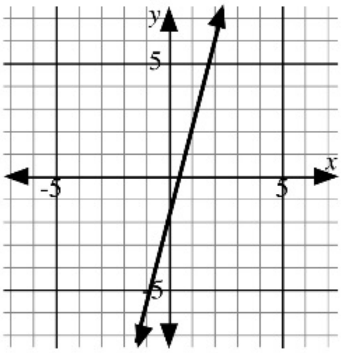
Equation 1:
$y - 6 = -4(x + 1)$

Equation 2:
$x - 4y - 8 = 0$

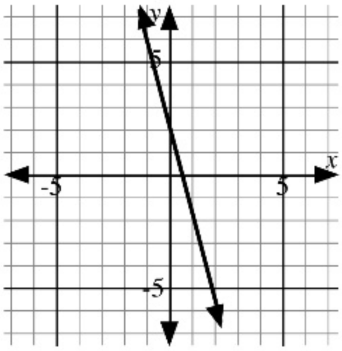
Equation 3:
$4x - y - 2 = 0$

Equation 4:
$y = -\frac{1}{4}x + 2$

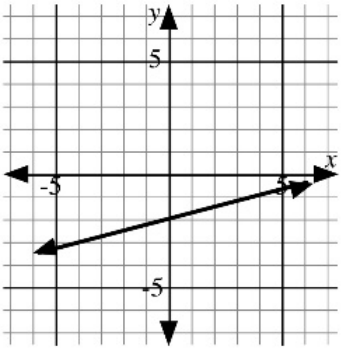
Graph A



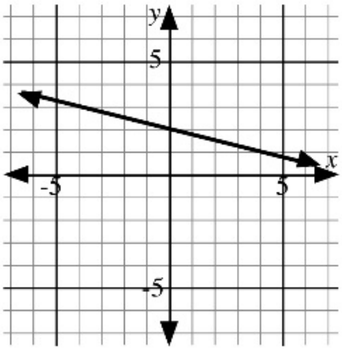
Graph B



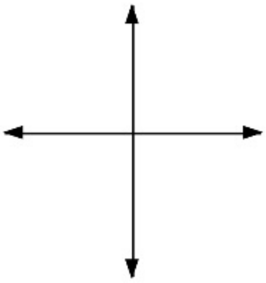
Graph C



Graph D



6. a) Explain the strategy used to graph (with technology) the linear relations
$x + 5y + 10 = 0$ and $y - 3 = \frac{1}{3}(x + 6)$.



b) State an appropriate window to show x- and y-intercepts, and draw the graph of both linear relations on the grid.

c) Determine the x- and y-intercepts of each graph.

7. Consider the following data points expressed in a table of values

a)

x	−1	0	1	2	3
y	−3	−1	1	3	5

b)

x	−6	−3	0	3	6
y	6	5	4	3	2

c)

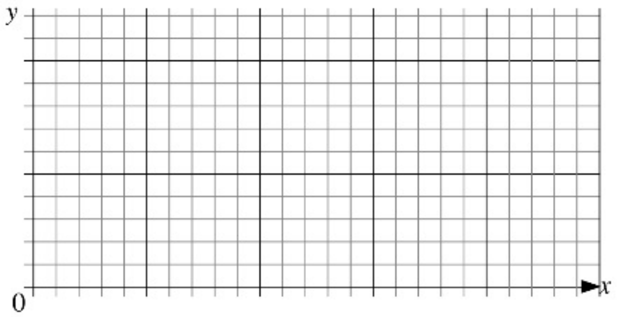
x	0	1	2	3	4
y	1	4	9	16	25

In each case, determine the equation of the linear relationship if it exists.

8. The following data is taken from a continuous linear relationship involving two quantities x and y.

x	32	41	50	59	68	77	86	95
y	0	5	10	15	20	25	30	35

a) Plot the data on the grid and obtain, in general form, the equation of the linear relation which is represented by the data.



b) Rewrite the equation of the linear relation

i) in terms of x (i.e. $y = ...$)

ii) in terms of y (i.e. $x = ...$)

c) The formulas in **b)** are well known in the scientific field. Can you suggest what scientific variables are represented by x and y?

Use the following information to answer the questions #9 and #10.

The following data is taken from a continous linear relationship involving two quantities, x and y.

x	-10	–5	5	10	20
y	-4	–3	–1	0	2

Multiple Choice **9.** When the data is graphed on a grid, the slope of the line is

A. 5

B. -5

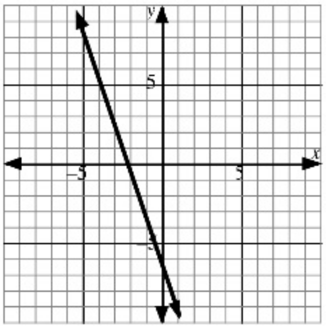
C. $\frac{1}{5}$

D. $-\frac{1}{5}$

Numerical Response **10.** The equation of the linear relationship can be written in the form $Ax - 5y + C = 0$. The value of $A - C$ is _____ .

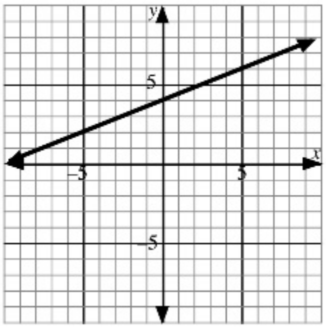
Answer Key

1.



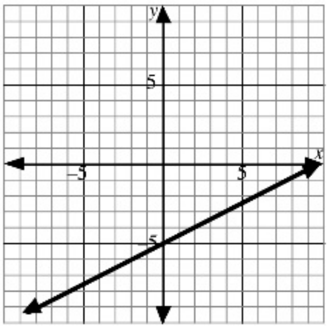
Plot the y-intercept $(0, -6)$. Since the slope, -3, equals rise over run, move 3 up and 1 left and plot another point. Repeat for two more points and draw a line through the points.

2.



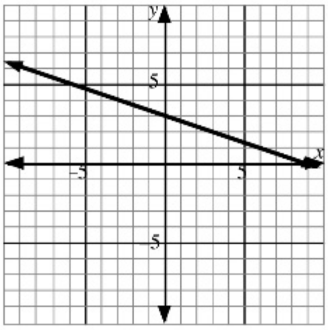
Plot the x-intercept $(-10, 0)$ and the y-intercept $(0, 4)$. Draw a line through these two points.

3.

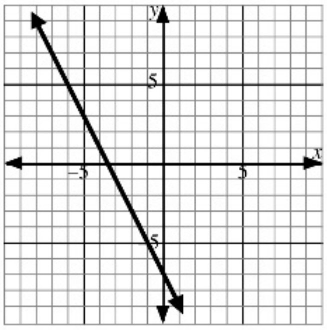


Plot the point $(2, -4)$. Since the slope equals 1/2, move 1 up and 2 right and plot another point. Repeat for two more points and draw a line through the points.

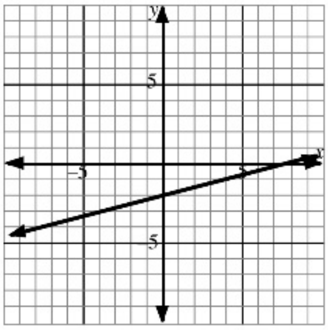
4. a)



b)



c)



5. 1B, 2C, 3A, 4D.

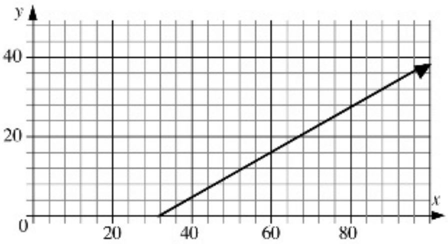
6. a) Solve each equation for y. Then input Y_1 and Y_2 into the equation editor of the graphing calculator. Press Graph.

b) x:[–20, 10, 5] y:[–6, 10, 2], answers may vary.

c) For $x + 5y + 10 = 0$, x-int $= -10$, y-int $= -2$, and for $y - 3 = \frac{1}{3}(x + 6)$, x-int $= -15$, and y-int $= 5$.

7. a) $y = 2x - 1$ **b)** $y = -\dfrac{1}{3}x + 4$ **c)** not linear

8. a) $5x - 9y - 160 = 0$



b) i) $y = \dfrac{5}{9}x - \dfrac{160}{9}$, or $y = \dfrac{5}{9}(x - 32)$

ii) $x = \dfrac{9}{5}y + 32$

c) x is temperature in °F, and y is temperature in °C.

9. C

10.

1	1		

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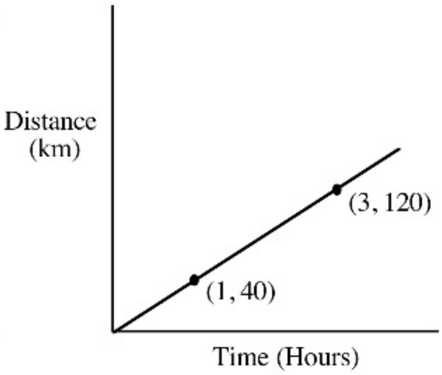
Equations of Linear Relations Lesson #7:
Slope as a Rate of Change

Rate of Change

Part One

The graph shown represents the distance travelled by a car as a linear function of time.



a) Complete the following to calculate the slope of the line.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} =$$

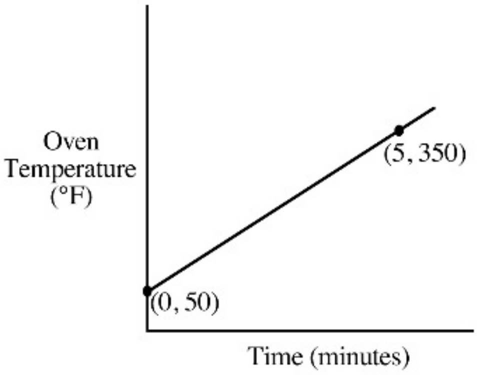
b) The slope represents a **rate of change**: a change in distance divided by a change in time.

Complete the following statement by choosing the correct alternative and filling in the blank.

The distance is (increasing/decreasing) at the rate of ______ km per hour.

Part Two

The graph shown represents the temperature of an oven as a linear function of time.



a) Calculate the slope of the line.

b) The slope represents a **rate of change**: a change in temperature divided by a change in time.

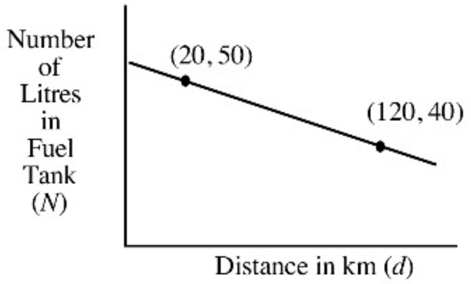
What units are used to represent this rate of change?

c) Complete the following statement.

The temperature is (increasing/decreasing) at the rate of ______ ____ per ______ .

Part Three

The graph shown represents the amount of fuel in the gas tank of a car as a function of the distance travelled.



a) Calculate the slope of the line.

b) The slope represents a **rate of change**: a change in the amount of fuel in the fuel tank divided by a change in distance.

What units are used to represent this rate of change?

c) Complete the following statements.

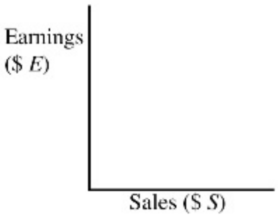
The amount of fuel in the tank is (increasing/decreasing). Circle one.

The rate of change of fuel in the fuel tank is ______ ____ per ______ .

The amount of fuel is decreasing at the rate of ______ ____ per ______ .

d) Determine the equation of the line in the form $N = mD + b$.

Tyrone is paid a base salary per week plus commission for selling electrical appliances. Last week his sales totalled $3 500, and he earned $620. This week he earned $680 for sales of $4 250.



a) On a grid, plot ordered pairs to represent this information.

b) Calculate the slope of the line segment joining the ordered pairs.

c) Explain what the slope of the graph represents.

d) State as a percent, the rate of commission which Tyrone is paid.

e) Calculate his weekly base salary.

f) How does the answer to **e)** relate to the graph?

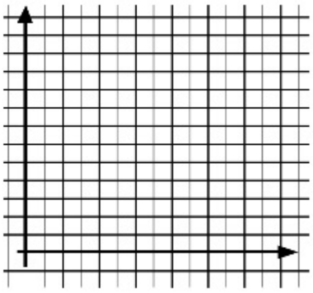
g) Write the equation in the form $E = mS + b$.

h) Two weeks ago Tyrone earned $486. Calculate his sales for that week.

Average Speed

Biyen is taking part in a long distance car race. After 3 hours he had travelled 270 km, and after 6 hours he had travelled 630 km.

a) On a grid, plot ordered pairs to represent this information.



b) Calculate the slope of the line segment joining the ordered pairs.

c) The slope of the line segment represents a rate of change: **a change in distance divided by a change in time**.
This rate is the **average speed** between the two points.
State the average speed of the car from 3 hr to 6 hr using appropriate units.

d) On the grid, plot the point $(0, 0)$ and determine the average speed of the car during the first 3 hr of the race.

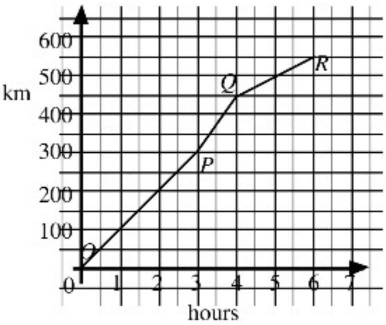
e) By looking at the grid and without doing any calculations, how can we tell that the average speed during the first 3 hr was less than the average speed during the next 3 hr?

Note On a graph of distance as a function of time, the slope of a line segment joining two points represents the average speed between the two points.

Complete Assignment Questions #1 - #10

Assignment

1. The distances and times are recorded at certain points on a journey. Calculate the average speed



a) between O and P

b) between P and Q

c) between Q and R

d) for the whole journey

2. Absolute Value Computer Company was formed in 2014. By Jan 2016, the company had sold 520 000 computers and by July 2016, the company had sold 610 000 computers. Calculate the average rate of change stating appropriate units.

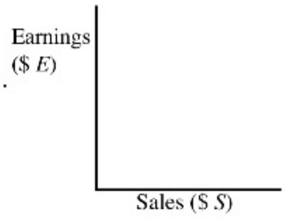
3. In 2017, the transit authority in a large city reported 17 678 465 passenger journeys. In 2012, the number of passenger journeys was 21 520 075.
Calculate the average rate of change stating appropriate units.

4. Shanna is paid a base salary per month plus commission for working in a clothing store. In January her sales totalled \$10 500 and she earned \$2 460. In February her sales totalled \$9 350, and she earned \$2 322.

a) On a grid, plot ordered pairs to represent this information.

b) Calculate the slope of the line segment joining the ordered pairs.

c) Explain what the slope of the graph represents.



d) State, as a percent, the rate of commission which Shanna is paid.

e) Calculate her monthly base salary.

f) How does the answer to **e)** relate to the graph?

g) Write the equation in the form $E = mS + b$.

h) In March, her sales were \$11 200. Calculate her earnings for March.

i) In April, her rate of commission was increased by 1%. If her earnings were \$70 less than in March, calculate the value of her sales in April.

5. Water is leaking out of the bottom of a barrel at a constant rate. After 3 min the water level is 58 cm and after 8 min the water level is 23 cm.

a) On a grid, plot ordered pairs (time, water level) to represent this information.

b) Calculate the slope of the line segment joining the ordered pairs.

c) Explain what the slope of the graph represents.

d) Complete the following.
The water level is changing at the rate of ______ ____ per ______.

e) Determine an equation to represent this information in the form $W = mt + b$, where W represents the water level in cm and t represents the time in minutes.

f) After how many minutes will the barrel be empty?

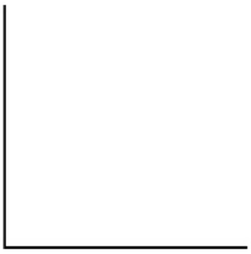
g) What was the water level in the barrel when it started leaking?

h) The barrel is cylindrical in shape with a radius of 20 cm.

i) Calculate the volume (in terms of π) of water in the barrel after 3 min and after 8 min.

ii) Calculate the rate (in terms of π) at which water is leaking out of the barrel.

6. Jack rented a car from Absolute Value Rent-a-Car Company. After driving for two hours, the odometer reading was 21 328 and after five hours the odometer reading was 21 604. He completed his journey after six hours.



a) On a grid, plot ordered pairs to represent this information.

b) Calculate the slope of the line segment joining the ordered pairs.

c) Explain what the slope of the graph represents.

d) Assuming a constant rate of driving for the whole journey, determine the odometer reading at the start of the journey.

e) Determine an equation to represent this information in the form $f(t) = mt + b$, where $f(t)$ represents the odometer reading after t hours.

f) State an appropriate domain and range for f.

7. To test the gas consumption of a new SUV, Jana filled up the gas tank of 58 L and drove the SUV until it was empty. She drove the SUV for 464 km.

a) Sketch the graph on the grid provided with distance travelled on the horizontal axis.

b) Write an equation in the form $y = mx + b$ which represents the volume of fuel in the tank as a function of distance.

c) State the slope of the graph, and explain what it represents.

d) Determine the distance travelled after using 12 litres of gas.

e) How many litres of gas are used by the SUV when it has travelled 200 km?

8. After 7 days of heavy rainstorms, the water level in a river peaked at 2.85 m above the regular level. Four days later, the water dropped to 2.25 m above the regular level.

a) Assuming the water level falls at a constant rate, determine a function $h(t)$, which describes the height of the river above regular level as a function of time. Take $t = 0$ at peak water level.

b) State the slope of the graph of the function and explain what it represents.

c) Determine an appropriate domain and range for h.

Multiple Choice **9.** A repair company charges a fixed call-out fee for any service call, plus a fixed rate per hour for the length of the repair. A three hour repair costs \$155, and a four and a half hour repair costs \$215. The fixed call-out fee and the cost of a seven hour repair are respectively

A. \$35 and \$285
B. \$40 and \$285
C. \$35 and \$315
D. \$40 and \$315

Numerical Response **10.** The temperature at the top of a mine shaft is 18° C. 250 metres below the surface, the temperature is 18.8° C. To the nearest tenth, the rate of temperature increase in °C per km is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. **a)** 100 km/h **b)** 150 km/h **c)** 50 km/h **d)** $91\frac{2}{3}$ km/h

2. 15 000 computers per month

3. –768 322 passenger journeys per year

4. **b)** 0.12 **c)** the rate of commission (earnings per sales)
d) 12% **e)** \$1200
f) it is the intercept on the vertical axis **g)** $E = 0.12S + 1200$ **h)** \$2544 **i)** \$9800

5. **b)** –7
c) the rate at which the water level is changing in cm/min
d) –7 cm/min **e)** $W = -7t + 79$ **f)** $11\frac{2}{7}$ min
g) 79 cm **h)** **i)** 23200π cm^3, 9200π cm^3 **ii)** 2800π cm^3/min

6. **b)** 92
c) the average speed between 2h and 5h is 92 km/h
d) 21144 **e)** $f(t) = 92t + 21144$
f) domain $0 \le t \le 6, t \in R$, range $21144 \le f(t) \le 21696, f(t) \in R$

7. **b)** $y = -\frac{1}{8}x + 58$
c) slope $= -\frac{1}{8}$, the volume of fuel in the gas tank is decreasing at the rate of $\frac{1}{8}$ L/km
d) 96 km **e)** 25 litres

8. **a)** $h(t) = -0.15t + 2.85$
b) slope = –0.15, it represents the rate at which the water level is changing in metres per day
c) domain $0 \le t \le 19, t \in R$, range $0 \le h(t) \le 2.85, h(t) \in R$

9. C **10.**

3	.	2	

Equations of Linear Relations Lesson #8: Practice Test

1. The slope of the line with equation $3y = 2x - 12$ is

A. 2
B. $\frac{2}{3}$
C. -4
D. -12

2. The y-intercept of the graph of the line with equation $y = 5x - 10$ is

A. 2
B. 5
C. 10
D. -10

3. Which equation represents a line with a slope of 3 and a y-intercept of -4?

A. $y = -4x + 3$
B. $y = -\frac{1}{3}x - 4$
C. $y = 3x - 4$
D. $y = 3x + 4$

4. Which of the following is the equation of a line perpendicular to $5y + x + 6 = 0$?

A. $y = 5x$
B. $y = x$
C. $y = \frac{1}{5}x$
D. $y = -\frac{1}{5}x$

5. Which of these ordered pairs can be found on the graph of the line $3x - 5y - 4 = 0$?

i) $(8, 4)$ **ii)** $(-3, 1)$ **iii)** $(0, -0.8)$ **iv)** $(-2, 2)$

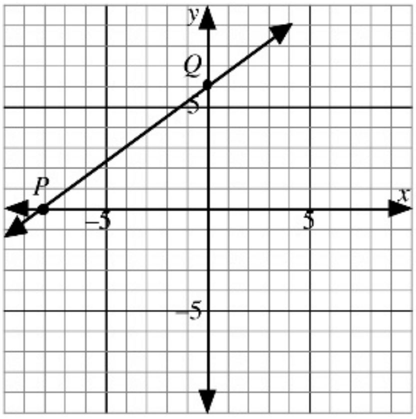
A. **i)** and **ii)** only
B. **i)** and **iii)** only
C. **i)**, **ii)**, and **iii)** only
D. some other combination of **i)**, **ii)**, **iii)**, and **iv)**

6. The point of intersection of the line $9x - 3y + 9 = 0$ and the y-axis is

A. $(0, 9)$
B. $(0, 3)$
C. $(0, -1)$
D. $(0, -3)$

*Use the following information to answer the next **two** questions.*

The x- and y-intercepts of the graph shown are integers.



7. The equation of the line PQ is

A. $3x + 4y + 24 = 0$
B. $3x + 4y + 32 = 0$
C. $3x - 4y + 24 = 0$
D. $3x - 4y + 32 = 0$

Numerical Response **1.** Given that the line above passes through $(7.2, k)$, the value of k, to the nearest tenth, is ________ .

(Record your answer in the numerical response box from left to right)

8. If the lines $ax + by + c = 0$ and $dx + ey + f = 0$ are parallel, then

A. $ae - bd = 0$
B. $ae + bd = 0$
C. $ad - be = 0$
D. $ad + be = 0$

Numerical Response **2.** Given that the line joining the points $(2, 3)$ and $(8, -q)$ and where $q \in W$, is perpendicular to the line $3x - 2y - 5 = 0$, then the value of q is ________ .

(Record your answer in the numerical response box from left to right) ☐☐☐☐

9. The equations of four straight lines are
1) $7x - y = 0$ **2)** $7x + y - 6 = 0$ **3)** $x - 7y + 4 = 0$ **4)** $x + 7y - 2 = 0$

Which pairs of lines are perpendicular?

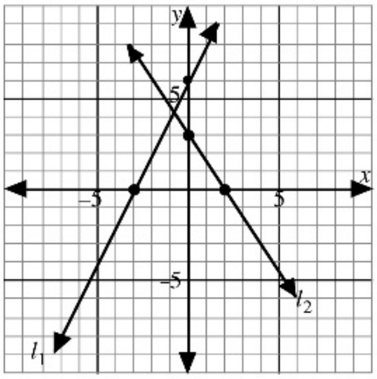
A. **1)** and **2)** only
B. **1)** and **4)** only
C. both **1)** and **4)** and **2)** and **3)**
D. both **1)** and **2)** and **2)** and **3)**

10. The line passing through the points $(-5, -2)$ and $(-2, -1)$ has equation

A. $x + y + 3 = 0$
B. $x + 3y + 5 = 0$
C. $x - 3y + 1 = 0$
D. $x - 3y - 1 = 0$

Numerical Response **3.** The lines $3x - y + 2 = 0$ and $5x - By + 26 = 0$ and where $B \in \mathrm{W}$, intersect on the y-axis. The value of B is _____.
(Record your answer in the numerical response box from left to right) ☐☐☐☐

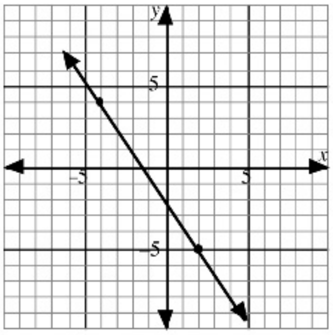
11. Which equation represents a line which is perpendicular to line l_1 and has the same x-intercept as line l_2?



A. $x + 2y - 2 = 0$
B. $x + 2y + 2= 0$
C. $2x + y - 4 = 0$
D. $2x + y + 4 = 0$

Numerical Response

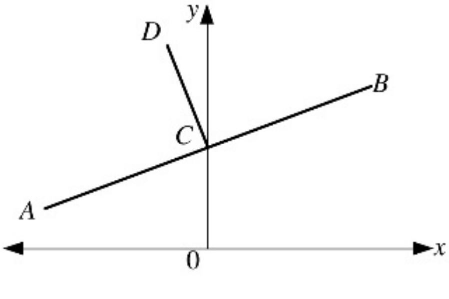
4. The equation of the line shown in the diagram is $Ax + 2y + C = 0$. The value of $\frac{A}{C}$, to the nearest hundredth, is _____.



(Record your answer in the numerical response box from left to right)

12. The equation of AB is $x - 2y + 4 = 0$.
AB cuts the y-axis at C.
CD is perpendicular to AB.

The equation of CD is



A. $x + 2y - 2 = 0$
B. $2x + y - 2= 0$
C. $2x - y + 2 = 0$
D. $2x + y - 4 = 0$

13. Consider $\angle PQR$ in which side PQ has slope $\frac{1}{3}$ and R has coordinates $(-4, 7)$. The equation of the altitude from R to PQ (the line drawn from R to PQ, perpendicular to PQ), is

A. $x + 3y = 25$
B. $3x + y = 19$
C. $3x + y = -5$
D. $3x + y = -19$

14. Which of the following lines is/are perpendicular to the line $9x + y + 2 = 0$?

i) $9y + x = 2$ **ii)** $9y - x = 2$ **iii)** $y = 9x + 2$ **iv)** $9y = x - 2$

A. **i)** and **iii)** only
B. **ii)** only
C. **iv)** only
D. some other combination of **i)**, **ii)**, **iii)**, and **iv)**

15. The line l_1 passes through the points $(-3, 5)$ and $(-2, -1)$.

Which of the following statements is true?

i) l_1 passes through $(4, -37)$.
ii) l_1 has an x–intercept of $-\frac{13}{6}$.
iii) l_1 is perpendicular to $y = \frac{1}{6}x + 2$.

A. **i)** and **ii)** only
B. **i)** and **iii)** only
C. **ii)** and **iii)** only
D. **i)**, **ii)**, and **iii)**

Numerical Response **5.** The temperature at sea level is 12.1 °C. At the top of a mountain, 6 400 m above sea level, the temperature is –29.5 °C. To the nearest tenth, the rate of temperature decrease, in °C per km, is _____ .

(Record your answer in the numerical response box from left to right)

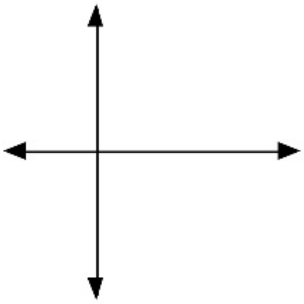
Written Response - 5 marks

1. Consider the points $P(-7,-2)$, $Q(2,1)$, $R(-2,-7)$, and $S(8,3)$.

- Show that the equation of the line, L_1, through S and perpendicular to PQ is $y=-3x+27$.

- Determine the equation of the line, L_2, through R and parallel to PQ. Give the answer in in slope y-intercept form.

- Draw both lines on the grid, and state a suitable window which shows x- and y-intercepts for each graph.



Answer Key

1. B **2.** D **3.** C **4.** A **5.** B
6. B **7.** C **8.** A **9.** C **10.** D
11. A **12.** B **13.** C **14.** D **15.** D

Numerical Response

1.	1	1	.	4
2.	1			
3.	1	3		
4.	0	.	7	5
5.	6	.	5	

Written Response

1. • $y=-3x+27$ • $y=\frac{1}{3}x-\frac{19}{3}$ • $x{:}[-5, 25, 5]$ and $y{:}[-10, 35, 5]$

Systems of Linear Equations Lesson #1:
Solving Systems of Linear Equations by Graphing

Overview of Unit

In this unit, we solve problems that involve systems of linear equations in two variables. We do this by inspection, by graphing, and algebraically using the method of substitution and the method of elimination.

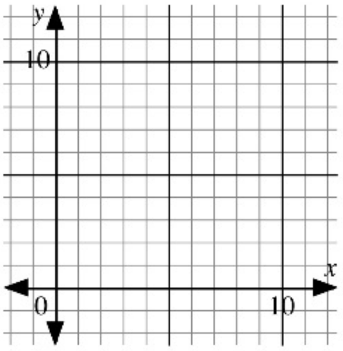
Exploring a Meaning of Two Intersecting Lines

The Smith family and the Harper family are going to a book fair which is raising money for charity. Mr. Smith pays an entry fee of \$11 for three adults and one child. Mrs. Harper pays an entry fee of \$12 for two adults and three children.

We can determine the cost of an adult ticket and the cost of a child ticket by forming two linear equations and graphing them.

Let \$$x$ be the entry fee for an adult ticket and let \$$y$ be the entry fee for a child. The information about the Smith family can be modelled by the equation $3x + y = 11$, and information about the Harper family can be modelled by the equation $2x + 3y = 12$.

a) Draw the graphs of the equations $3x + y = 11$ and $2x + 3y = 12$ on the grid without using technology.



b) The graphs of the equations intersect at a point. State the coordinates of this point and explain what the coordinates represent in the context of the question.

Systems of Equations

In the exploration on the previous page, we worked with the equation $3x + y = 11$.
There are many values for x and y which satisfy this equation, e.g. $x = 1$ and $y = 8$, or $x = 2$ and $y = 5$, or $x = 3$ and $y = 2$, etc.

We also worked with the equation $2x + 3y = 12$.
There are also many values for x and y which satisfy this equation, e.g. $x = 0$ and $y = 4$, or $x = 3$ and $y = 2$, or $x = 4.5$ and $y = 1$, etc.

If we consider both of these equations simultaneously, there is only one solution, $x = 3$ and $y = 2$.

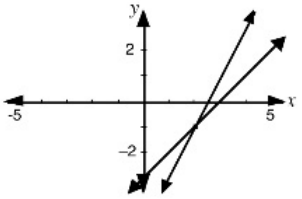
The equations $3x + y = 11$ and $2x + 3y = 12$, considered at the same time, are called a **system of equations**.

The **solution** to this system of equations is $x = 3$ and $y = 2$.
This is because $x = 3$ and $y = 2$ **satisfy** each equation in the system.

Graphically, the solution to the system is the point of intersection of the two lines.

Class Ex. #1

A system of equations has been represented on the grid.
The system has an integral solution.



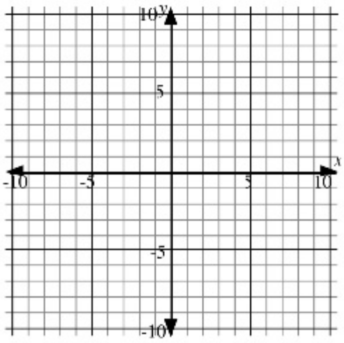
a) State the solution $x =$ _____ , $y =$ _____

b) Write the solution as an ordered pair.

Class Ex. #2

Consider the system of equations $2x + y = 2$, $x - 3y = 15$.

a) Graph the system of equations without using technology.



b) State the solution to the system of equations.

c) Algebraically verify the solution by replacing the values in the original equations.

Conplete Assignement Questions #1 and #2

Solving a System of Equations using a TI Graphing Calculator

Check that the calculator is in "Function" mode.
Use the following procedure to find the solution to a system of equations.

1. Write each equation in terms of y.

2. Access the "Y= editor" by pressing the **Y=** key.

3. Enter one equation in **Y_1**.

4. Enter the other equation in **Y_2**.

5. Press the **GRAPH** key to display the graphs.

6. Access the intersect command by pressing **2nd** then **TRACE** and scroll down to "**intersect**". The calculator will return to the display window with the graphs.

7. The calculator will display "**First curve?**". Use the cursor key, if necessary, to select the first graph and then press **ENTER**.

8. The calculator will display "**Second curve?**". Use the cursor key, if necessary, to select the second graph and then press **ENTER**.

9. The calculator will display "**Guess?**". Press **ENTER**.

Consider the system of equations from the exploration at the beginning of this lesson.

$$\begin{aligned}3x + y &= 11\\ 2x + 3y &= 12.\end{aligned}$$

a) Rewrite each equation in slope y-intercept form.

b) Use a graphing calculator to graph each equation.

c) State a suitable window which shows both sets of x- and y-intercepts and the point of intersection.

d) Solve the system of equations using the features of the graphing calculator. Confirm the amount of the entry fees established in the exploration.

If a decimal value appears for the x and/or y coordinates, then the x and/or y value can be converted to an exact value (as long as it is not irrational and is within the limitations of the calculator) by using the following steps.

For the *x*-coordinate

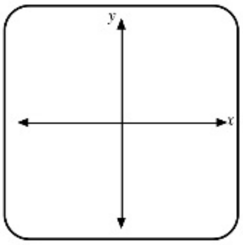
1. Exit the graphing screen by pressing **CLEAR** twice.
2. Press **X,T, θ, n** key, then press **ENTER** to import the x-coordinate.
3. To display the exact value,
   Press **MATH**, select "**Frac** ", then press **ENTER**.

For the *y*-coordinate

Except for step 2, the instructions to import the y-coordinate are the same as above.
For step 2, press **ALPHA** **1** **ENTER** to import the y-coordinate value.
Then proceed to step 3 above.

a) Solve the following system of equations using a graphing calculator.

$$6a + 7b = 5$$
$$3a = 14b$$



b) List the answers as exact values using the technique above.

c) Algebraically verify the solution.

Complete Assignment Questions #3 - #7

Assignment

1. Consider the system of equations $x - 2y = 3$, $x + y = 0$.

a) Write each equation in slope y-intercept form.

b) Complete the table of values for each equation.

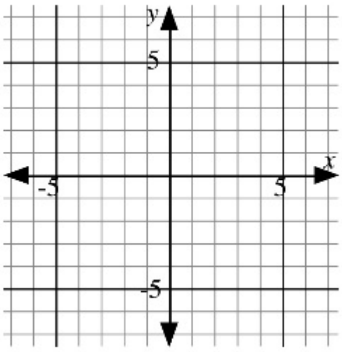
$x - 2y = 3$

x	y
-3	
-1	
1	
3	
5	

$x + y = 0$

x	y
-4	
-2	
0	
2	
4	

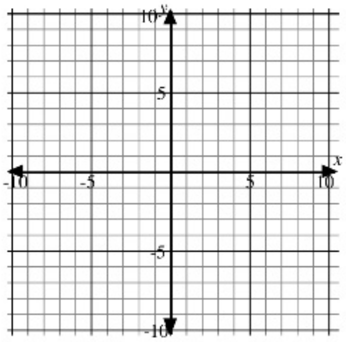
c) Draw the lines on the grid and state the solution to the system.



d) Verify the solution.

2. The following system of equations is given: $x - y = 7$, $x + 5y = -5$

a) Without using technology, graph each equation and hence solve the system.



b) Verify the solution.

3. In each case, solve the system of equations using technology. Verify the solution by replacing the values in the original equations.

a) $y = 3x - 7$
$y = -x + 9$

b) $y = -x$
$y = -\frac{1}{3}x + 3$

c) $y = x - 2$
$y = \frac{3}{4}x - 4$

d) $3x + 2y = 5$
$x - y = 1$

e) $4a - b = 6$
$3a + b = 1$

f) $0.6p - 0.8q = 2.6$
$5p + 6q = 9$

4. Solve the following systems of equations using technology.
List the answers as exact values.

a) $4x - y + 6 = 0, \quad y = x + 2$

b) $8x - 3y = 5, \quad 5x + 3y = 2$

Multiple Choice **5.** The ordered pair (x, y) which satisfies the system of equations $x - 2y = 6$, $x + 6y = 22$ is

A. $(-10, 2)$

B. $(2, 10)$

C. $(10, -2)$

D. $(10, 2)$

Numerical Response **6.** If $7x - 5y = 19$ and $2x + 3y = 17$, then the value of x, to the nearest tenth, is _____ .

(Record your answer in the numerical response box from left to right)

7. A pear costs 24 cents less than two apples. Four apples cost the same as three pears. In order to determine the cost of each piece of fruit, Courtney graphs the equations $y = 2x - 24$ and $4x = 3y$. The cost of a pear, in cents, is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $y = \frac{1}{2}x - \frac{3}{2}$, $y = -x$ **b)**

x	y
–3	**–3**
–1	**–2**
1	**–1**
3	**0**
5	**1**

x	y
–4	**4**
–2	**2**
0	**0**
2	**–2**
4	**–4**

c) $x = 1$, $y = -1$

2. a) $x = 5, y = -2$

3. a) $x = 4, y = 5$ **b)** $x = -4.5, y = 4.5$ **c)** $x = -8, y = -10$
d) $x = 1.4, y = 0.4$ **e)** $a = 1, b = -2$ **f)** $p = 3, q = -1$

4. a) $x = -\frac{4}{3}, y = \frac{2}{3}$ **b)** $x = \frac{7}{13}, y = -\frac{3}{13}$

5. D **6.**

4	.	6	

7.

4	8		

Systems of Linear Equations Lesson #2:
Determining the Number of Solutions to a System of Linear Equations

Exploration *Determining the Number of Solutions to a System of Linear Equations*

As part of a high school work experience course, three students have been placed in three different burger restaurants. Deja has been placed at Burger Shack, Shelly has been placed at Big's Burgers, and John has been placed at The Burger Haven.

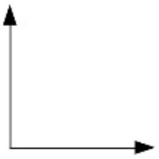
Detailed below are the first two orders taken by each student and the total cost calculated by the student for each order.

Burger Shack	For the first order Deja charges \$28 for 4 burgers and 2 salads. For the second order she charges \$34 for 6 burgers and a salad.
Big's Burgers	Shelly charges \$18 for a burger and 3 salads, and \$54 for 3 burgers and 9 salads.
The Burger Haven	John charges \$32 for 2 burgers and 4 salads, and \$42 for 3 burgers and 6 salads.

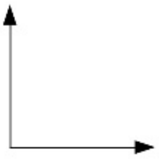
a) Let x dollars be the cost of a burger and y dollars be the cost of a salad. Write a system of equations for each scenario.

Burger Shack Big's Burgers The Burger Haven

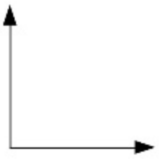
b) Consider the equations for the Burger Shack. Write each equation in terms of y and use a graphing calculator to determine the cost of a burger and the cost of a salad.



c) Repeat part **b)** for Big's Burgers. Can you determine the cost of a burger and the cost of a salad? Explain.



d) Repeat part **b)** for The Burger Haven. Can you determine the cost of a burger and the cost of a salad? Explain how you can tell that the student must have made an error in at least one of the calculations.



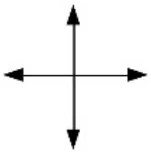
Number of Solutions to a System of Equations

In all of the examples in Lesson #1, each system of equations had one unique solution. However, we have seen on the previous page that a system of two linear equations may have no solution, only one solution or an infinite number of solutions.

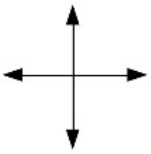
Class Ex. #1

Graph each system of equations on the grid provided. State the number of solutions for each system.

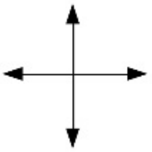
a) $y = x + 5$
$y = -x + 1$



b) $y = 2x - 8$
$y = 2x + 6$

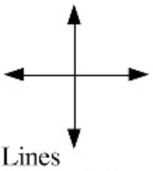
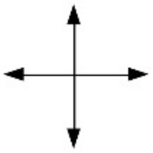
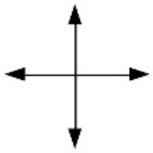


c) $y = 3x - 2$
$6x - 2y - 4 = 0$



The number of solutions can be determined from the graph as above, or directly from the equations if they are expressed in slope y-intercept form.

d) Complete the following chart.

Number of Solutions			infinitely many
Graphical Example	 Lines intersect at one point	 Lines are parallel	 Lines are coincident
Slopes and Intercepts		Slopes are equal and intercepts are different	

Class Ex. #2

Without graphing, analyze each system to determine whether the system has one solution, no solution, or infinitely many solutions.

a) $3x + 5y = 15,\ y = -\frac{3}{5}x$ **b)** $x - 4y + 8 = 0,\ y = \frac{1}{4}x + 2$ **c)** $7x + y = 12,\ x - 6y = 5$

Complete Assignment Questions #1 - #5

Solving a System Graphically by Changing the Calculator Window

Often, the solution to a system of equations will not be visible using the default window of the graphing calculator. When this occurs, we must change the window settings.

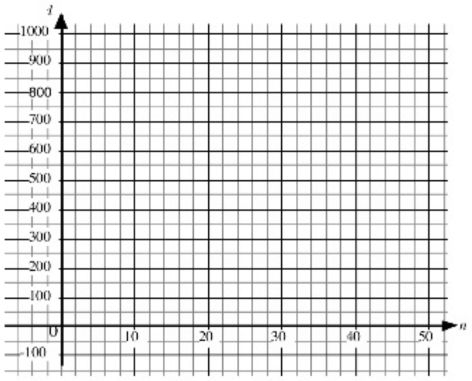
We use the following graphing calculator window format:

x:[$x_{\text{min}}, x_{\text{max}}, x_{\text{scl}}$] $\quad$ y:[$y_{\text{min}}, y_{\text{max}}, y_{\text{scl}}$]

At a local High School, the Students' Council decided to sell sweaters to students. The cost of designing the sweaters included a fixed cost of \$600 plus \$5 per sweater.

The Students' Council planned to sell the sweaters for \$25 each. The cost and revenue can be represented by the following system of equations where d represents the dollar cost and n represents the number of sweaters sold.

Cost of sweaters in dollars $d = 600 + 5n$
Revenue of sweaters in dollars $d = 25n$



a) Use a graphing calculator to graph each equation and sketch each graph on the grid provided.

b) How much profit or loss is made if
i) twenty sweaters are sold? **ii)** fifty sweaters are sold?

c) The break even point is the point where no profit or loss is made.
Mark the break-even point on the grid.

d) Use a graphing calculator to determine the number of sweaters which must be sold in order to break even.

e) If all 850 students in the school purchased a sweater, how much profit would the Students' Council make?

Complete Assignment Questions #6 - #11

Assignment

1. How can you tell by graphing a system of linear equations whether the system has no solution, one solution, or infinitely many solutions?

2. Graph each system and determine whether the system has no solution, one solution, or infinitely many solutions.

a) $x = 2y - 5$
$y = \frac{1}{2}(x + 5)$

b) $6x - y = 5$
$y = 6x + 7$

c) $2x - 5y = 10$
$3x - 4y = 24$

3. How can you tell by writing a system of linear equations in the form $y = mx + b$ whether the system has no solution, one solution, or infinitely many solutions?

4. Rearrange each equation into the form $y = mx + b$ and state whether the system has no solution, one solution, or infinitely many solutions.

a) $6x - y = 1$
$y = 6x + 1$

b) $8x - y = 13$
$x - 8y = 13$

c) $5y + x - 10 = 0$
$y = -\frac{1}{5}x + 2$

5. Write an equation which forms a system with the equation $3x - y = 9$ so that the system has

a) no solution **b)** one solution **c)** an infinite number of solutions

6. All 480 tickets for a school concert were sold. Seats in the front part of the hall cost \$6 each, and seats in the back part of the hall cost \$4 each. The receipts totalled \$2 530.

Information from the number of tickets can be represented by $f + b = 480$.

a) State an equation which can be formed from the costs of the tickets.

b) Graph the system to determine the number of tickets sold for each part of the hall.

c) State the graphing window used.

d) Verify the solution.

7. Six pencils and four crayons cost \$3.40. Three similar pencils and ten similar crayons cost \$4.90. **Describe** a method to determine how much you would expect to pay for a set of eight pencils and twelve crayons and then calculate the cost.

Multiple Choice

8. The solution to the system $\begin{cases} 4x - 3y = 9 \\ 8x - 6y = 81 \end{cases}$ has

A. no solution

B. one solution

C. two solutions

D. infinitely many solutions

Numerical Response

9. If $3x + 2y = 48$ and $2x + 3y = 12$, then the value of $x - 2y$, to the nearest tenth, is _____ .

(Record your answer in the numerical response box from left to right)

10. The value of k, $k \in N$, for which the system of equations $10x + ky = -8$ and $-15x - 6y = 12$ has an infinite number of solutions, is _____ .

(Record your answer in the numerical response box from left to right)

11. The value of a, $a \in N$, for which the system of equations $ax + 5y = 10$ and $6x + 2y = 7$ has no solution, is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. If the lines are parallel, there is no solution.
If the lines intersect, there is one solution.
If the lines are coincident, there are infinitely many solutions.

2. **a)** infinitely many solutions **b)** no solution **c)** one solution

3. If the values of m are identical but the values of b are different, there is no solution.
If the values of m are different, there is one solution.
If the values of m are identical and the values of b are identical, there are infinitely many solutions.

4. **a)** no solution **b)** one solution **c)** infinitely many solutions

5. **a)** e.g. $3x - y = 4$ **b)** e.g. $7x - 3y = 6$ **c)** e.g. $6x - 2y = 18$

6. **a)** $6f + 4b = 2530$ **b)** Front 305 tickets, Back 175 tickets **c)** x:[−100, 500, 100] y:[−100, 700, 100]

7. Let \$$x$ be the cost of a pencil and \$$y$ be the cost of a crayon. Form two equations from the given information. These are $6x + 4y = 3.4$ and $3x + 10y = 4.9$ Write each equation in slope y-intercept form. Graph the system of equations and determine the coordinates (x, y) of the intersection point. The answer is $8x + 12y = 8(0.3) + 12(0.4) = 7.2$ *Answer* = \$7.20

8. A

9.

4	8	.	0

10.

4			

11.

1	5		

Systems of Linear Equations Lesson #3:
Solving Systems of Linear Equations by Inspection and by Substitution

Method of Inspection

In some simple cases, a system of linear equations can be solved by mentally trying different values for the variables until a correct solution is reached. This is called the method of inspection and is really only practical if the equations are very simple.

The sum of two numbers is 14 and the difference between the numbers is 2. Form two equations in two variables and determine the numbers by inspection.

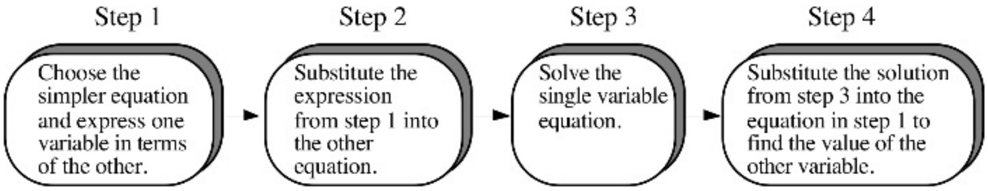
Solve the system $x + 2y = 12$ and $x + 3y = 17$ by inspection.

Complete Assignment Questions #1 - #3

Method of Substitution

If the equations are too complex to be solved by inspection, then algebraic procedures such as the method of substitution and the method of elimination (next lesson) may be used.

When using the method of substitution, there are four steps which are shown in the flowchart below.


Step 1
Choose the simpler equation and express one variable in terms of the other.
Step 2
Substitute the expression from step 1 into the other equation.
Step 3
Solve the single variable equation.
Step 4
Substitute the solution from step 3 into the equation in step 1 to find the value of the other variable.

Consider the following system of equations:

$$\begin{aligned} x + 4y &= 17 \\ 2x - y &= 7 \end{aligned}$$

a) Solve the system using the method of substitution by rewriting the first equation in the form $x = \; .\;.\;.\;.\;.$

b) Solve the system using the method of substitution by rewriting the first equation in the form $y = \; .\;.\;.\;.\;.$

c) Which method, a) or b), was simpler?

d) Verify that the solution satisfies both equations.

e) Check the solution using a graphing calculator.

Class Ex. #4

Consider the following system of equations: $4x + 3y = 0,\ 8x - 9y = 5.$

a) Solve and verify the system using the method of substitution.

b) Check the solution using a graphing calculator.

Class Ex. #5

Consider the following system of equations: $5(2a - 3) + b = 5, \quad 6a - 2(b - 4) = 20.$

a) Solve the system using the method of substitution.

b) Verify algebraically that the solution satisfies both equations.

Complete Assignment Questions #4 - #10

Assignment

1. Solve the following linear systems by method of inspection.

a) $x + y = 9,\ x - y = 1$ **b)** $x + y = 12,\ x - y = 0$ **c)** $x + y = 4,\ x - y = 6$

2. At the Little River Pow Wow, a vendor sells a salmon burger and two cans of cola for \$8. If two salmon burgers and two cans of cola sell for \$14, then determine the cost of

a) a salmon burger **b)** a can of cola

3. Tickets are on sale for a music concert. Three adult tickets and two child tickets cost \$90. Three adult tickets and four child tickets cost \$120.

a) Write a system of equations in two variables to represent the above information.

b) Determine the total cost of two adult tickets and three child tickets.

4. In each of the following systems:

- solve the system using the method of substitution
- verify the solution satisfies both equations
- check the solution by graphing

a) $y = x + 2,\quad 3x + 4y = 1$ **b)** $x - 2y = 10,\quad x + 5y + 4 = 0$

5. Solve each of the following systems by substitution. Check each solution.

a) $4p + q = 0, \quad 7p + 4q = 3$ **b)** $6u - 3v + 4 = 0, \quad 3u = 3v - 5$

c) $2x - 5y = -7$
$\frac{1}{2}x - y = 3$

d) $2(x + 2) + y = 8$
$7x - 2(y - 3) + 24 = 0$

6. The straight line $px + qy + 14 = 0$ passes through the points $(-3, 1)$ and $(-4, 6)$.

a) Substitute the x and y-coordinates of the two points into the equation of the line to form two equations in p and q.

b) Solve this system of equations by substitution to determine the values of p and q and write the equation of the line.

c) Verify the equation in **b)** using the slope formula and the point-slope equation of a line formula.

7. Solve the following systems by substitution. Explain the results.

a) $y = 3x - 7$
$6x - 2y = 14$

b) $x = 3y + 2$
$2x - 6y = 5$

Multiple Choice

8. If $x + 2y = 10$ and $x - 2y = 2$, then $x + y$ is equal to

A. 8

B. 12

C. 13

D. -2

9. When solving a system of equations, one of which is $\frac{x}{2} - \frac{y}{3} = 1$, a substitution which can be made is

A. $x = \frac{1}{3}(2y + 1)$

B. $y = \frac{1}{2}(3x - 1)$

C. $x = \frac{1}{2}(3y + 6)$

D. $y = \frac{1}{2}(3x - 6)$

Numerical Response

10. If $s - 8t + 20 = 5s - 7t + 1 = 0$, then the value of $s + t$, to the nearest tenth, is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $x = 5, y = 4$ **b)** $x = 6, y = 6$ **c)** $x = 5, y = -1$

2. a) \$6 **b)** \$1

3. a) $3x + 2y = 90$, $3x + 4y = 120$ **b)** \$85

4. a) $x = -1, y = 1$ **b)** $x = 6, y = -2$

5. a) $p = -\frac{1}{3}, q = \frac{4}{3}$ **b)** $u = \frac{1}{3}, v = 2$ **c)** $x = 44, y = 19$ **d)** $x = -2, y = 8$

6. a) $-3p + q + 14 = 0$, $-4p + 6q + 14 = 0$ **b)** $p = 5, q = 1$ $5x + y + 14 = 0$

7. a) There are an infinite number of solutions of the form $x = a, y = 3a - 7, a \in R$ because the equations are identical, (the resulting equation reduces to $0 = 0$).

b) There are no solutions since the graphs of the equations are parallel lines, (the resulting equation reduces to $4 = 5$).

8. A **9.** D **10.**

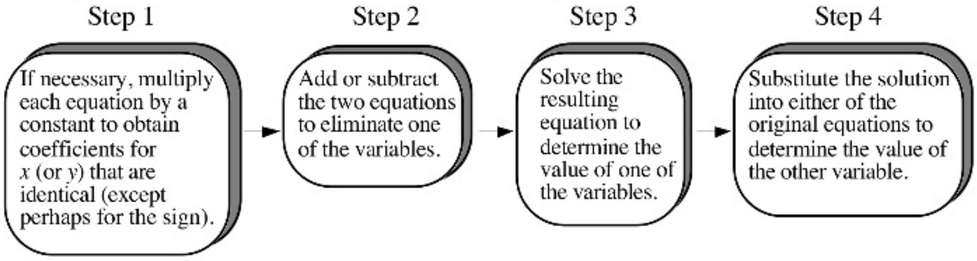
7	.	0	

Systems of Linear Equations Lesson #4:
Solving Systems of Linear Equations by Elimination

So far we have used three methods to solve systems of equations: graphing, inspection, and substitution. In this lesson we will learn another algebraic technique: the method of elimination. This method is particularly useful when the equations involve fractions.

Method of Elimination

In using the method of elimination, there are four steps which are shown below.



Step 1

If necessary, multiply each equation by a constant to obtain coefficients for x (or y) that are identical (except perhaps for the sign).

Step 2

Add or subtract the two equations to eliminate one of the variables.

Step 3

Solve the resulting equation to determine the value of one of the variables.

Step 4

Substitute the solution into either of the original equations to determine the value of the other variable.

Consider the system of equations:

$$\begin{aligned} 2x + 7y &= 13 \\ 3x - 7y &= 2 \end{aligned}$$

a) Add the two equations.
This will eliminate the variable y.

b) Use the equation in **a)** to determine the value of x and hence solve the system.

c) Verify the solution satisfies both equations.

Consider the system of equations:

$$\begin{aligned} 2x + 6y &= 6 \\ 2x + 3y &= 4.5 \end{aligned}$$

a) Subtract the two equations.
This will eliminate the variable x.

b) Use the equation in **a)** to determine the value of y and hence solve the system.

c) Verify the solution satisfies both equations.

Complete Assignment Questions #1 - #3

Consider the system of equations: $2x + 3y = 4$
$4x - y = 22$

a) Does adding or subtracting the equations eliminate either of the variables?

b) Multiply the second equation by 3 and then add the two equations.

c) Solve and verify the system.

d) Consider the original system. Multiply the first equation by an appropriate number which will eliminate x by addition or subtraction. Solve the system.

Consider the system of equations: $5a + 3b = 3$
$3a - 7b = 81$

a) Choose appropriate whole numbers to multiply each equation so that the system can be solved by eliminating b.

b) Solve and verify the system by eliminating b.

c) Choose appropriate whole numbers to multiply each equation so that the system can be solved by eliminating a.

d) Solve the system by eliminating a.

Class Ex. #5

Solve the following system using elimination. $4x + 2y - 13 = 0$, $3x = 5y + 26$

Class Ex. #6

Solve the following system using elimination.

$$\frac{x-2}{3} - \frac{y+2}{5} = 2, \qquad \frac{3}{5}(x+1) - \frac{4}{5}(y-3) = \frac{21}{2}$$

Complete Assignment Questions #4 - #12

Assignment

1. In each of the following systems:

- solve the system using the method of elimination by adding the equations.
- verify the solution satisfies both equations.

a) $8x - y = 10$
$4x + y = 14$

b) $x + 2y = 3$
$-x + 3y = 2$

c) $4a - 3b = 2$
$-4a - b = 6$

2. In each of the following systems:

- solve the system using the method of elimination by subtracting the equations.
- verify the solution satisfies both equations.

a) $7x + y = 15$
$3x + y = 3$

b) $5m + 3n = 10$
$5m - 2n = -15$

c) $4a - 3b = -18$
$-2a - 3b = -9$

3. Solve and verify each of the following systems using the method of elimination.

a) $-10p + 10q = 3$
$10p + 5q = 6$

b) $x + 4y = -0.5$
$5x + 4y = 2.3$

c) $4x + 2y - 31 = 0$
$-4x + 6y - 13 = 0$

4. Solve each of the following systems by elimination. Check each solution.

a) $2a + 5b = 16$
$a - b = 1$

b) $4x - 3y = 9$
$2x - 5y = 1$

c) $5x - 2y = 0.6$
$2x + y = 1.5$

5. Solve each of the following systems by elimination. Check each solution.

a) $2x + 4y = 7, \ \ 4x - 3y = 3$

b) $5x = 8y, \ \ 4x - 3y + 17 = 0$

c) $7e + 4f - 1 = 0, \ \ 5e + 3f + 1 = 0$

d) $3x + 2y - 6 = 0, \ \ 9x = 5y + 18$

6. Consider the system of equations $x - 2y + 1 = 0$, $2x + 3y = 12$. Solve the system by

a) elimination **b)** substitution

Which method do you prefer?

7. Consider the system of equations: $11x + 3y + 2 = 0$, $11x - 5y - 62 = 0$.

Solve the system by

a) elimination **b)** substitution

Which method do you prefer?

8. Solve each of the following systems by elimination. Explain the results.

a) $-2x + 6y - 1 = 0, \quad 5x - 15y + 2.5 = 0$ **b)** $2x - 4y = 7, \quad -7x + 14y = -21$

9. Solve each of the following systems by elimination.

a) $3x - \frac{1}{2}y = 5$
$\frac{1}{3}x + \frac{1}{4}y = 3$

b) $\frac{m}{2} - \frac{n-4}{4} = 2$
$\frac{3m}{4} - \frac{n}{5} = 5$

Multiple Choice

10. When b is eliminated from the equations $2x + b = 8$ and $5x + 2b = 2$, we obtain

- **A.** $7x = 10$
- **B.** $9x = 18$
- **C.** $x = -14$
- **D.** $3x = -6$

11. The solution to the systems of equations $x + y = 0$, $\frac{1}{2}x + \frac{1}{3}y = 1$ is

- **A.** $x = 6, y = -6$
- **B.** $x = 1, y = -1$
- **C.** $x = 0, y = -0$
- **D.** $x = -6, y = 6$

Numerical Response

12. If $\frac{1}{3}x + 5 = \frac{2}{3}y$ and $\frac{1}{2}x + \frac{1}{3}y = \frac{1}{3}$, then the value of $y - \frac{1}{2}x$, to the nearest tenth, is ______ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. a) $x = 2,\ y = 6$ **b)** $x = 1,\ y = 1$ **c)** $a = -1,\ b = -2$

2. a) $x = 3,\ y = -6$ **b)** $m = -1,\ n = 5$ **c)** $a = -\frac{3}{2},\ b = 4$

3. a) $p = \frac{3}{10},\ q = \frac{3}{5}$ **b)** $x = 0.7,\ y = -0.3$ **c)** $x = 5,\ y = \frac{11}{2}$

4. a) $a = 3,\ b = 2$ **b)** $x = 3,\ y = 1$ **c)** $x = 0.4,\ y = 0.7$

5. a) $x = \frac{3}{2},\ y = 1$ **b)** $x = -8,\ y = -5$ **c)** $e = 7,\ f = -12$ **d)** $x = 2,\ y = 0$

6. $x = 3,\ y = 2$ **7.** $x = 2,\ y = -8$

8. a) There are an infinite number of solutions of the form $x = a, y = \frac{1}{6}(2a + 1), a \in R$ because the equations are identical (the resulting equation reduces to $0 = 0$).

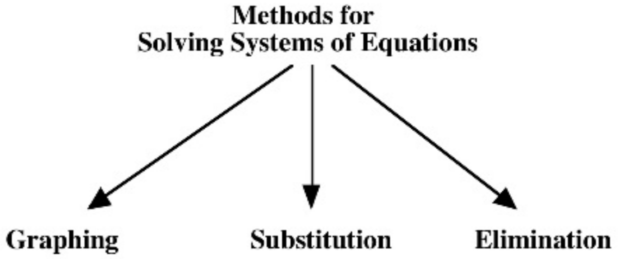
b) There are no solutions since the graphs of the equations are parallel lines (the resulting equation reduces to e.g. $0 = 7$).

9. a) $x = 3,\ y = 8$ **b)** $m = 12,\ n = 20$ **10.** C **11.** A **12.**

7	.	5	

Systems Of Linear Equations Lesson #5:
Number and Money Applications

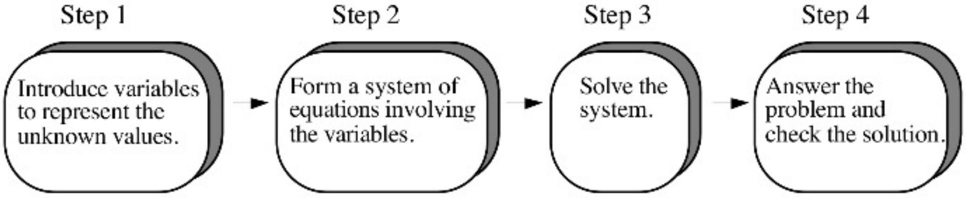
We have discussed three different methods for solving systems of equations.


Methods for
Solving Systems of Equations
Graphing
Substitution
Elimination

In this lesson we apply these methods in problem solving.

Problem Solving

We can solve a variety of types of problems using a system of equations. There are four general steps to problem solving which are shown in the flowchart below.


Step 1
Introduce variables to represent the unknown values.
Step 2
Form a system of equations involving the variables.
Step 3
Solve the system.
Step 4
Answer the problem and check the solution.

Number Applications

The difference between two numbers is 9. The larger number, 3, is more than twice the smaller number. Find the numbers.

The perimeter of a rectangle is 40 metres. The width is 4 metres less than the length. Find the dimensions of the rectangle.

Money Applications

Gary had a total of \$260 in five-dollar bills and ten-dollar bills. If he has 33 bills in total, how many of each denomination does he have?

Complete Assignment questions #1 - #10

Assignment

In problems #1 - #7 use the following procedure:

a) Introduce variables to represent the unknown values.
b) Form a system of equations involving the variables.
c) Solve the system.
d) Answer the problem and check the solution.

1. A rectangle is to be drawn with perimeter 64 cm. If the length is to be 14 cm more than the width, determine the area of the rectangle.

2. The sum of two numbers is 3, and twice the larger number is 36 more than three times the smaller number. Find the numbers.

3. Five pencils and four pens cost $6.15. Three similar pencils and eight similar pens cost $9.85. How much would you expect to pay for a set of eight pencils and seven pens?

4. The perimeter of a rectangle is 40 cm. If the length were doubled and the width halved, the perimeter would be increased by 16 cm. Find the dimensions of the original rectangle.

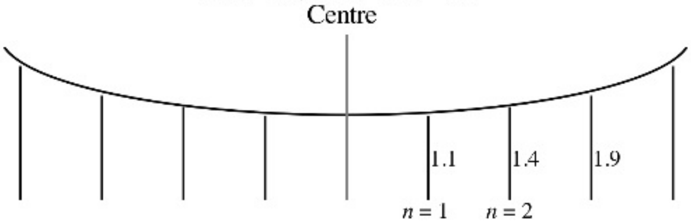
5. A small engineering company has an old machine which produces 30 components per hour, and has recently installed a new machine which produces 40 components per hour. Yesterday, both machines were in operation for different periods of time. If 545 components were produced when the total number of hours of operation was 15 hours, determine for how many hours each machine was operating.

6. In a hockey arena, a seat at rink level costs three times as much as a seat in the upper level. If five seats at rink level cost $112 more than eight seats in the upper level, find the cost of a seat at rink level.

7. Rachel had been saving quarters and dimes to buy a new toy. She had 103 coins and had saved \$21.40. How many coins of each type had she saved?

8. The heights, in metres, of the vertical rods of a suspension bridge, as you move out from the centre of the bridge, form the sequence,

1.1, 1.4, 1.9, 2.6, . . .



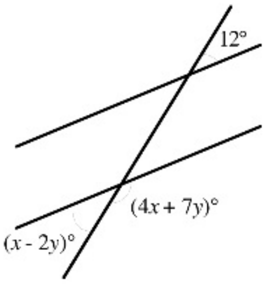
a) Without a calculator determine the next two terms in the sequence.

b) The height, h metres, of the n^{th} rod is given by the formula $h = a + bn^2$. Using the terms of the sequence given to form a system of equations, determine the values of a and b and state the formula.

c) Use this formula to verify the answers in **a**).

Numerical Response

9. The diagram shows two parallel lines and a transversal.



The value of $x + y$, to the nearest whole number, is _____ .

(Record your answer in the numerical response box from left to right) ☐☐☐☐

10. A number consists of two digits whose sum is 11. If the digits are reversed, the original number is increased by 27. The original number is _____ .

(Record your answer in the numerical response box from left to right) ☐☐☐☐

Answer Key

1. 207 cm^2 **2.** 9, −6 **3.** \$10.50 **4.** 12 cm by 8 cm

5. $5\frac{1}{2}$ hours old and $9\frac{1}{2}$ hours new **6.** \$48 **7.** 74 quarters, 29 dimes

8. **a)** 3.5, 4.6, **b)** $a = 1$, $b = 0.1$, $h = 1 + 0.1n^2$

9.

3	6		

10.

4	7		

Systems of Linear Equations Lesson #6:
Mixture and Percentage Applications

Mixture Applications

Cashew nuts costing \$22/kg are mixed with Brazil nuts costing \$16/kg.
The mixture weighs 50 kg and sells for \$18/kg.

a) How much does it cost to buy the whole mixture?

b) Form a system of equations and solve it to determine the number of kilograms of each type of nut used in the mixture.

Lora invested her inheritance of \$48 000 in two different mutual funds. At the end of one year one fund had earned 10.5% interest and the other fund had earned 12% interest. If she received a total of \$5520 in interest, how much did she invest in each mutual fund?

Earl the chemist has to make 180 mL of 60% hydrochloric acid (HCl) solution. He has available a one litre bottle of 45% HCl solution and a one litre bottle of 70% HCl solution by volume. How many mL of each solution are mixed to make the 60% HCl solution?

Complete Assignment questions #1 - #8

Assignment

In problems #1 - #8 use the following procedure.

a) Introduce variables to represent the unknown values.
b) Form a system of equations involving the variables.
c) Solve the system.
d) Answer the problem and check the solution.

1. Candy costing \$6 per kg is mixed with candy costing \$4.50 per kg to produce 112 kg of candy worth \$ 612. How many kg of each type of candy were used?

2. Chad invested $\frac{3}{4}$ of his \$56 000 lottery winnings in two different mutual funds. At the end of the year the *Balanced Fund* had earned 6.5% interest, but the *Emerging Markets Fund* had lost 3%. If the value of Chad's funds increased by \$1 590, determine the amount invested in each fund.

3. Shoji invested \$7 000, part at 9% interest and part at 6% interest. The interest obtained from the 6% investment was half of the interest obtained from the 9% investment. How much was invested at each rate?

4. 300 grams of Type A Raisin Bran is mixed with 500 grams of Type B Raisin Bran to produce a mixture which is 11% raisins. Type A Raisin Bran has twice as many raisins per kilogram as Type B. What percentage of raisins are in each type of Raisin Bran?

5. A scientist has to make 800 ml of 61% sulfuric acid solution . He has available a one litre bottle of 40% sulfuric acid solution and a one litre bottle of 75% sulfuric acid solution by volume.

a) How many ml of each solution are mixed to make the 61% sulfuric acid solution?

b) What is the maximum volume, rounded down to the nearest ml, of 61% sulfuric acid solution which the scientist could mix with the original bottles of sulfuric acid?

6. One year a man saved $5000. The next year his income increased by 10% and his expenditure decreased by 16%. He was able to save $14 600. Calculate his income in the second year.

7. Pure gold (24-carat) is often mixed with other metals to produce jewellery. 12-carat gold is $^{12}/_{24}$ or 50% gold, 6-carat gold is $^{6}/_{24}$ or 25% gold, etc.
A jeweller has some 1- carat gold and some 21-carat gold and wants to produce 90 grams of 75% gold.

a) What percentage of gold is 21-carat?

b) How many grams of 12-carat gold and of 21 carat gold are needed to produce the mixture?

Multiple Choice

8. A shopkeeper wishes to mix two types of tea together. One type sells at $8 per kg and the second type sells at $12 per kg. He wishes to make 100 kg of the mixture to sell at $11 per kg. The number of kg of the first type of tea in this mixture should be

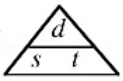
A. 25
B. $33\frac{1}{3}$
C. 50
D. 75

Answer Key

1. 72 kg of $6/kg candy, 40 kg of $4.50/kg candy
2. $30 000, $12 000
3. $3000 at 6% and $4000 at 9%
4. 16% in type A, 8% in type B
5. **a)** 320 ml of 40% solution, 480 ml of 75% solution **b)** 1666 ml
6. $44 000
7. **a)** 87.5% **b)** 30 g of 12-carat gold, 60 g of 21-carat gold
8. A

Systems of Linear Equations Lesson #7:

Distance, Speed, and Time Applications



A student drove the 1245 km from Edmonton to Vancouver in $16\frac{1}{2}$ hours. This included a one hour stop in Golden and a 30 minute stop in Kamloops. She averaged 100 km/h on the divided highways and 75 km/h on the non-divided mountainous roads. How much time did she spend on the divided highways?

A small cruise boat took 3 hours to travel 36 km down a river with the current. On the return trip it took 4 hours against the current. Find the speed of the current and the speed of the small cruise boat in still water.

Complete Assignment questions #1 - #6

Assignment

In problems #1 - #6 use the following procedure.

a) Introduce variables to represent the unknown values.
b) Form a system of equations involving the variables.
c) Solve the system.
d) Answer the problem and check the solution.

1. A cycle road test consists of a series of uphill and downhill sections.
Li Na averaged 20 km/hr on the uphill sections and 40 km/hr on the downhill sections.
If she completed the 35 km course in 1.3 hours, determine the length of the downhill sections.

2. A train travels 315 km in the same time that a car travels 265 km. If the train travels, on average, 20 km/h faster than the car, find the average speed of the car and the time taken to travel 265 km.

3. A small plane flying into a wind takes 3hr to travel the 780 km journey from Victoria to Prince Rupert. At the same time, a similar plane leaves Prince Rupert and reaches Victoria in $2\frac{1}{2}$ hr. If the planes have the same cruising speed in windless conditions, determine the speed of the wind.

4. A cyclist leaves home at 7:30 am to cycle to school 7 km away. He cycles at 10 km/h until he has a puncture; then he has to push his bicycle the rest of the way at 3 km/h. He arrives at school at 8:40 am. How far did he have to push his bicycle?

Multiple Choice **5.** Chris walks at 8 km/h and runs at 12 km/h. One day he walks and runs on the way from his house to the library. It takes him 20 minutes.
On his way back from the library he again walks and runs, but he runs twice as far as he did on the way to the library. The journey home takes $17\frac{1}{2}$ minutes.
The distance between his house and the library is

A. 3 km

B. 4 km

C. 5 km

D. 6 km

Numerical Response **6.** Raj left home at 1 pm to travel 675 km to visit his sister. He averaged 110km/h for the first part of the trip during which he had a 1 hour rest, and 90 km/h for the second part of the trip during which he had a 30 minute rest. He reached his destination at 9 pm. The number of minutes taken for the first part of the trip, to the nearest minute, was ______ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. 18 km **2.** 106 km/h, $2\frac{1}{2}$ hr **3.** 26 km/h **4.** 2 km **5.** A **6.**

2	7	0	

Systems of Linear Equations Lesson #8:
Practice Test

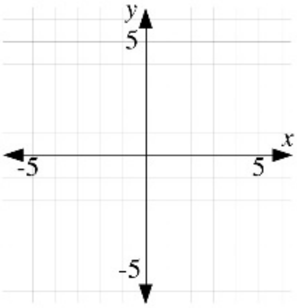
Numerical Response **1.** Two students worked together to solve a system of equations which had integer solutions. Tara made a table of values for the first equation and Jorge made a table of values for the second equation.

Tara

x	y
-2	-6
0	-2
2	2
4	6

Jorge

x	y
-3	-2
-1	0
1	2
3	4
5	6



Using the students' results to determine the solution to the system, the value of $x + y$ is _____ .

(Record your answer in the numerical response box from left to right)

1. The ordered pair (x, y) which satisfies the system of equations $x - 3y = 8$, $x + 4y = -13$, is

A. $(-1, 3)$

B. $(3, -1)$

C. $(-1, -3)$

D. $(3, 1)$

2. The number of solutions to the system of equations $6x - 2y = 24$, $5y = 15x - 64$, is

A. zero

B. one

C. two

D. infinite

Numerical Response **2.** Alyssa graphs the equations $x - y = -4$ and $x + 2y = 4$. The y-coordinate, to the nearest hundredth, of the point of intersection is _____ .

(Record your answer in the numerical response box from left to right)

3. Consider the following two systems of equations.

a) $y = \frac{2}{3}x + 1, \quad y = \frac{1}{2}x - 2$ **b)** $4x + 5y = 18, \quad 2x + 3y = 1$

Solve the systems of equations using a graphing calculator and determine which one of the following statements is true.

A. In system **a),** $x + y = 29$.

B. In system **b),** $x + y = 40.5$.

C. One of the values of x is 42.5 more than the other.

D. One of the values of y is 42.5 more than the other.

4. The system $y = 4x - 8, \; x = by + c,$ has an infinite number of solutions if

A. $b = 0.25$ and $c = -2$

B. $b = 0.25$ and $c = 2$

C. $b = 0$ and $c = 0$

D. $b = 4$ and $c = -8$

5. The graphs of $y = ax + b$ and $y = cx + d$ are parallel.
The number of solutions to the system $y = ax + b, \; y = cx + d$ is

A. zero **B.** one **C.** two **D.** infinite

6. For which system of equations graphed on a grid is the point $(-2, -1)$ a solution?

A. $x - 2y = 3, \quad x + 5y = -11$

B. $3x - 3y = -9, \quad x - 4y = -6$

C. $2x - 10y = 6, \quad x + 5y = -3$

D. $x + y = -3, \quad -2x + 5y = -1$

7. When solving a system of equations, one of which is $\frac{x}{4} - \frac{y}{3} = 2$, a substitution which can be made is

A. $x = \frac{1}{4}(3y + 2)$

B. $x = \frac{1}{3}(4y + 6)$

C. $x = \frac{1}{4}(3y + 24)$

D. $x = \frac{1}{3}(4y + 24)$

8. In solving the system $3a - 2b = 14,\ 2a + b = 7$ by elimination, an equation which arises could be

A. $-7b = 49$

B. $-b = 7$

C. $5a = 21$

D. $7a = 28$

9. If $x + y = 12$ and $x - y = 2$, then $x + 2y$ is equal to

A. 10

B. 17

C. 19

D. 34

10. If $3(x - 2) + y = 7$ and $4x - 3(y - 1) = 16$, then y is equal to

A. $-\frac{15}{13}$

B. $-\frac{5}{13}$

C. 1

D. 4

11. Solve the following system using elimination.

$$\frac{2p}{3} - \frac{3q}{4} = \frac{11}{2}, \qquad \frac{5p}{9} + \frac{q}{6} = 3$$

The value of p is

A. 6

B. $\frac{75}{38}$

C. $\frac{48}{11}$

D. $\frac{7}{24}$

Numerical Response **3.** If $m - 2n - 30 = 2m - n - 39 = 0$, then the value of $m - n$, to the nearest tenth, is _____ .

(Record your answer in the numerical response box from left to right)

12. If $\frac{2x + y}{3} - 5 = 0$ and $\frac{3x - y}{5} = 1$, then the value of y is

A. 4

B. 7

C. $\frac{13}{5}$

D. $\frac{23}{25}$

Numerical Response **4.** The straight line $ax + y = b$ passes through the points $(-1, 1)$ and $(-5, 4)$. The value of ab, to the nearest hundredth, is _____.

(Record your answer in the numerical response box from left to right)

Use the following information to answer the next two questions.

> Lisbeth cycled 100 km from Calgary to Canmore. On the uphill sections her average speed was 12 km/h, and on the rest of the trip her average speed was 28 km/h.
>
> She cycled for a total of 5 hours on the journey.

13. If x km represents the distance travelled uphill and y km represents the distance travelled on the rest of the trip, which of the following systems could be used to determine the values of x and y?

A. $x + y = 100, \quad 12x + 28y = 5$

B. $x + y = 100, \quad \frac{x}{12} + \frac{y}{28} = 5$

C. $x + y = 5, \quad 12x + 28y = 100$

D. $x + y = 5, \quad \frac{x}{12} + \frac{y}{28} = 100$

Numerical Response **5.** The distance travelled uphill, to the nearest km, is _____.

(Record your answer in the numerical response box from left to right)

14. For a birthday gift Isabelle was given an electronic piggy bank containing loonies and toonies. The display showed that the bank contained 26 coins with a value of \$40. The value of the toonies in the piggy bank, in dollars, was

A. 28

B. 24

C. 14

D. 12

15. Susan solves the system of equations $\frac{2}{x} + \frac{3}{y} = 2, \quad \frac{8}{x} - \frac{9}{y} = 1$ by first substituting a for $\frac{1}{x}$ and b for $\frac{1}{y}$. The value of xy is

A. 6

B. $\frac{1}{6}$

C. $-\frac{49}{720}$

D. $-\frac{720}{49}$

Written Response - 5 marks

1. Erika plans to set up an internet connection with *Y2K Internet Company*. There are three plans to choose from.

- Plan 1 costs $20 per month and includes a user fee of 40¢ per hour.
- Plan 2 costs $15 per month and includes a user fee of 80¢ per hour.
- Plan 3 costs $60 per month for unlimited use.

- What factor would determine which plan is most economical?

- Let y = total cost per month in dollars and x = number of hours of use per month. Write a linear equation for each of the three plans.

- Use a graphical method to determine when plans 1 and 2 are equally economical to use. State the graphing window used.

- Algebraically verify the solution in the bullet above algebraically.

- For each of plans 1 and 2, determine the number of hours of use which could be obtained for $60.

- Devise a simple rule which would determine which plan is most economical depending on the expected number of hours of internet use per month.

Answer Key

1. C **2.** A **3.** C **4.** B **5.** A **6.** D **7.** D **8.** D

9. B **10.** C **11.** A **12.** B **13.** B **14.** A **15.** A

Numerical Response

1.

7			

2.

2	.	6	7

3.

2	3		

4.

0	.	1	9

5.

3	0		

Written Response

1.
- The expected number of hours of internet use per month.
- $y = 20 + 0.4x$, $y = 15 + 0.8x$, $y = 60$
- 12.5 hours, eg. x:[0, 50, 10], y:[0, 50, 10]
- 12.5 hours
- 100 hours, $56\frac{1}{4}$ hours
- Plan 2 for up to 12.5 hours, Plan 1 for between 12.5 and 100 hours, Plan 3 for more than 100 hours

Arithmetic Sequences Lesson #1:
Investigating Patterns and Sequences

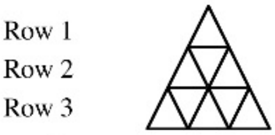
Overview

In this unit we investigate patterns used to define types of sequences. We then focus on arithmetic sequences and apply formal language to increasing and decreasing linear patterns. We derive a rule for determining the general term of an arithmetic sequence and explore problems relating to arithmetic growth and decay.
As an extension we explore arithmetic series.

Investigation 1

Jesse is making a tower using playing cards.
The top three rows of the tower are shown.

The top row (Row 1) requires three playing cards.



The second row (Row 2) requires six additional playing cards.

Continue the pattern for two more rows, and complete Tables A and B below.

Table A

Row Number	1	2	3	4	5
Number of Additional Cards in the Row	3	6			

Table B

Row Number	1	2	3	4	5
Number of Triangles in the Row	1	3			

Investigation 2

Each row in the triangle, named after French mathematician Blaise Pascal, begins and ends with the number 1.

Apart from the ones, every other number in a particular row can be determined by adding the two numbers diagonally above it.

	Row
1	1
1 1	2
1 2 1	
1 3 3 1	

Continue the pattern for rows 5, 6, and 7, and complete Table C.

Table C

Row Number	1	2	3	4	5	6	7
Sum of the Numbers in the Row	1	2	4				

Investigation 3

Triangle 1, shown in Diagram 1, is an equilateral triangle with sides of length 32 cm. A smaller triangle, Triangle 2, is placed inside Triangle 1 by joining the midpoints of Triangle 1 (as illustrated in Diagram 2). The pattern is continued as illustrated in Diagram 3.

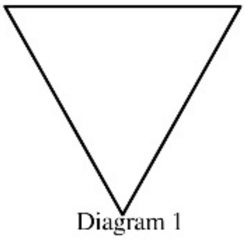


Diagram 1

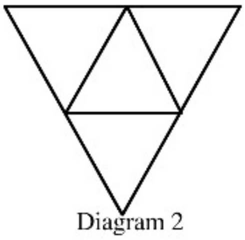


Diagram 2

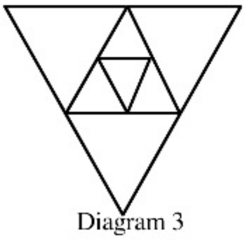


Diagram 3

a) Diagrams 4 and 5 in the sequence have not been completed. Complete each diagram.

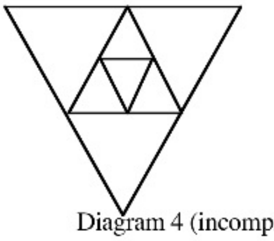


Diagram 4 (incomplete)

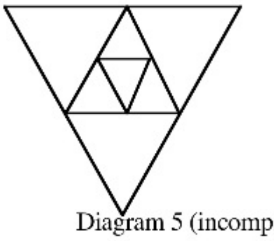


Diagram 5 (incomplete)

b) Complete Tables D and E below.

Table D

Triangle Number	1	2	3	4	5
Length of Side of Triangle (cm)	32				

Table E

Diagram Number	1	2	3	4	5
Number of Triangles of any Size in the Diagram	1	5			

Investigation 4

The mean daily temperature in Prince George on Dec 1 was 8°C. For the next six days, the mean daily temperature decreased by 4°C each day.

Complete Table F below.

Table F

Day Number	1	2	3	4	5
Temperature (°C)	8				

In each of the tables A - F, the top row consists of the **natural numbers** 1, 2, 3, 4, etc., and the bottom row consists of a **sequence** of numbers related to the natural numbers in a specific order.

The first table you completed was **Table A**

Row Number	1	2	3	4	5
Number of Cards in the Row	3	6	9	12	15

The sequence formed is 3, 6, 9, 12, 15. The sequence consists of **terms**.

The first term, written t_1, is equal to 3. The second term, t_2, is equal to 6; $t_3 = 9$ etc.

In each of the following:

a) Complete the table using the information from Tables A - F.

b) Complete the statement, explaining how to find the next term in the sequence from the previous term using only addition or multiplication.

c) State the next two terms of the sequence.

Table A

n	1	2	3	4	5
t_n	3	6			

The next term can be calculated by
________________ the previous term.
$t_6 =$ ________ and $t_7 =$ _______ .

Table B

n	1	2	3	4	5
t_n					

The next term can be calculated by
________________ the previous term.
$t_6 =$ ________ and $t_7 =$ _______ .

Table C

n	1	2	3	4	5	6	7
t_n							

The next term can be calculated by
________________________________ .
$t_8 =$ ________ and $t_9 =$ _______ .

Table D

n	1	2	3	4	5
t_n					

The next term can be calculated by
_____________________________.
$t_6 =$ ________ and $t_7 =$ _______ .

Table E

n	1	2	3	4	5
t_n					

The next term can be calculated by
________________ the previous term.
$t_6 =$ ________ and $t_7 =$ _______ .

Table F

n	1	2	3	4	5
t_n					

The next term can be calculated by
________________ the previous term.
$t_6 =$ ________ and $t_7 =$ _______ .

Types of Sequences

There are different types of sequences.

A sequence in which the next term is formed by adding a constant (positive or negative) to the previous term is called an **arithmetic sequence**.

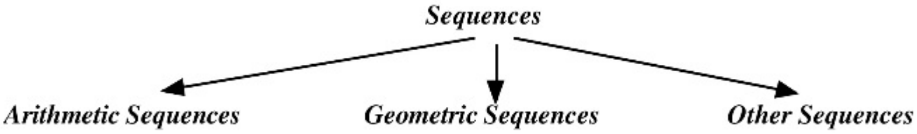
A sequence in which the next term is formed by multiplying the previous term by a constant (positive or negative) is called a **geometric sequence**.

There are other sequences, which are not arithmetic or geometric.

For example: Sequence of Prime Numbers 2, 3, 5, 7, 11, ...

A Fibonacci Sequence, 1, 1, 2, 3, 5, 8, 13, ... , is a special type of sequence which occurs in nature in such things as seed growth, leaves on stems, petals on flowers, etc.

In the remaining lessons in this unit, we focus on arithmetic sequences.
Geometric sequences will be studied in a later course.



Classify the sequences in Tables A - F as arithmetic sequences or geometric sequences.

Finite and Infinite Sequences

Finite Sequence - a sequence that has a specific number of terms.

eg. 4, 10, 16, 22, 28 or 2, 4, 8, 16, ... 256.

Infinite Sequence - a sequence that has an unlimited number of terms.

eg. 4, 10, 16, 22, 28 ... or 2, 4, 8, 16, ...

A Sequence as a Function

A sequence can be regarded as a **function** relating the set of natural numbers to the terms of the sequence.

The **domain** of the function is the set of natural numbers.

The **range** of the function is the set of terms of the sequence.

Some sequences can be represented by linear functions, and some can be represented by non-linear functions.

Class Ex. #3

Using the information in **Tables A - F,** plot the points (n, t_n) on the grids. In each case, state whether the function represented by the sequence is linear or non-linear.

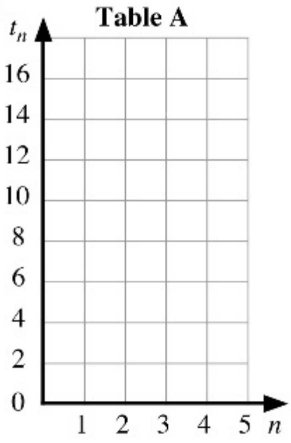


Table A

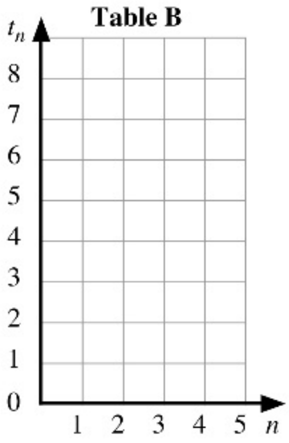


Table B

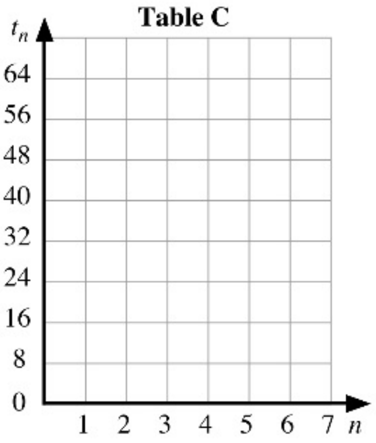


Table C

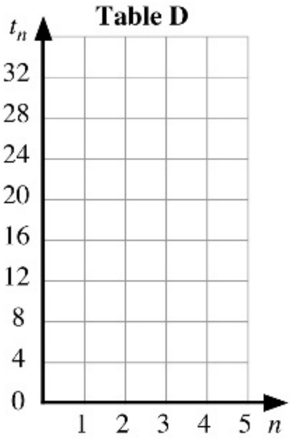


Table D

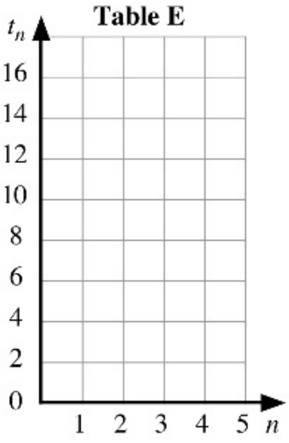


Table E

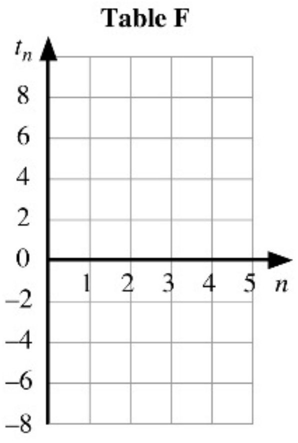


Table F

Circle the correct alternative in the following statements.

a) A sequence in which the next term is determined by adding a constant to the previous term is an arithmetic / geometric sequence.

The sequence can be represented by a linear / non-linear function.

b) A sequence in which the next term is determined by multiplying the previous term by a constant is an arithmetic / geometric sequence.

The sequence can be represented by a linear / non-linear function.

Complete Assignment Questions #1 - #12

Assignment

1. Consider the pattern of squares within squares shown below.

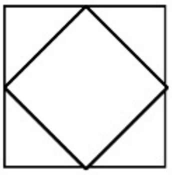


Diagram 1

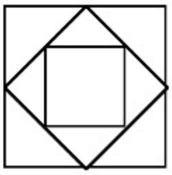


Diagram 2

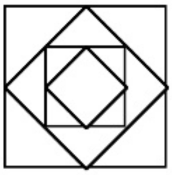


Diagram 3

a) Draw the next diagram in the pattern in the space above.

b) Complete the table.

Diagram Number	1	2	3	4
Number of Triangles in the Diagram	4			

c) Consider the sequence of triangles in the table above.

(i) Complete the statement explaining how to find the next term in the sequence from the previous term.

The next term can be calculated by ____________________________________.

(ii) State the value of the following terms. t_1 = ___ t_5 = ___ t_6 = ___

(iii) Circle the correct alternatives.
The sequence is arithmetic / geometric and can be represented by a linear / non-linear function.

2. Consider the following pattern of squares.

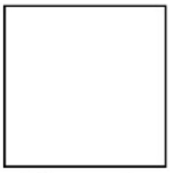


Diagram 1

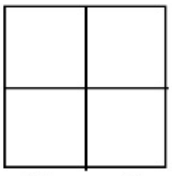


Diagram 2

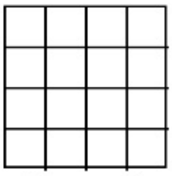


Diagram 3

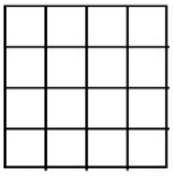


Diagram 4 (incomplete)

a) Complete diagram 4 in the pattern.

b) Complete the table.

Diagram Number	1	2	3	4
Number of Congruent Squares in the Diagram				

c) Consider the sequence of squares in the table above.

(i) Complete the statement explaining how to find the next term in the sequence from the previous term.

The next term can be calculated by ____________________________________.

(ii) State the value of the following terms. $t_4 =$ ___ $t_5 =$ ___ $t_6 =$ ___

(iii) Circle the correct alternatives.
The sequence is an arithmetic / geometric sequence and can be represented by a linear / non-linear function.

3. For each of the following sequences, determine if the sequence is arithmetic, geometric, or neither.

a) $1, 2, 3$

b) $-1, 1, -1, \ldots$

c) $12, 60, 300, \ldots$.

d) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots, \frac{1}{2187}$

e) $1, 1, 2, 3, \ldots$

f) $250, 200, 150, \ldots$

4. Classify each sequence in question **3.** as a finite sequence or an infinite sequence.

a)

b)

c)

d)

e)

f)

5. Write the next three terms of each of the infinite sequences in question **3**.

6. For each of the following sequences of numbers:

- Identify the type of sequence as arithmetic or geometric.
- Write the next two terms of the sequence.
- Describe a rule which can be used to form the sequence using addition or multiplication.

a) $5, 10, 15, \ldots$ **b)** $5, 10, 20, \ldots$ **c)** $-8, -5, -2, \ldots$

d) $80, 40, 20, \ldots$ **e)** $25, 18, 11, \ldots$ **f)** $-8, -11, -14, \ldots$

7. A fractal is a fragmented geometric shape that can be subdivided into parts, each of which is a scaled down copy of the whole. There is increased complexity of the shape at each step. An example is shown below.

Diagram 1

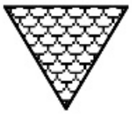


Diagram 2

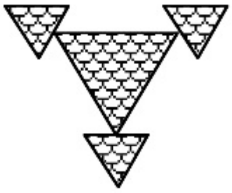
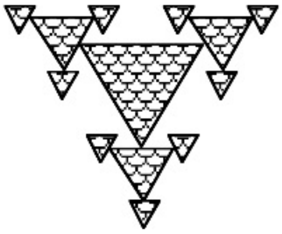


Diagram 3



a) Diagram 4 has been started below. Complete the diagram.

b) Complete the table below.

Diagram 4

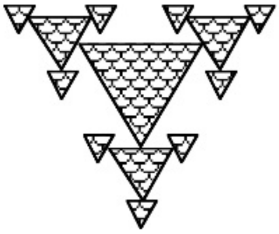


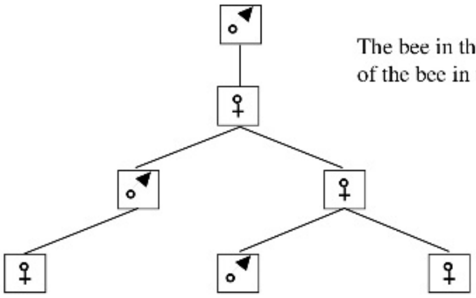
Diagram Number	1	2	3	4
Number of New Triangles in the Diagram	1			

c) Classify the sequence as arithmetic, geometric, or neither.

d) Determine the eighth term of the sequence.

8. The reproduction of bees in nature follows an interesting sequence. Whereas the female bee has two parents (a mother and a father), the male bee has only one parent (a mother).

a) Complete three more generations of the family tree for the male bee below.



The bee in the first row is the child of the bee in the second row.

b) Complete the following table.

Row Number	1	2	3	4	5	6	7
Number of Bees in the Row	1	1					

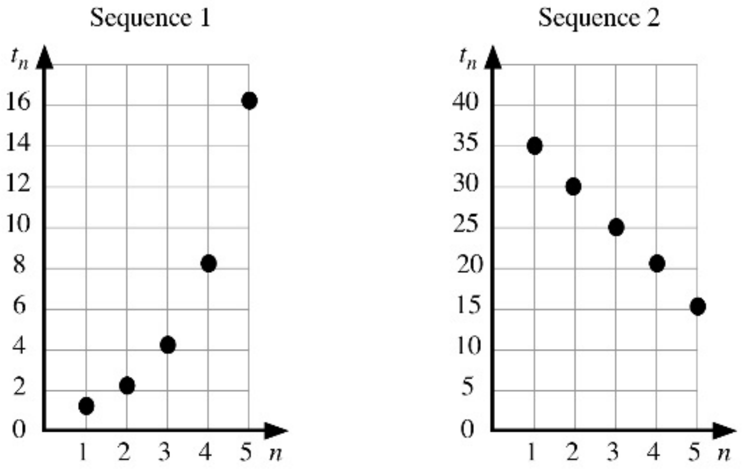
c) Is the sequence represented in the table arithmetic, geometric or neither?

d) How many female bees are in row 7?

e) How many bees are in the eighth row?

Use the following information to answer the next four questions.

The graphical representations of two sequences are shown in the grids below.



Sequence 1

Sequence 2

9. Explain how you can determine which one of the sequences is arithmetic.

10. For each sequence, describe a rule for determining the next term from the previous term.

Multiple Choice **11.** The eighth term of Sequence 2 is

A. 5

B. 0

C. –5

D. –10

Numerical Response **12.** The eight term of Sequence 1 is _____ .

(Record your answer in the numerical response box from left to right.)

Group Investigation

The following investigation can be used as a lead-in to the next lesson. The solution can be obtained by drawing a grid and using counters or coins; however, greater understanding may be obtained by having students act out the situation.

	A	B	C
1	S	S	S
2	S	S	S
3	S	S	

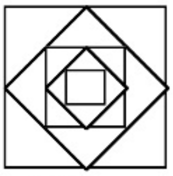
Eight students are arranged in a 3 × 3 grid. Students can move either left/right or up/down into an empty cell. A "move" consists of any one student moving into an adjacent cell.

a) Determine the smallest number of "moves" required for the student in cell A1 to end up in cell C3.

b) Extend your thinking by developing a strategy for determining the smallest number of "moves" required to move a student from cell A1 to cell Z26 in a 26 × 26 grid.

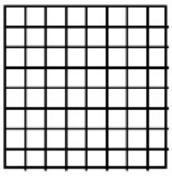
Answer Key

1. a)



b) 8, 12, 16
c) **i)** adding four to the previous term
ii) $t_1 = 4, t_5 = 20, t_6 = 24$
iii) arithmetic, linear

2. a)



b) 1, 4, 16, 64
c) **i)** multiplying the previous term by 4
ii) $t_4 = 64, t_5 = 256, t_6 = 1024$
iii) geometric, non-linear

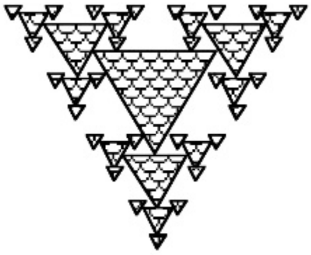
3. a) arithmetic **b)** geometric **c)** geometric **d)** geometric **e)** neither **f)** arithmetic

4. a) finite **b)** infinite **c)** infinite **d)** finite **e)** infinite **f)** infinite

5. b) 1, –1, 1 **c)** 1 500, 7 500, 37 500 **e)** 5, 8, 13 **f)** 100, 50, 0

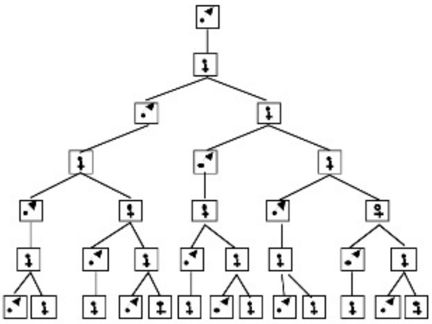
6. a) Arithmetic. 20, 25. Add 5 to the previous term.
b) Geometric. 40, 80. Multiply the previous term by 2.
c) Arithmetic. 1, 4. Add 3 to the previous term.
d) Geometric. 10, 5. Multiply the previous term by $\frac{1}{2}$.
e) Arithmetic. 4, –3. Add –7 to the previous term.
f) Arithmetic. –17, –20. Add –3 to the previous term.

7. a)



b) 3, 9, 27 **c)** Geometric **d)** 2187

8. a)



b) 2, 3, 5, 8, 13 **c)** Neither **d)** 8 **e)** 21

9. If the graph is linear, the sequence is arithmetic. Sequence 2 is arithmetic.

10. For sequence 1, multiply the previous term by 2. For sequence 2, add –5 to the previous term.

11. B

12.

1	2	8	

Group Investigation

a) 13
b) 2 x 2 grid → 5 moves. 3 x 3 grid → 13 moves. 4 x 4 grid → 21 moves.
Extend the pattern: 26 x 26 grid → 197 moves.

Arithmetic Sequences Lesson #2:

Arithmetic Sequences

Arithmetic Sequence

An **arithmetic sequence** is a sequence in which each term is formed from the preceding term by ***adding*** a constant (positive or negative).

Complete the following for the sequence 7, 10, 13, 16, ...

- Each term is determined by adding ____ to the previous term.
- Calculate the differences: $t_2 - t_1 =$ ____ $t_3 - t_2 =$ ____ $t_4 - t_3 =$ ____

Notice that there is a **common difference** between successive terms.

The common difference in this example is ____.

Finding a Common Difference

To find a common difference in an arithmetic sequence, we can subtract any term from the term after it.

For example $t_2 - t_1 =$ common difference, or
$t_5 - t_4 =$ common difference, etc.

$$\boxed{\text{common difference} = t_n - t_{n-1}}$$

Consider the sequence 16, 13, 10, 7, ...

The **common difference** in the sequence is ______ .

For each of the following :

- Determine which sequences are arithmetic.
- Find the common difference for those sequences which are arithmetic .

a) 2, 4, 6, 8, ... **b)** $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \ldots$ **c)** −10, −4, 2, 8, ... **d)** 4, 8, 16, 32, ...

In an arithmetic sequence we often use the following terminology.

The **first term** in an arithmetic sequence is represented by t_1, or a, and the **common difference** is represented by d.

Class Ex. #3

State the values of a and d in the following sequences:

i) $-8, 2, 12, 22, \ldots$ **ii)** $15, 10, 5, 0 \ldots.$

Investigation

Investigating the Formula for the General Term of an Arithmetic Sequence

Consider the sequence $2, 12, 22, 32, 42 \ldots.$

a) State the following
$t_1 =$ $t_2 =$ $t_3 =$ $t_4 =$ $t_5 =$ $a =$ $d =$

b) Complete the following pattern which describes each term in the sequence in terms of the first term, a, and the common difference, d.

$t_1 = 2$	$t_1 = a$
$t_2 = 2 + 1(10) = 12$	$t_2 = a + (1)d$
$t_3 = 2 + 2(10) = 22$	$t_3 = a + 2d$
$t_4 =$	$t_4 =$
$t_5 =$	$t_5 =$
$t_{30} =$	$t_{30} =$
$t_n =$	$t_n =$

The Formula for the General Term of an Arithmetic Sequence

The formula for the general term of an arithmetic sequence is

$$\boxed{t_n = t_1 + (n-1)d} \quad \text{or} \quad \boxed{t_n = a + (n-1)d}$$

where, t_n is the general term of the arithmetic sequence
$a = t_1$, is the first term
d is the common difference
n is the position of the term in the sequence

Note

The general arithmetic sequence is $a, a + d, a + 2d, a + 3d, \ldots a + (n-1)d.$

Class Ex. #4

Consider the arithmetic sequence $-6, -1, 4, 9, \ldots$.

a) Determine the formula for the general term of the sequence.

b) Determine the value of the twelfth term of the sequence.

Class Ex. #5

Find the number of terms in the arithmetic sequence $3, -1, -5, \ldots, -117$.

Complete Assignment Questions #1 - #9

Arithmetic Means

The terms placed between two non-consecutive terms of an arithmetic sequence are called **arithmetic means**. For example, in the sequence $5, 10, 15, 20$, the numbers 10 and 15 are arithmetic means between 5 and 20. In order to determine arithmetic means between two given terms, it is helpful to think of the two given terms as the **first** and **last** terms of a sequence.

Class Ex. #6

Place three arithmetic means between -4 and 8.

Solving Sequence Problems Where Both "a" and "d" are Unknown

Consider the sequence $x + 2, 3x - 1, 2x + 1$.

a) Determine the value of x such that $x + 2, 3x - 1$, and $2x + 1$ form an arithmetic sequence.

b) Determine the numerical value of the three terms.

The next two class examples show two different ways of solving the same problem. Class Example #8 uses arithmetic means, and Class Example #9 uses a system of linear equations.

The third and eighth terms of an arithmetic sequence are 12 and –18, respectively.

a) Use arithmetic means to determine the fifth term of the sequence.

b) State the first term, a, and the common difference, d, of the sequence.

c) Complete the following: $\dfrac{t_8 - t_3}{8 - 3} = \underline{\qquad\qquad} =$

d) Write t_3 and t_8 in terms of a and d and prove that $\dfrac{t_8 - t_3}{8 - 3} = d$.

e) Suggest a formula for finding the common difference of a sequence if you are given the value of the p^{th} term and the q^{th} term.

Class Ex. #9

The third and eighth terms of an arithmetic sequence are 12 and –18, respectively. Use a system of linear equations to determine the values of the first term and the common difference. Hence, determine the fifth term of the sequence.

Complete Assignment Questions #10 - #16

Assignment

1. For the following arithmetic sequences:

i) Determine the common difference. **ii)** Find the next three terms of the sequence.

a) 8, 14, 20, ... **b)** –5, 7, 19, ... **c)** 70, 53, 36, 19, ...

d) 7.1, 4.2, 1.3, ... **e)** $\frac{2}{3}, \frac{1}{15}, -\frac{8}{15}, \dots$ **f)** $-2x + 3y, -5x + y, -8x - y, \dots$

2. In each of the following sequences, the value of one term is given. Write the missing terms of the sequence if the common difference is as indicated.

a) ____, ____, 0, ____, ____: $d = 3$ **b)** ____, ____, ____, –3, ____: $d = -7$

c) ____, ____, 1, ____, ____: $d = -2$ **d)** ____, ____, ____, ____, 15: $d = 2.5$

3. Calculate the first four terms of the arithmetic sequences with given term and common difference, d.

a) $t_1 = 5, d = 6$ **b)** $t_3 = 15, d = -2$ **c)** $t_5 = 20, d = -1$

4. Consider the sequence $12, 5, -2, -9, \dots$.

a) Determine the formula for the general term of the sequence.

b) Determine the nineteenth term of the sequence.

c) Which of the numbers –268 and –350 are terms of the sequence?

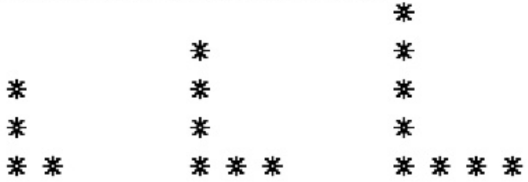
5. Determine the indicated terms in each arithmetic sequence.

a) $-1, -4, -7, -10, \dots . t_5,\ t_{24},\ t_n$

b) $-21, -6, 9, 24, \dots . t_{10},\ t_{90},\ t_n$

c) $-b, 2a - b, 4a - b, 6a - b, \dots . t_{12},\ t_n$

6. Consider the pattern of L-shapes shown.



a) State the number of stars in each of the first four patterns.

b) How many stars are in the 34th pattern?

7. Determine the number of terms in each sequence.

a) $4, 7, 10, ..., 49$

b) $-52, -56, -60, ..., -148$

8. How many multiples of 5 are there from 25 to 315, inclusive?

9. Consider the sequence of multiples of 7 between 51 and 275.

a) State the first and last terms of the sequence.

b) How many multiples of 7 are there between 51 and 275?

10. Determine the 4th, 5th, 6th, and 7th terms of the arithmetic sequence in which $t_3 = -24$ and $t_8 = -94$.

11. The terms $2x + 3$, $3x + 1$, and $8x - 1$ are consecutive terms in an arithmetic sequence. Calculate the value of x, and state the three terms.

12. The terms $x + 3$, $3x - 1$, and $7x - 2$ are consecutive terms in an arithmetic sequence. Calculate the value of x, and determine the general term of the sequence.

13. In an arithmetic sequence, the seventh term is 3 and the sixteenth term is 9.

a) Use arithmetic means to determine the common difference and the first term of the sequence.

b) Use a system of equations to determine the common difference and the first term of the sequence.

c) Calculate t_{19} and determine the general term of the sequence.

14. Use linear systems to determine a, d, and t_n for the sequences in which the following two terms are given.

a) $t_5 = 21, t_{10} = 41$ **b)** $t_4 = -9, t_{15} = -31$

Multiple Choice **15.** Which of the following represents an arithmetic sequence with a common difference of –4?

A. 8, 4, 2, 1 . . .
B. 20, 24, 28, 32 . . .
C. 32, –8, 2, –0.5 . . .
D. 20, 16, 12, 8 . . .

Numerical Response **16.** Twenty-seven arithmetic means are inserted between the first and last terms of a sequence. The number of terms in the sequence is _______ .

(Record your answer in the numerical response box from left to right.)

Answer Key

1. a) i) 6 ii) $t_4 = 26, t_5 = 32, t_6 = 38$ **b)** i) 12 ii) $t_4 = 31, t_5 = 43, t_6 = 55$
c) i) -17 ii) $t_5 = 2, t_6 = -15, t_7 = -32$ **d)** i) -2.9 ii) $t_4 = -1.6, t_5 = -4.5, t_6 = -7.4$
e) i) $-3/5$ ii) $t_4 = -17/15, t_5 = -26/15, t_6 = -7/3$
f) i) $-3x - 2y$, ii) $t_4 = -11x - 3y, t_5 = -14x - 5y, t_6 = -17x - 7y$

2. a) $-6, -3, 0, 3, 6$ **b)** $18, 11, 4, -3, -10$ **c)** $5, 3, 1, -1, -3$ **d)** $5, 7.5, 10, 12.5, 15$

3. a) 5, 11, 17, 23 **b)** 19, 17, 15, 13 **c)** 24, 23, 22, 21

4. a) $t_n = 19 - 7n$ **b)** -114 **c)** -268 is the 41st term

5. a) $-13, -70, t_n = 2 - 3n$ **b)** $114, 1314, t_n = 15n - 36$ **c)** $22a - b, t_n = 2an - 2a - b$

6. a) 4, 6, 8, 10 **b)** 70

7. a) 16 **b)** 25 **8.** 59 **9. a)** 56 and 273 **b)** 32

10. $-38, -52, -66, -80$

11. $x = 0$; 3, 1, -1 **12.** $x = -\frac{3}{2}$; $t_n = \frac{17}{2} - 7n$

13. a) $d = \frac{2}{3}, t_1 = -1$; **b)** $d = \frac{2}{3}, a = -1$; **c)** $t_{19} = 11$, $t_n = \frac{2}{3}n - \frac{5}{3}$

14. a) $a = 5, d = 4$, $t_n = 4n + 1$ **b)** $a = -3, d = -2$, $t_n = -2n - 1$

15. D **16.**

2	9		

Arithmetic Sequences Lesson #3:
Arithmetic Growth and Decay

Generating Number Patterns Exhibiting Arithmetic Growth

Many real-life scenarios can be represented by a pattern of numbers which exhibit arithmetic growth.

BP Birthplace Forest program in Calgary enables parents to honour their children by planting a tree when their child is born. At the same time, this shows concern for the urban environment by encouraging the growth of city forests. Some of the trees which have been planted are evergreen trees which grow an average of 12 to 18 inches per year. The program was launched in the year 2000.

a) An evergreen tree, 6 inches tall was planted in June 2007 and has a growth rate of 15 inches per year. Two students were asked to determine the height of the tree in June 2018. Joel formed an arithmetic sequence beginning $6, 21, 36, \ldots$ and Jenna formed an arithmetic sequence beginning $21, 36, 51 \ldots$. Use the formulas in the previous lesson to determine each student's answer to the problem.

b) Determine the formula for the general term of Joel's arithmetic sequence.

c) Determine the formula for the general term of Jenna's arithmetic sequence.

d) Explain why the formulas in **b)** and **c)** are different.

e) If the tree continues to grow at the same rate, in which year will it reach a height of 28 ft?

Generating Number Patterns Exhibiting Arithmetic Decay

Many real-life scenarios can be represented by a pattern of numbers which exhibit arithmetic decay or arithmetic depreciation.

A printing press was bought in the year 1999. It depreciates in value by the same amount each year. Five years after its purchase, the printing press had a value of $311 000.
It had a scrap value of $2900 in the year 2017.

a) Use an arithmetic sequence to determine the annual depreciation.

b) Determine the purchase price of the printing press.

Relating Arithmetic Sequences to Linear Functions

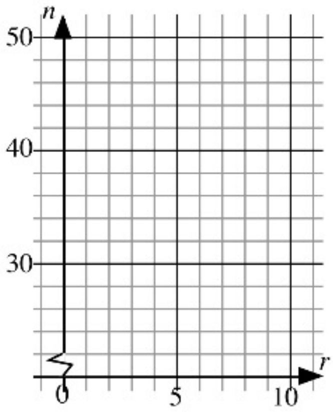
We can relate arithmetic sequences to linear functions over the natural numbers. Consider the following example:

A pile of bricks is arranged in rows. There are 28 bricks in the first row, and the number of bricks in each successive row is two more than in the previous row.

a) Complete the table of values showing the number of bricks in each of the first 10 rows.

Row Number, r	Number of Bricks, n

b) Plot the ordered pairs on the grid. and classify the relationship as linear or non-linear.



c) Determine if the range is an arithmetic sequence.

d) State the domain of the relationship.

e) Explain why the graph does not have an intercept on the vertical axis?

f) Write the equation for the number of bricks in a row, n, as a function of the row number, r.

Complete Assignment Questions #1 - #8

Assignment

1. A contractor charges \$68 for the first hour of work. This includes a rate for one hour of work and a fixed travel fee. For a two hour job, the contractor charges \$110, and for a three hour job, the contractor charges \$152.

a) Using these three numbers as the first three terms of an arithmetic sequence, determine the values of a and d.

b) State the rate for each additional hour of work.

c) Determine the amount of the travel fee.

d) Determine the total cost for a six hour job.

2. For a forthcoming horticultural exhibition, bulbs were planted in rows. The number of bulbs in each row forms an arithmetic sequence. There are 58 bulbs in the eighth row and 107 bulbs in the fifteenth row.

How many bulbs in total are in the first three rows?

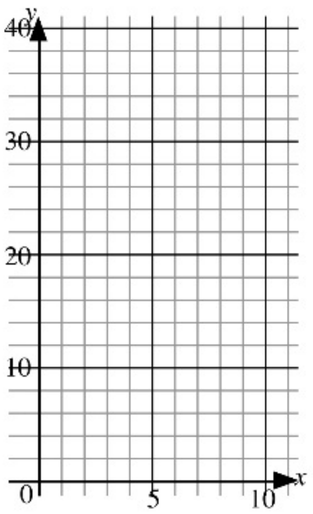
3. Consider the linear function with equation $y = 3x + 5$.

a) Sketch the graph of the linear function on the grid.

b) Restrict the domain to the set of natural numbers. Mark with dots points on the graph which represent the function on the restricted domain.

c) Write the first five elements of the range in numerical order.

d) Show that the elements of the range form an arithmetic sequence and state the common difference.



4. Charity starts a new job as a geologist in the mining sector. Her rate of pay for the first year is $36 000, with an increase of $2 750 per year thereafter.

a) Calculate her rate of pay in the seventh year.

b) In which year will she first earn more than $60 000?

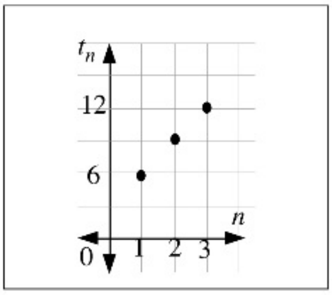
5. Chairs in an auditorium are arranged in rows in such a way that the first two rows each have the same number of chairs. The third and fourth rows each have three more chairs than the first and second row; the fifth and sixth rows each have three more chairs than the third and fourth row, etc. The sequence of number of chairs for every second row forms an arithmetic sequence. The first two rows each have 27 chairs, and the last two rows each have 114 chairs.

a) How many rows of chairs are there?

b) How many chairs are in the **i)** thirteenth **ii)** thirtieth row?

6. A sports utility vehicle sells for \$35 000. The vehicle depreciates \$5000 the first year and \$2400 each year thereafter. Calculate the value of the vehicle at the end of the eleventh year.

Use the following graph to answer questions #7 and #8.



Multiple Choice **7.** The graph above represents arithmetic growth. If the graph continues indefinitely, then the fifteenth term of the sequence is

A. 48
B. 51
C. 90
D. 153

Numerical Response **8.** The formula for this sequence is $t_n = mn + b$.
The value of b is ______ .
(Record your answer in the numerical response box from left to right.)

Answer Key

1. **a)** $a = 68$, $d = 42$ **b)** \$42 **c)** \$26 **d)** \$278 **2.** 48
3. **c)** 8, 11, 14, 17, 20 **d)** common difference = 3
4. **a)** \$52500 **b)** year 10 **5.** **a)** 60 **b)** **i)** 45 **ii)** 69 **6.** \$6000
7. A **8.** 3

Arithmetic Sequences Lesson #4:
Extension: Arithmetic Series

Arithmetic Series

When the terms of an arithmetic sequence are added, the result is known as an **arithmetic series**.

For example $3, 5, 7, 9, 11$ → arithmetic sequence.

$3 + 5 + 7 + 9 + 11$ → arithmetic series.

The symbol, S_n is used to represent the sum of n terms of an arithmetic series.

In the example above $S_5 =$ _____ .

Investigation *Investigating the Sum of an Arithmetic Series*

To illustrate the method for determining a formula for the sum of n terms of an arithmetic series, the story of the great mathematician Karl Gauss (1777-1855) is frequently told.

> When Karl was about 10 years old, he was placed in Master Buttner's arithmetic class. Master Buttner often gave his class long arithmetic problems to keep them quiet for a time. On one particular day, Master Buttner asked his class to add the whole numbers from 1 to 100. While all the students began to work madly on this assignment, Karl laid his slate on the desk and informed Master Buttner he was finished. Master Buttner asked Karl what his answer was. To Master Buttner's astonishment, Karl gave the correct answer of 5050.

Here we apply a method similar to his to determine the answer.

$$S_{100} = 1 + 2 + 3 + 4 + 5 + \dots + 96 + 97 + 98 + 99 + 100$$

$$S_{100} = 100 + 99 + 98 + 97 + 96 + \dots + 5 + 4 + 3 + 2 + 1$$

Add the rows and complete the work to show that $S_{100} = 5050$.

$$2\,S_{100} = 101 + 101 +$$

Formulas for the Sum of an Arithmetic Series

In an arithmetic series of n terms,

the first term is a, the second term is $a + d$, and the last term, $t_n = a + (n - 1)d$.

The sum of n terms of the series can be written as

$$S_n = a + (a + d) + (a + 2d) + \ \ldots \ + (a + (n - 3)d) + (a + (n - 2)d) + (a + (n - 1)d$$

OR

$$S_n = (a + (n - 1)d) + (a + (n - 2)d) + (a + (n - 3)d.+ \ldots + (a + 2d) + (a + d) + a$$

Adding these two lines together gives

$$2S_n = (2a + (n - 1)d) + (2a + (n - 1)d) + (2a + (n - 1)d) + \ldots + (2a + (n - 1)d) + (2a + (n - 1)d) + (2a + (n - 1)d)$$

$$2S_n = n(2a + (n - 1)d)$$

Dividing by 2 gives the **formula for the sum of *n* terms of an arithmetic series.**

$$\boxed{S_n = \frac{n[2a + (n - 1)d]}{2}}$$

This formula connects a, d, n, and S_n.
If any three of these values are known, the fourth can be determined.

However, when the common difference of the series is not known, another **formula for the sum of *n* terms of an arithmetic series** can be formed by replacing $a + (n - 1)d$ by t_n to give

$$\boxed{S_n = \frac{n(a + t_n)}{2}}$$

The above formula can be thought of as the average of the first and last terms multiplied by the number of terms.

Class Ex. #1

Determine the sum of the first fourteen terms of the arithmetic series $9 + 15 + 21 + \ldots$.

Class Ex. #2

Determine the sum of 22 terms of an arithmetic sequence with $t_1 = -18$ and $t_{22} = 45$.

Class Ex. #3

Find the sum of the terms in the sequence 17, 12, 7, ... −38.

Class Ex. #4

Tarvarus starts work at a salary of \$16 000 per annum. He receives annual increases of \$850. He works for the firm for twelve years.

a) Calculate his salary in the twelfth year.

b) How much has he earned in total over the twelve years?

Complete Assignment Questions #1 - #4

Investigation #2 *Investigating an Arithmetic Series Defined in Terms of S_n.*

Corrie was given two questions on a sequences and series assignment. The first question is listed below.

"Find the first four terms of the series defined by $S_n = 2n^2 - n$."

a) Complete her work below to find the first four terms.

$S_n = 2n^2 - n$

$S_1 = 2(1)^2 - 1 = 1 \qquad \Rightarrow t_1 = S_1 \qquad \therefore t_1 = 1$

$S_2 = 2(2)^2 - 2 = 6 \quad \Rightarrow S_2 = S_1 + t_2 \quad \Rightarrow t_2 = S_2 - S_1 \quad \Rightarrow t_2 = 6 - 1 \quad \therefore t_2 =$

$S_3 = 2(3)^2 - 3 = 15 \quad \Rightarrow S_3 = \_ + \quad \Rightarrow t_3 = \_ - \_ \quad \Rightarrow t_3 = \quad - \quad \therefore t_3 =$

$S_4 = \qquad = \quad \Rightarrow S_4 = \_ + \_ \quad \Rightarrow t_4 = \quad \Rightarrow t_4 = \quad \therefore t_4 =$

b) Express t_{10} in terms of S.

c) Express t_n in terms of S.

d) The second question Corrie received was "Find t_n if $S_n = 2n^2 - n$."

i) Find t_n using the formula $t_n = a + (n - 1)d$.

ii) Find t_n using the formula in **c)**.

Remember the formula $t_n = S_n - S_{n-1},\ n \geq 2, n \in N$

Class Ex. #5

For a certain arithmetic series, $S_n = \frac{1}{2}n(11 - n)$.
Determine the first four terms of the corresponding arithmetic sequence.

Complete Assignment Questions #5 - #10

Assignment

1. Find the sum of each series.

a) $2 + 3 + 4 + ...$ (first 30 terms)

b) $(-8) + (-4) + 0 + ...$ (first 27 terms)

c) $2.5 + 2.7 + 2.9 + ...$ to 16 terms

d) $\frac{5}{2} + \frac{11}{6} + \frac{7}{6} + \frac{1}{2} + ...$ to 12 terms

2. Find the sum of each arithmetic series given the first and last terms.

a) $a = 8, t_{15} = 120$ **b)** $t_1 = -11, t_{23} = -253$

3. Find the sum of each series.

a) $11 + 23 + 35 + 47 + \ldots + 179$ **b)** $29 + 21 + 13 + 5 + \ldots -27$

4. As payment for her yard work, a father agrees to give his daughter an allowance of \$3.50 in the first week of the year with an increase of 50 cents each week until the last week of the year.

a) How much money did she receive for an allowance in the last week of the year?

b) What was the total amount of money her father gave her in allowances for the year?

5. On a math assignment, Bob "Bubbles" Burble was asked to find the sum of eight arithmetic means placed between –15 and 12. Bubbles proceeded to find the eight arithmetic means and determine the sum. Along came Sally "Sequential" Sequence and asks Bubbles why he first found all the means. Sally told Bubbles he does not have to find the means at all! Is Sally correct in stating that Bubbles does not have to find all the means? If so, show clearly Sally's method, and determine the sum.

6. Consider the series defined by $S_n = 3n - 1$.

a) Find the first four terms of the series.

b) Is the sequence arithmetic? Explain.

7. Consider the series defined by $S_n = 3n^2 - n$.

a) Find the first four terms of the series.

b) Determine the eighth term of the corresponding sequence.

Multiple Choice

8. Which of the following is an arithmetic series?

A. $1 + 4 + 9 + 16 + 25$
B. $1, 3, 5, 7, \ldots$
C. $6 + 2 + (-2) + (-6)$
D. $8, -8, 8, -8$

9. The sum of the first 100 terms of the arithmetic series $3 + 1 + (-1) + (-3) + \ldots$ is

A. -1020
B. -1005
C. -9705
D. -9600

Numerical Response

10. If the nth term of an arithmetic series is $3n - 7$, then the sum of the first 18 terms is _____ .

(Record your answer in the numerical response box from left to right.)

Answer Key

1. **a)** 495 **b)** 1188 **c)** 64 **d)** -14
2. **a)** 960 **b)** -3036 **3.** **a)** 1425 **b)** 8
4. **a)** $29, **b)** $845
5. Sally is correct. We are given: i) the first and last terms, ii) the number of terms, 10 (eight arithmetic means plus the first and last term) and iii) the series is arithmetic. ∴ given this information, all Bubbles has to do is substitute the appropriate numbers in the general arithmetic series formula $S_n = \dfrac{n(a + t_n)}{2}$ to get the answer S_{10}. Subtract the first and last terms to get the answer of -12.
6. **a)** 2, 3, 3, 3 **b)** no, because there is no common difference between successive terms.
7. **a)** 2, 8, 14, 20 **b)** 44 **8.** C **9.** D **10.**

3	8	7	

Arithmetic Sequences Lesson #5:

Practice Test

1. An example of a finite sequence is

A. $3, 8, 13, 18, \ldots 5n-2, \ldots\ 498,\ \ n \in N$ **B.** $3, 8, 13, 18, \ldots 5n-2, \ldots\ \ n \in N$

C. $3 + 8 + 13 + 18 + \ldots$ **D.** $3 + 8 + 13 + 18$

2. Which of the following statements regarding the sequence $2, 4, 7, 11, 16, \ldots$ is correct?

A. The sequence is arithmetic.

B. The sequence is geometric.

C. The sequence is both arithmetic and geometric.

D. The sequence is neither arithmetic nor geometric.

Numerical Response **1.** On a particular street, the house numbers on one side of the street form an arithmetic sequence. If the first two houses are numbered 8991 and 8995, and the last house is numbered 10039 then the number of houses on this side of the street is _____ .

(Record your answer in the numerical response box from left to right.) ☐☐☐☐

3. The common difference of the arithmetic sequence defined by $t_n = \frac{1}{3}(7-2n),\ n \in N,$ is

A. $\frac{7}{3}$ **B.** $-\frac{7}{3}$

C. $-\frac{2}{3}$ **D.** $\frac{2}{3}$

4. As part of a new training routine, Jesse does some burpees and push-ups. On the first day he does eight burpees and ten push-ups. Each day he increases the number of burpees by three and the number of push-ups by four. How many burpees and push-ups does he do on the nineteenth day?

A. 57 and 76

B. 62 and 82

C. 65 and 86

D. 68 and 90

5. If the 15th and 16th terms of an arithmetic sequence are 99 and 92 respectively, the 5th term is

A. 29

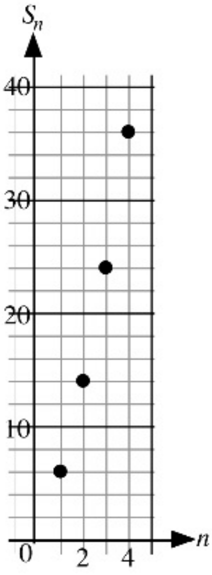
B. 36

C. 162

D. 169

Numerical Response **2.** The graph shown plots the sum of n terms of an arithmetic sequence as a function of $n \in N$. Each point on the graph is of the form (n, S_n), where S_n is the sum of n terms of the sequence.

The sum of the second and third terms of the sequence is _____.



(Record your answer in the numerical response box from left to right.)

6. $p - 1,\ p + 3,\ 3p - 1$, in that order, form an arithmetic sequence.
Which of the following is/are true about p?

1. p is even **2.** p is odd **3.** p is a perfect square

A. 1 only
B. 1 and 3 only
C. 2 only
D. 2 and 3 only

7. Two students are asked to write the first four terms of an arithmetic sequence.
Rob writes the sequence $-14, -6, 2, 10 \ldots$.
Jason writes the sequence $166, 162, 158, 154 \ldots$..

Which statement is true about the fifteenth term of these sequences?

A. t_{15} is the same in each sequence

B. t_{15} is smaller in Rob's sequence

C. t_{15} is smaller in Jason's sequence

D. there is not enough information to answer the question

Use the following information to answer questions #3 - #4.

A child arranges animal blocks in rows on a floor. There are 64 animal blocks in the fifth row and 92 animal blocks in the ninth and last row. Assume that the number of animal blocks from row to row form an arithmetic sequence.

Numerical Response **3.** The number of animal blocks in the first row is _____ .
(Record your answer in the numerical response box from left to right.)

Numerical Response **4.** The total number of blocks used in the arrangement is _____ .
(Record your answer in the numerical response box from left to right.)

8. If $x+2,\ 3x-4,$ and $7x-6$ are the first three terms of an arithmetic sequence, then the first term of the sequence has a numerical value of

A. 0
B. –2
C. 2
D. 4

9. The number of multiples of 12 between 179 and 892 is

A. 59
B. 60
C. 61
D. 62

10. The first term of an arithmetic sequence is 20 and the last term is –76. If there are seven terms in the sequence, then the middle term

A. –12
B. –20
C. –28
D. –44

11. As part of a fitness program, Melissa walks for 35 minutes on day 1 and increases the walking time by 8 minutes each day. She completes the fitness program on the day she first spends more than 3 hours walking. The program is completed on day _____ .

A. 17
B. 18
C. 19
D. 20

Use the following information to answer questions #12 and #13.

Mary Ann was a statistician. She was paid according to a salary grid with annual increases from year one to year ten. The sequence of salaries from years one to ten formed an arithmetic sequence. After year ten she reached her maximum salary. She earned $65 328 in the fifth year and $81 276 in the ninth year. She worked as a statistician for twelve years.

12. Mary Ann's pay in her first year of work was

A. $49 380

B. $53 367

C. $57 354

D. none of the above

13. The total amount that Mary Ann earned as a statistician was

A. $85 263

B. $673 215

C. $843 741

D. $1 346 430

14. Joe, from Perfection Millworks, has fourteen counter tops to deliver to fourteen floors in an office building. Because of the size of the counter tops, he can only get one counter top into the elevator at a time. He starts at the main floor and takes the first counter top to the first floor, returns to the main floor, picks up the second counter top and takes it to the second floor, and so on. He continues in this way until all fourteen countertops have been delivered. If the distance between floors is exactly 7 m, how far has the elevator travelled when Joe has delivered all the counter tops and returned to the main floor?

A. 637 m
B. 735 m
C. 1274 m
D. 1470 m

Numerical Response **5.** The side lengths of a triangle form an arithmetic sequence. The shortest side has a length of 10.5 cm and the perimeter of the triangle is 45 cm. The length, in cm, of the longest side is _____ .

(Record your answer in the numerical response box from left to right.)

15. All the terms in a particular arithmetic sequence are whole numbers. If the first three terms can be represented by $2x + 10, 4x + 30$, and $8x + 60$, then the sum of the first four terms of the corresponding series is

A. 340

B. 170

C. 60

D. 30

Written Response - 5 marks

1. The manager of a condo development receives a base salary of $15 000 per year plus $800 for every condo unit sold.

- Write the first four terms of the arithmetic sequence for the manager's earnings if 1, 2, 3, . . . units are sold.

- Determine the formula for the general term of the arithmetic sequence in the form $t_n = a + (n - 1)d$.

- Use the general term formula to calculate his earnings if

  i) 23 units are sold **ii)** 54 units are sold

- Write the equation for the manager's earnings, E, as a linear function of the number, n, of units sold. Write the equation in the form $E = mn + b$.

- Use the linear function from the previous bullet to calculate his earnings if

  i) 23 units are sold **ii)** 54 units are sold

Answer Key

1. A	**2.** D	**3.** C	**4.** B	**5.** D	**6.** B	**7.** B	**8.** A
9. B	**10.** C	**11.** D	**12.** A	**13.** B	**14.** D	**15.** C	

Numerical Response

1.	2	6	3	
2.	1	8		
3.	3	6		
4.	5	7	6	
5.	1	9	.	5

Written Response

1.
- 15,800, 16,600, 17,400, 18,200,
- $t_n = 15800 + (n - 1)(800)$
- i) \$33,400 ii) \$58,200
- $E = 800n + 15000$
- i) \$33,400 ii) \$58,200

Financial Literacy Lesson #1:
Income

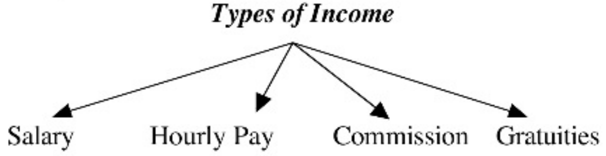
Gross and Net Income

Many people in Canada are employed on a full-time or part-time basis for which they receive an income.

Gross income is the amount of money earned through employment before **deductions**.

Net income (or "take-home" pay) is the amount of money after deductions.

Types of Income



Salary

A person who receives an annual salary is usually paid on a monthly basis and will not usually receive extra income for working extra hours, eg. teacher.

Hourly Pay

A person who is paid by the hour usually works for a fixed number of hours per week and will receive overtime (usually at a greater rate per hour) for any extra hours worked, eg. cashier.

Commission

A person who works on commission is paid according to how much product they sell. A person can be paid in straight commission, graduated commission, or by salary and commission, and may have to meet a predetermined sales quota before any commission is paid, eg. salesperson.
A variation of commission is piecework, where a person is paid according to the number of items they make.

Gratuities

A person receives "tips", or gratuities, for the work they do. This may be in addition to an hourly rate of pay, eg. server.

Some people will earn income through a combination of the above types.

Mrs. Cardinal is an elementary teacher who earns an annual salary of $62 400.
What is her gross monthly income?

Mr. Alpine is an electronics salesperson who is paid a graduated commission. He is paid 5% of the first $4 000 of sales, 6% of the next $6 000, and 10% on any further sales. Last month his total sales amounted to $13 520. Calculate his gross income for the month.

Betty Lou works full-time at the local convenience store. She is paid \$18 per hour for a 40 hour workweek. Overtime is paid at a rate of time and a half.

Last week she worked the hours shown in the table. On a day when she works both in the morning and in the afternoon she receives a one hour unpaid lunch break. Calculate her gross income for the week.

Monday	9:00 a.m. - 5:30 p.m.
Tuesday	9:00 a.m. - 5:30 p.m.
Wednesday	6:00 p.m. - 10:00 p.m.
Thursday	9:30 a.m. - 6:30 p.m.
Friday	9:00 a.m. - 5:30 p.m.
Saturday	9:00 a.m. - 5:30 p.m.
Sunday	off

Ms. Baitress works once a week as a server at Sloppy Burgers restaurant. Her income consists of an hourly salary (\$15.50/hr) plus tips. She noticed that her tips usually average to about 12% of the total food revenue of her assigned wing of the restaurant. If she works from 7:00 p.m. to 11:00 p.m. on a night when her wing of the restaurant has earned \$650 in food revenue, calculate her gross income for the night.

Complete Assignment Questions #1 - #9

Assignment

1. Talulah is a research engineer. She has an annual salary of \$77 304. Determine her monthly gross income.

2. Elizabeth works part time at the dry cleaners. She is paid \$2.25, \$2.75, and \$3.25, for every shirt, pair of slacks, and suit she cleans, respectively.
Last week she laundered 45 shirts, 22 pair of slacks, and 18 suits. Calculate Elizabeth's gross income for the week.

3. Dominic works as a head waiter for a downtown restaurant. Last week he worked five, 8 hour shifts at \$15.75 per hour. His gratuities for those shifts were as follows:

\$78.25, \$75.50, \$81.50, \$97.50, and \$104.25.

Calculate Dominic's gross weekly income.

4. Determine the gross weekly income for the hours worked at the given rate. After working for more than 36 hours in a week, the worker is paid "double-time".

a) 40 hrs at \$18.75 **b)** 52.5 hrs at \$20.75

5. Determine the gross weekly income for each employee.

a) Tara-Lee works as a sales clerk for a major clothing chain. She earns \$17.75 per hour. Last week she worked for 40 hours.

b) When Madison works overtime at an ice cream parlour, she receives time-and-a-half pay for overtime. This week she worked a total of 36 hours including 11 hours of overtime. Her hourly rate is \$15.50.

c) Kelsey works as a waitress which pays her \$13.35/hr for each shift. This week she worked four, 7-hour shifts and her tips averaged \$81 per shift.

d) Zachary works in a bicycle shop where he receives \$450 a week plus 2.5% commission on bicycles sold. Last week his bicycle sales totalled \$2500.

e) Abigail sells snowboards, which pays \$16.50/hr for a 30 hour workweek. She also receives 3% commission on sales, which during the snow season average \$4500 per week.

6. Joshua, a Senior Fund Manager is paid a graduated commission. He is paid 3.5% of the first $3 000 of sales, 5% of the next $5 000 and 12% on any further sales. Last month his total sales amounted to $35 500. Calculate his gross income for the month.

7. Mr. Martinez works as an Assistant Manager. He is paid $23 per hour for 36 hour workweek. Overtime is paid at a rate of time-and-a-half. If his shift starts in the morning and continues into the afternoon, he receives a one hour unpaid lunch break from noon to 1:00 pm. This week he will work the hours shown. Calculate his gross income for the week.

Monday	07:00 to 15:30
Tuesday	08:00 to 14:30
Wednesday	15:00 to 21:00
Thursday	09:30 to 19:00
Friday	10:00 to 18:00
Saturday	08:00 to 18:00
Sunday	Day Off

Multiple Choice **8.** Mrs. Rodriguez has been offered a job as a technical sales representative with Cyber Software where four different salary options were presented. The payment option which would generate the highest gross income is

A. A yearly salary of $65 800

B. A weekly wage, payable for 46 weeks, at $31 per hour for a forty hour workweek. Overtime will average four hours per week, calculated at time and a half.

C. A monthly income of $5 450.

D. A weekly wage, payable for 46 weeks, at $26 per hour for a forty hour workweek with overtime, averaged at four hours per week, calculated at time-and-a-half. In addition, 3% commission on sales expected to average S8 200 per week.

Numerical Response **9.** Mrs. Parker works as a Chief Administrative Officer with a gross annual income of $77 300. She works 46 weeks in a year. If she works an average of 40 hours per week, her hourly rate of pay in dollars per hour, to the nearest dollar, is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key

1. S6 442 **2.** $220.25 **3.** $1 067

4. a) $825 **b)** S1 431.75 **5. a)** $710 **b)** $643.25 **c)** S697.80 **d)** S512.50 **e)** $630

6. S3 655 **7.** $1 086.75 **8.** D **9.** 4 2

Financial Literacy Lesson #2:
Deductions From Income Part One - CPP, EI, etc.

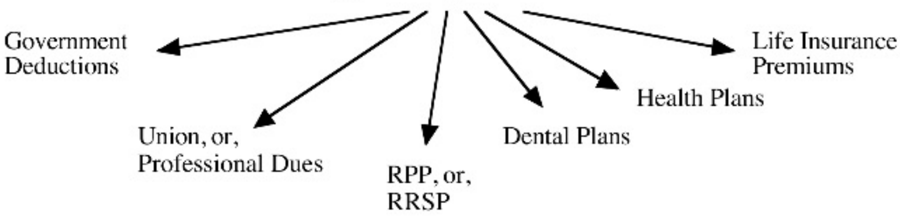
Net Income

Job postings in the newspaper or on the internet usually list the income an employee will earn. This income is the gross income and is not the income the prospective employee will actually receive. The amount the employee receives (or the "take-home" pay) is the **net income** - the income left after deductions are taken off the gross income.

Payroll Deductions

There are many different types of payroll deductions which vary from year to year. The flowchart illustrates some of them.

Some Types of Payroll Deductions



Government Deductions

- ***Canada Pension Plan (CPP)***

  The Canada Pension Plan is designed to provide an employee with pension income when they retire. The government collects money from both employee and employer at the same rate and puts it towards the employee's future pension. In the year 2018 the government collects CPP contributions from an employee at the rate of 4.95% of gross annual income between $3 500 and $55 900. The government does not collect CPP from anyone who has a gross income of $3 500 or less.

- ***Employment Insurance (E.I. Premiums)***

  Employment Insurance is an insurance plan which provides the employee with some income during periods of unemployment. The government collects EI Premiums from both the employee and the employer (the employer contributes premiums at 1.4 times the amount charged to the employee). In the year 2018 the government collects EI premiums from the employee at the rate of 1.66% of the employee's gross yearly income up to $51 700.

- ***Income Tax***

  Income tax is a tax that the federal government and the provincial and territorial governments impose on income generated by all businesses and individuals. Income tax is a key source of revenue that the governments use to fund their activities and serve the public. We will discuss this topic in the next lesson.

Other Types of Payroll Deductions

- *Union or Professional Dues*
- *Child Care Expenses*
- *RPP (Registered Pension Plan)*
- *RRSP (Registered Retirement Savings Plan)*
- *Dental Plans, Health Plans, Life Insurance Premiums, etc.*

Class Ex. #1

Calculate the annual CPP contribution for the following employees:

a) Sheila, who has an annual salary of $29 500.

b) A student who works part-time for15 weeks, earning $175 per week.

c) A civil servant who earns $5 760 per month.

d) A service manager who earns $37 600 per year.

Note

The maximum CPP contribution for the year 2018 is ________ .

Class Ex. #2

Calculate the annual EI Premium for the following employees:

a) Ranjit who earns $35 236 per year.

b) A technician with a gross monthly salary of $4 025.

Note

The maximum EI Premiums for the year 2018 is ________ .

Tax-Exempt Deductions

Tax-Exempt Deductions are deductions which are subtracted from a person's gross income to determine the person's taxable income.

Tax-exempt deductions include the following:

- Union or Professional Dues
- RPP
- RRSP
- Child Care Expenses
- Others

Taxable Income

Taxable Income is income on which income tax is calculated.

Taxable Income = gross income – tax-exempt deductions.

Helen is a chemical engineer who works for a research chemical company. She has a gross monthly income of $5 750. She pays union dues of $65 per month, contributes $500 per month to a Registered Retirement Savings Plan, and has child care expenses of $200 per month. Calculate her annual taxable income.

Summary of Payroll Deductions

Government Payroll Deductions Year 2018

Canada Pension Plan (CPP)
- 4.95% of gross annual income between $3 500 and $55 900
- maximum annual contribution is $2 593.80

Employment Insurance (E.I. Premiums)
- 1.66% of gross annual income up to $51 700
- maximum annual contribution is $858.22

Income Tax

Other Possible Payroll Deductions

Tax-Exempt	*Taxable*
• Union or Professional Dues	• Dental Plans, Health Plans,
• RPP (Registered Pension Plan)	• Life Insurance Premiums
• RRSP (Registered Retirement Savings Plan)	• Others
• Child Care Expenses	
• Others	

The rates of CPP Contributions and EI Premiums change according to government policy. You may wish to use the current rates when doing the assignment questions. Current rates can be obtained by contacting the Government of Canada website shown below.

https://www.canada.ca/en/revenue-agency/services/tax/businesses/topics/payroll/calculating-deductions.html **or** perform the web search "Payroll deductions and Contributions Canada".

Government Payroll Deductions for the Year ________

Canada Pension Plan (CPP)

- ________ % of gross annual income between _________ and _________

- maximum annual contribution is $_________

Employment Insurance (E.I. Premiums)

- ________ % of gross annual income up to ___________

- maximum annual contribution is $_________

Complete Assignment Questions #1 - #7

Assignment

1. Calculate the annual CPP contribution for the following employees:

a) Mrs. Makowichuk, who has an annual salary of $37 700.

b) Lauren, a junior accountant who works 25 weeks part-time for $475 per week.

c) Mr. Marshall, a city manager who earns $3 850 per month.

d) Mr. Foster, a zoning enforcement officer who earns $61 500 per year.

2. Calculate the annual EI Premium for the following employees:

a) Cheyenne, a substitute teacher who earns $38 350 per year.

b) Rebecca, a consultant who earns a monthly salary of $5 240.

3. Siddhu, a web designer, has a weekly gross income $850. Find his weekly

a) Canada Pension Plan contribution **b)** Employment Insurance Premium

4. Lee Huang, a Marketing Manager, worked 42 hours last week. He is paid $28.50 for the first thirty-five hours and double-time for overtime.

a) Calculate his gross earnings for the week.

b) Assuming he works 42 hours per week for forty weeks and has 12 weeks of vacation, calculate his annual CPP and EI deductions.

5. Mr. McDaniel, a Project Manager, earns a gross monthly income of $6 250. He pays union dues of $105 per month, contributes $780 per month to a Registered Retirement Savings Plan, and pays $185 per month into a health plan. Calculate his annual taxable income.

6. Luigi is a lawyer who has an annual salary of $83 000. He pays professional association fees of $120/month, contributes $1 150 per month to a RPP, and pays life insurance premiums of $175 per month. His annual CPP contribution is $2 593.80 and his annual EI premiums are $858.22. Calculate his annual taxable income.

Numerical Response **7.** Marlyn is a loans officer at the local bank. She has a gross monthly income of $4 979 per month. She contributes $470 per month to a Registered Retirement Savings Plan, and has weekly child care expenses of $162. Her monthly mortgage payment is $1 375. Her monthly taxable income, to the nearest dollar, is _____ .

(Record your answer in the numerical response box from left to right)

Answer Key (based on the year 2018)

1.a) $1 692.90 **b)** $414.56 **c)** $2 113.65 **d)** $2 593.80

2.a) $636.61 **b)** $858.22 **3. a)** $38.74 **b)** $14.11

4.a) $1 396.50 **b)** CPP 2 591.82 EI $858.22

5. $64 380 **6.** $67 760 **7.** 4 3 4 7

Financial Literacy Lesson #3:
Deductions from Income Part Two - Income Tax

Income Tax

Income tax is a tax that the federal, provincial, and territorial governments impose on income generated by all businesses and individuals. Income tax is a key source of revenue that the governments use to fund their activities and serve the public.

Indviduals file an income tax return every year to determine whether they owe any taxes or are eligible for a tax refund, e.g.. they have had too much tax deducted from their earnings. In general, higher income earners will pay tax at a higher rate. The Canada Revenue Agency (CRA) administers tax laws for the federal government and for most provinces and territories.

Individual Income Tax

Individual Income Tax is composed of two parts: **Basic Federal Tax** and **Provincial Tax**.

Basic Federal Tax

Individuals do not pay tax on all of their income. There are various deductions available which taxpayers subtract from their gross income to determine their **taxable income**. For example, if a taxpayer earns $100 000 in income and qualifies for $20 000 in tax-exempt deductions, then the taxpayer is only taxed on the remaining $80 000. As indicated in the previous lesson, tax-exempt deductions include union or professional dues, RPP or RRSP contributions, child care expenses, and others.

The amount of **Federal Tax** due can be calculated from the Federal Tax Rate Table. The bands and rates will vary from year to year and the rate table for 2018 is shown.

Federal Tax is calculated on taxable income and not on gross income.

As an example using the table, for a taxable income of $47 605, the Federal Tax would be 15% of $46 605 plus 20.5% of $1 000.

Federal Tax Rate Table for Year 2018

2018 Taxable Income	Tax Rate
first $46 605	15%
over $46 605 up to $93 208	20.5%
over $93 208 up to $144 489	26%
over $144 489 up to $205 842	29%
over $205 842	33%

Once the amount of tax due has been calculated, there are **tax credits** available which will reduce the actual amount of tax which has to be paid.

Tax credits include a **basic personal amount**, CPP contributions, EI premiums, some health and dental expenses, charitable donations, disability credits, and others.

The basic personal amount for the year 2018 is $11 809.

The amount of tax payable to the Federal Government is called the **Basic Federal Tax** and is given by the following formula:

Basic Federal Tax = Federal Tax – 15% of an Individual's Tax Credits

Provincial Tax

The amount of provincial tax owed varies between province/territory to province/territory. It is calculated using a provincial rate table with different tax credits than those used in the federal rate table. More information can be found on the Government of British Columbia website.

To simplify matters in this book, the Provincial Tax payable by individuals in British Columbia will be calculated as 40% of the Basic Federal Tax. This gives a reasonable approximation in most cases.

As can be seen, the calculation of an individual's Federal Tax and Provincial Tax can be very complicated. It is based on all types of income e.g.. earned, interest, dividend, capital gains, etc., and on the numerous types of tax-exempt deductions and tax credits available. Professional tax accountants can help individuals and businesses with the preparation of income tax returns.

In this book, we will only consider earned income and we will use the following simplified guide to determine an individual's total income tax.

Calculating an Individual's Total Income Tax

Step 1: Calculate the individual's annual taxable income
Annual Taxable Income = Gross Income - Tax-Exempt Deductions

Step 2: Calculate the individual's tax credits
Tax Credits = CPP + EI Premium + Basic Personal Amount + others

Step 3: Calculate the individual's **Federal Tax** using the Federal Tax Rate Table for the current year.

Step 4: Calculate the individual's Basic Federal Tax
Basic Federal Tax = Federal Tax – 15% of Tax Credits

Step 5: Calculate the individual's Provincial Tax as 40% of the Basic Federal Tax.

Step 6: Add the Basic Federal Tax and the Provincial Tax to calculate the **total income tax** the individual must pay.

Net Annual Income - Take Home Pay

An individual's net annual income is the income remaining after all government deductions, income tax, and other voluntary deductions are subtracted from the gross income.

Net Annual Income = Gross Income – CPP – EI Premiums – Federal and Provincial Tax – Others

Melissa works as a customs officer in Vancouver. In the year 2018 she earned a gross income of $67 400. She paid 4% of her gross income into an RRSP, and paid union dues of $50 per month. The union dues were a direct deduction from her salary, but the RRSP was purchased after receiving her take-home pay. She had additional deductions from her salary of $32.40 per month.

a) Complete the steps below to determine her total income tax for the year 2018.

Step 1: Calculate Melissa's annual taxable income.

Gross Income = Tax Exempt Deductions =

Annual Taxable Income = Gross Income - Tax Exempt Deductions

Step 2: Calculate Melissa's Tax Credits.

CPP =

EI Premium = Basic Personal Amount =

Melissa's Tax Credits = CPP + EI Premium + Basic Personal Amount

Step 3: Calculate Melissa's Federal Tax using the Federal Tax Rate Table for the year 2018.

Step 4: Calculate Melissa's Basic Federal Tax.

Melissa's Basic Federal Tax = Federal Tax − 15% of Melissa's Tax Credits

Step 5: Calculate Melissa's Provincial Tax as 40% of her Basic Federal Tax.

Step 6: Calculate Melissa's total income tax.

b) Calculate Melissa's monthly take-home pay.

Complete Assignment Questions #1 - #4

Assignment

1. Antonio, an animal control officer in the district of Squamish, had a gross annual income of $46 880 in the year 2018. After receiving his take-home pay he paid $170 per month into an RRSP, and had child care expenses of $150 per month, both of which were paid after receiving his take-home pay.

a) Complete the steps below to determine his total income tax for the year 2018.

Step 1: Calculate Antonio's annual taxable income.

Gross Income =

Tax-Exempt Deductions =

Annual Taxable Income = Gross Income - Tax-Exempt Deductions

Step 2: Calculate Antonio's tax credits.

CPP =

EI Premium =

Basic Personal Amount =

Antonio's Tax Credits = CPP + EI Premium + Basic Personal Amount

Step 3: Calculate Antonio's Federal Tax using the Federal Tax Table for the year 2018.

Step 4: Calculate Antonio's Basic Federal Tax.

Antonio's Basic Federal Tax = Federal Tax – 15% of Antonio's Tax Credits

Step 5: Calculate Antonio's Provincial Tax as 40% of his Basic Federal Tax.

Step 6: Calculate Antonio's total income tax.

b) Calculate Antonio's monthly take-home pay if he has additional direct deductions from his salary amounting to $27.50 per month.

2. Dr. Fuller is a physician in Burnaby whose gross monthly salary was $9 650 in the year 2018. Her monthly deductions included: union dues $108, RRSP $1 300, health insurance plan $175, and dental plan $145. Her annual deduction for CPP and EI totalled $3 452.02 . She had an annual tax credit of $1 230 for allowable medical expenses.

Calculate her total annual income tax.

3. John worked 46 hours last week. He was paid $16.50 per hour for the first 38 hours and time-and-a-half for overtime.

a) Calculate John's gross earnings for the week.

b) Calculate his total deductions for the week using the following rates.

CPP at 4.95% of (gross earnings - $67.30)

EI at 1.66% of gross earnings

Income tax at 14.1% of gross earnings after CPP and EI have been deducted

c) Calculate John's net pay for the week.

4. Scott is a computer analyst who is earning a monthly income of $6 800. Deductions from his salary are CPP, EI, and an RRSP contribution of 8% of his gross income. Calculate his net annual income, if he has a total of $97.89 per month of other non tax-exempt deductions. Use the following tax information in your calculations.

Federal Tax Rate Table

Taxable Income From	To	Tax Rate
$0	$31 800	15%
$31 800	$54 600	22%
$54 600	$73 000	29%
$73 000		35%

Provincial Tax Rate = 10% of taxable income

Basic Personal Amount = $8 240

CPP = 4.2% of gross annual income
between $4 100 and $39 100

EI = 1.9% of gross annual income up to $40 000

Basic Federal Tax = Federal Tax - 15% of Tax Credits

Answer Key

1. **a)** Step 1: $43 040, Step 2: $14 734.52, Step 3: $6 456, **b)** $3 139.19
Step 4: $4 245.82, Step 5: $1 698.33 Step 6: $5 944.15
2. Taxable Income: $98 904 Tax Credits: $15 261.02 Federal Tax: $18 025.33
Basic Federal Tax: $15 736.18 Provincial Tax: $6 294.47 Total Tax: $22 030.65
3. **a)** $825 **b)** $160.31 **c)** $664.69
4. Taxable Income: $75 072 Tax Credits: $10 470 Federal Tax: $15 847.20
Basic Federal Tax: $14 276.70 Provincial Tax: $7 507.20 Total Tax: $21 783.90
Net Annual Income: $49 883.42

Financial Literacy Lesson #4:
Income Project

Choose an occupation you are interested in and carry out some research under the following:

- Pay Period and Annual Income
- Payroll Deductions
- Tax Credits
- Federal and Provincial Taxes
- Net Annual Income

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Notes Page 2

Notes Page 3

Notes Page 4

Notes Page 5